

ΤΥΧΟΛΟΓΙΟ. Γενικός τύπος Green,  $\Phi(\vec{x}) = \int_V \rho(\vec{x}') G(\vec{x}, \vec{x}') d\vec{x}' - \frac{1}{4\pi} \int_{\partial V} \Phi(\vec{x}') \frac{\partial G}{\partial n'} da' + \frac{1}{4\pi} \int_{\partial V} G \frac{\partial \Phi}{\partial n'} da'$   
 Green σφαιρικού φλοιού  $G(\vec{x}, \vec{x}') = \frac{1}{|\vec{x} - \vec{x}'|} - \frac{a}{x' |\vec{x} - \frac{a^2}{x'^2} \vec{x}'|}$

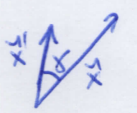
Πολικές συντεταγμένες,  $\vec{\nabla} = \frac{\partial}{\partial r} \vec{e}_r + \frac{1}{r} \frac{\partial}{\partial \theta} \vec{e}_\theta + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \vec{e}_\phi$ ,  $\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial \Phi}{\partial r}) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Phi}{\partial \phi^2} = 0$

$\Phi(r, \theta, \phi) = A_0 + B_0 \ln r + \sum (A_n r^n + B_n r^{-n}) (C_n \cos n\theta + D_n \sin n\theta) + \sum_{lm} (A_{lm} r^l + B_{lm} r^{-l-m}) Y_{lm}(\theta, \phi)$   
 Σφαιρικές,  $\vec{\nabla} = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \vec{e}_\phi \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}$ .  $\Phi(r, \theta, \phi) = \sum_{lm} (A_{lm} r^l + B_{lm} r^{-l-m}) Y_{lm}(\theta, \phi)$

Πολυώνυμα Legendre,  $P_0(x) = 1$ ,  $P_1(x) = x$ ,  $P_2(x) = \frac{1}{2}(3x^2 - 1)$ .  $\int_{-1}^1 P_l(x) P_m(x) dx = \frac{2}{2l+1} \delta_{lm}$

Σφαιρικές αρμονικές,  $Y_{lm}(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos \theta) e^{im\phi}$ ,  $P_l^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$

Ορθοκανονικότητα  $Y_{lm}$ ,  $\int_0^\pi \int_0^{2\pi} \sin \theta d\theta d\phi Y_{lm}^*(\theta, \phi) Y_{lm}(\theta, \phi) = \delta_{lm} \delta_{lm'}$



$\cos \gamma = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\phi - \phi')$

Ορθοκανονικότητα καρτεσιανών,

$\int_0^L \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \sqrt{\frac{2}{L}} \sin\left(\frac{m\pi x}{L}\right) dx = \delta_{nm}$

Πολικές,  $\int_0^{2\pi} \sin(n\phi) \sin(m\phi) d\phi = \eta \cdot \delta_{nm}$