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Abstract

This article highlights key findings from the small body of research on mathematics difficulties (MD) relevant to early identification and early intervention. The research demonstrates that (a) for many children, mathematics difficulties are not stable over time; (b) the presence of reading difficulties seems related to slower progress in many aspects of mathematics; (c) almost all students with MD demonstrate problems with accurate and automatic retrieval of basic arithmetic combinations, such as 6 + 3. The following measures appear to be valid and reliable indicators of potential MD in kindergartners: (a) magnitude comparison (i.e., knowing which digit in a pair is larger), (b) sophistication of counting strategies, (c) fluent identification of numbers, and (d) working memory (as evidenced by reverse digit span). These are discussed in terms of the components of number sense. Implications for early intervention strategies are explored.

Currently, the field of learning disabilities (LD) possesses a number of valid measures that can predict relatively well which students are likely to have trouble with learning to read. These measures are increasingly used for screening purposes, and they enable schools to provide additional support and relevant early intervention for children in kindergarten and first grade. The evolution of reliable, sound screening measures was the result of more than 20 years of interplay between research and theory (Liberman, Shankweiler, & Liberman, 1989). When the first author entered the field of research 25 years ago, the best predictive validity for reading readiness measures was .27; they now are routinely in the acceptable range of .6 to .7 (Schatschneider, Carlson, Francis, Foorman, & Fletcher, 2002).

Measurement of reading readiness evolved from a dynamic interaction among theoretical studies (Schatschneider et al., 2002) and intervention studies (O'Connor, Notari-Syverson, & Vadasy, 1996; Vellutino et al., 1996). Theories of early reading development were tested using a variety of measures, and, as the field evolved, findings from validation studies indicated which measures were the most powerful predictors, and which measures made the most sense to administer to children in kindergarten and first grade.

On the other hand, research on valid early screening measures of subsequent mathematics proficiency is in its infancy. The small set of studies conducted to date involves various disciplines, such as cognitive psychology, child development, and curriculum-based assessment (also known as general outcome measurement).

Our goal in this article is to highlight key findings from the diverse approaches taken by the researchers in the area of mathematics learning. In particular, we present what we currently know concerning (a) the nature of mathematics difficulties; (b) the role of number sense in young children; (c) valid screening measures for early detection of potential difficulties in mathematics; and (d) early intervention and instruction.

Although we allude to some of the theory that guided the empirical research, we do not intend this to be a theory-building article. In some respects, the field of mathematics instruction has been plagued with too much theory and theorizing and far too little programmatic, empirical research (Gersten et al., in press; National Research Council, 2001; Woodward & Montague, 2002). At this point, there is enough empirical research to suggest valid screening instruments for determining who is likely to need extensive support in learning mathematics. Moreover, there is enough convergence in findings to begin to understand the trajectories of students with mathematics difficulties (MD) and to understand areas where these students need intensive support. Both these bodies of knowledge can inform the nature of instruction provided to young children who are struggling with learning mathematical concepts. Our hope is to stimulate research that examines the effectiveness of various approaches to interventions for young students likely to have trouble in mathematics.

The Nature of Mathematical Difficulties

In the present article, we use the term *mathematics difficulties* rather than mathematics *disabilities*. Children who exhibit mathematics difficulties include those performing in the low average range (e.g., at or below the 35th percentile) as well as those performing well below average (Fuchs, Fuchs, & Prentice, 2004; Hanich, Jordan, Kaplan, & Dick, 2001). Using higher percentile cutoffs increases the likelihood that young children who go on to have serious math problems will be picked up in the screening (Geary, Hamson, & Hoard, 2000; Hanich et al., 2001). Moreover, because mathematics achievement tests are based on many different types of items, specific deficits might be masked. That is, children might perform at an average level in some areas of mathematics but have deficits in others.

In the past, approaches to studying children with mathematics difficulties (MD) often assessed their performance at a single point in time. Contemporary approaches determine a child's growth trajectory through longitudinal research, which is fundamental to understanding learning difficulties and essential for setting the stage for critical intervention targets (Francis, Shaywitz, Steubing, Shaywitz, & Fletcher, 1994). Measurement of growth through longitudinal investigations has been a major focus in the study of reading difficulties since the seminal research of Juel (1988). Although less longitudinal research has been devoted to mathematics difficulties than to reading difficulties, several studies have shown the advantages of longitudinal approaches.

The seminal researcher in MD was David Geary, although a good deal of subsequent research has been conducted by Nancy Jordan and her colleagues and other researchers such as Snorre Ostad (1998). These ambitious lines of longitudinal research studies have attempted to reveal the nature and types of mathematics difficulties that students experience in the elementary grades and to examine the extent to which these difficulties persist or change over time. For example, Geary et al. (2000) found that for many children, mathematics difficulties are not stable over time, identifying a group of "variable" children who showed mathematics difficulties on a standardized test in first grade but not in second grade. It is likely that some of these children outgrew their developmental delays, whereas others were misidentified to begin with.

Typically, the researchers in the MD area have examined longitudinal trajectories of students over periods of 2 to 5 years on different measures of mathematical proficiency. These studies explored the relationship between MD and reading difficulties and specify the nature of the deficits that underlie the various types of MD (Geary et al., 2000; Jordan, Hanich, & Kaplan, 2003).

Fluency and Mastery of Arithmetic Combinations

The earliest theoretical research on MD focused on *correlates* of students identified as having a learning disability involving mathematics. A consistent finding (Goldman, Pellegrino, & Mertz, 1988; Hasselbring, Goin, & Bransford, 1988) was that students who struggled with mathematics in the elementary grades were unable to automatically retrieve what were then called *arithmetic facts*, such as $4 + 3 = 7$ or $9 \times 8 = 72$. Increasingly, the term *arithmetic* (or number) *combinations* (Brownell & Carper, 1943) is used, because basic problems involving addition and subtraction can be solved in a variety of ways and are not always retrieved as "facts" (National Research Council, 2001). A major tenet in this body of research was that "the general concept of automaticity . . . is that, with extended practice, specific skills can reach a level of proficiency where skill execution is rapid and accurate with little or no conscious monitoring . . . attentional resources can be allocated to other tasks or processes, including higher-level executive or control function" (Goldman & Pellegrino, 1987, p. 145).

In subsequent longitudinal research, Jordan et al. (2003) compared two groups of children: those who possessed *low mastery* of arithmetic combinations at the end of third grade, and those who showed full mastery of arithmetic combinations at the end of third grade. They then looked at these children's development on a variety of math tasks between second and third grades over four time points. When IQ, gender, and income level were held constant, children in both groups performed at the same level in solving untimed story problems involving simple addition and subtraction operations and progressed at the same rate. Children who possessed combination mastery increased at a steady rate on timed number combinations, whereas children with low combination mastery made almost no progress over the 2-year period. What the research seems to indicate is that although students with MD often make good strides in terms of facility with algorithms and procedures and simple word problems when provided with classroom instruction, deficits in the retrieval of basic combinations remain (Geary, 2004; Hanich et al., 2001); this seems to inhibit their ability to understand mathematical discourse and to grasp the more complex algebraic concepts that are introduced. Our interpretation is that failure to instantly retrieve a basic combination, such as $8 + 7$, often makes discussions of the mathematical concepts involved in algebraic equations more challenging.

Maturity and Efficiency of Counting Strategies

Another major finding is the link between MD and efficient, effective counting strategy use (Geary, 1993, 2003; Geary et al., 2000). Jordan et al. (2003) found that a significant area of difference between students with number combination mastery and those without was the sophistication of their *counting strategies*. The poor combination mastery group continued to use their fingers to count on untimed problems between second and third grades, whereas their peers increasingly used verbal counting without fingers, which led much more easily to the types of mental manipulations that constitute mathematical proficiency.

Siegler and Shrager (1984) studied the development of counting knowledge in young children and found that children use an array of strategies when solving simple counting and computational problems. For example, when figuring out the answer to 3 + 8, a child using a very unsophisticated, inefficient strategy would depend on concrete objects by picking out first 3 and then 8 objects and then counting how many objects there are all together. A more mature but still inefficient counting strategy is to begin at 3 and "count up" 8. Even more mature would be to begin with the larger addend, 8, and count up 3, an approach that requires less counting. Some children will simply have this combination stored in memory and remember that it is 11.

Siegler's research reminds us that a basic arithmetic combination, such as 2 + 9, is at some time in a person's life a complex, potentially intriguing problem to be solved. Only with repeated use does it become a routine "fact" that can be easily recalled (Siegler & Shrager, 1984). If a child can easily retrieve some basic combinations (e.g., 6 + 6), then he or she can use this information to help quickly solve another problem (e.g., 6 + 7) by using decomposition (e.g., 6 + 6 + 1 = 13). The ability to store this information in memory and easily retrieve it helps students build both procedural and conceptual knowledge of abstract mathematical principles, such as commutativity and the associative law. Immature finger or object counting creates few situations for learning these principles.

Geary (1990) extended Siegler's line of research to MD by studying the counting strategy use of students with MD in the first grade. He divided the sample into three groups: (a) those who began and ended the year at average levels of performance, (b) those who entered with weak arithmetic skill and knowledge but benefited from instruction and ended the year at or above the average level, and (c) those who began and ended the year at low levels of number knowledge. The second group included students who had not received much in the way of informal or formal instruction in number concepts and counting; the third group of students were classified as having MD. The students who entered with weak number knowledge but showed reasonable growth during the first year of systematic mathematics instruction tended to perform at the same level as the group that entered at average achievement levels on the strategy choice measure. Those with minimal gains (i.e., the MD group) used slower, less mature strategies more often. Although first graders with MD tended to use the same types of strategies on addition combinations as average students, they made three or even four times as many errors while using the strategy. For example, when children in the MD group used their fingers to count, they were wrong half the time. When they counted verbally, they were wrong one third of the time. In contrast, the average-achieving students rarely made finger or verbal counting mistakes. Geary et al. (2000) noted that students with MD on the whole tend to "understand counting as a rote, mechanical activity. More precisely, these children appear to believe that counting . . . can only be executed in the standard way . . . from left to right, and pointing at adjacent objects in succession." (p. 238).

These studies suggest that maturity and efficiency of counting strategies are valid predictors of students' ability to profit from traditional mathematics instruction. Thus, they are of great importance for understanding methods for early screening of students for potential MD and the nature of potential interventions.

Mathematics Difficulties and Reading Difficulties

Jordan and colleagues have devoted a good deal of effort to distinguishing similarities and differences between children with specific mathematics difficulties (MD only) and those with both mathematics and reading difficulties (MD + RD). In many earlier studies, children with MD were defined as a single group of low achievers (e.g., Geary, 1993; Ostad, 1998). However, Jordan's work, as well as recent research by Fuchs et al. (2004), has suggested that children with MD who are adequate readers show a different pattern of cognitive deficits than children with MD who are also poor readers (Jordan et al., 2003; Jordan & Montani, 1997).

A distinguishing characteristic of this line of research involves the domains of mathematical cognition that are assessed. Much of the early research on children with MD was narrowly focused, emphasizing only one area of mathematical competence, namely arithmetic computation. Problem solving and number sense received less attention. Because mathematics has multiple cognitive requirements (National Research Council, 2001), Jordan et al. (2003) hypothesized that abilities would be uneven across areas of mathematical competence, especially among children with MD only.

Hanich et al. (2001) identified children with MD only, MD + RD, RD only, and typical achievement (TA) at the beginning of second grade. In a subsequent study with the same sample, Jordan, Kaplan, and Hanich (2002) examined each group's achievement growth in reading and in mathematics on the Woodcock-Johnson Psycho–educational Battery–Revised (Woodcock & Johnson, 1990) over a 2-year period. MD and RD were defined by performance at or below the 35th percentile, based on national norms. The MD-only group started at about the same math level as the MD + RD group (mean percentiles of 22 and 21, respectively) and the RD-only group at about the same math level as the TA group (mean percentile scores of 60 and 68, respectively).

Achievement growth of children with MD only was significantly more rapid than that of children with MD + RD. It is important to note that this difference persisted even when adjusted for IQ and income level. The RD-only group performed at about the same level in mathematics as children with MD only and at a lower level than TA children at the end of third grade. Difficulties in reading seem to have a negative influence on children's development in general mathematics achievement. In contrast, mathematics abilities did not influence reading growth. On reading measures, children with RD only achieved at the same rate as children with MD + RD when IQ and income level were held constant. Reading difficulties identified in second grade remained steady throughout the test period, regardless of whether they were specific (RD only) or general (MD + RD).

Gender and ethnicity did not predict growth in math or in reading. However, income level predicted growth in math but not in reading. Not surprisingly, special services were much more likely to be provided in reading than in math during second and third grades. Jordan et al. (2003) speculated that early reading interventions level the playing field for children with RD, making income less of a factor in primary school reading growth.

Over a 16-month period, Jordan and colleagues also investigated the development of specific mathematical competencies in the same group of children with MD only and MD + RD (Hanich et al., 2001; Jordan et al., 2003). The researchers looked at areas of mathematical cognition that are directly related to the teaching of mathematics (as opposed to more general cognitive competencies), including basic calculation, problem solving, and base-10 concepts. Children were assessed twice in second grade and twice in third grade. Children's ending level of performance was examined, as were their growth rates.

Jordan et al. (2003) showed that third graders with MD only had an advantage over their MD + RD peers in two areas of mathematical cognition: accuracy on arithmetic combinations and story problems. However, performance differences between the two MD groups in arithmetic combinations disappeared when IQ, gender, ethnicity, and income level were considered. The MD-only group performed better than the MD + RD group at the end of third grade on calculation principles (e.g., understanding of the relations between addition and subtraction and the commutativity principle), irrespective of predictor variables. The tasks that did not differentiate the MD-only group from the MD + RD group were timed number combinations, estimation, place value, and written computation. The two MD groups did not differ in their growth rates (from mid–second grade to the end of third grade) on any of the math tasks.

Difficulties of children identified as having MD + RD in early second grade are pervasive and steady over second and third grades, even when predictors such as IQ are held constant. Despite their weaknesses, however, children with MD + RD achieved at a rate that was similar to that of children in the other groups. Children with MD only appeared to have consistent difficulties with calculation fluency. They performed as low as children with MD + RD when required to respond to number combinations quickly and relied on their fingers as much as children with MD + RD, even at the end of third grade. However, children with MD only used finger counting strategies more accurately than children with MD + RD, suggesting better facility with counting procedures in the former group.

Jordan et al. (2003) speculated that weaknesses in spatial representations related to numerical magnitudes (rather than weaknesses in verbal representations) underpin rapid fact retrieval deficits. Such children might have difficulties manipulating visual (nonverbal) representations on a number line—a skill that may be critical for solving addition and subtraction problems. In fact, it was found that children with poor mastery of number combinations performed worse than children with solid mastery on nonverbal block manipulation and pattern recognition tasks. In contrast, the groups performed at about the same level on verbal cognitive tasks.

In summary, deficits in calculation fluency appear to be a hallmark of mathematics difficulties, specific or otherwise. The achievement growth patterns reported by Hanich et al. (2001) for students below the 35th percentile held even for children with extremely low performance (i.e., children who fell below the 15th percentile in achievement in mathematics; Jordan et al., 2003). In other words, when MD and RD were defined as below the 15th percentile, students with MD + RD tended to acquire mathematical competence at a slower rate than students with MD only. The deficits occurred in only two areas of arithmetic: accuracy on arithmetic combinations and story problems.

Number Sense

A very different developmental research tradition may also have implications for early identification and early intervention. This body of research emanates primarily from psychologists who are interested in the cognitive development of children; they focus on the concept of *number sense* (Bereiter & Scardamalia, 1981; Dehaene, 1997; Greeno, 1991; Okamoto & Case, 1996). Unfortunately, no two researchers have defined number sense in precisely the same fashion.

Even though in 1998, Case wrote that "number sense is difficult to define but easy to recognize" (p. 1), he did attempt to operationalize this concept (Okamoto & Case, 1996) in one of his last writings on the topic (Kalchman, Moss, & Case, 2001), noting that

> the characteristics of good number sense include: (a) fluency in estimating and judging magnitude, (b) ability to recognize unreasonable results, (c) flexibility when mentally computing, (d) ability to move among different representations and to use the most appropriate representation. (p. 2)

Case, Harris, and Graham (1992) found that when middle-income kindergartners are shown two groups of objects (e.g., a group of 5 chips and a group of 8 chips), most are able to identify the "bigger group" and know that the bigger group has more objects. Yet only those with well-developed number sense would know that 8 is 3 bigger than 5. Likewise, only those with developed number sense would know that 12 is a lot bigger than 3, whereas 5 is just a little bit bigger than 3. These findings highlight the need for differentiated instruction in mathematics in kindergarten.

In an attempt to identify the features of number sense, Okamoto (2000, cited in Kalchman et al., 2001, p. 3) conducted a factor analysis on students' performance and identified two distinct factors in children's mathematics proficiency in kindergarten. The first factor related to *counting*, a key indicator of the digital, sequential, verbal structure; the second to *quantity discrimination* (e.g., tell me which is more, 5 or 3?). For example, Okamoto and Case (1996) found that some students who could count to 5 without error had no idea which number was bigger, 4 or 2. They concluded that in children this age, the two key components of number sense are not well linked. Implicit in their argument is that these two factors are precursors of the other components of number sense specified previously, such as estimation and ability to move across representational systems. These abilities can be developed as students become increasingly fluent in counting and develop increasingly sophisticated counting strategies.

Further support for the importance of assessing *quantity discrimination* and its potential importance as an early screening measure emanates from the research of Griffin, Case, and Siegler (1994), who found that students entering kindergarten differed in their ability to answer quantity discrimination questions such as, "Which number is bigger, 5 or 4?" even when they controlled for student abilities in counting and simple computation. Another important finding was that the high–socioeconomic status (SES) children answered the question correctly 96% of the time, compared to low-SES children, who answered correctly only 18% of the time. This finding suggests that aspects of number sense development may be linked to the amount of informal instruction that students receive at home on number concepts and that some students, when provided with appropriate instruction in preschool, kindergarten, or first grade in the more complex aspects such as quantity discrimination, may quickly catch up with their peers. Geary's (1990) research with primarily low-income students found a substantial group of such students in the first grade. It may be that intervention in either pre-K or K would be very beneficial for this group.

In summary, number sense is, according to Case et al. (1992), a conceptual structure that relies on many links among mathematical relationships, mathematical principles (e.g., commutativity), and mathematical procedures. The linkages serve as essential tools for helping students to think about mathematical problems and to develop higher order insights when working on mathematical problems. Number sense development can be enhanced by informal or formal instruction prior to entering school. At least in 5- and 6-year-olds, the two components of number sense (counting/simple computation and sense of quantity/use of mental number lines) are not well linked. Creating such linkages early may be critical for the development of proficiency in mathematics. Children who have not acquired these linkages probably require intervention that builds such linkages. The two components of number sense also provide a framework for determining the focus of a screening battery, especially if some measure of working memory is also included.

Early Detection of MD and Potential Screening Measures

Using this framework, Baker, Gersten, Flojo, et al. (2002) explored the predictive validity of a set of measures that assess students' number sense and other aspects of number knowledge that are likely to predict subsequent performance in arithmetic. A battery of measures was administered to more than 200 kindergartners in two urban areas. Performance on these measures was correlated with subsequent performance on a standardized measure of mathematics achievement—the two mathematics subtests of the *Stanford Achievement Test–Ninth Edition* (SAT-9), Procedures and Problem Solving (Harcourt Educational Measurement, 2001).

The primary measure in the predictive battery was the *Number Knowledge Test,* developed by Okamoto and Case (1996). An item response theory reliability was conducted on the measure based on a sample of 470 students. Using a two-parameter model, the reliability was .93. The *Number Knowledge Test* is an individually administered measure that allows the examiner not only to appraise children's knowledge of basic arithmetic concepts and operations, but also to assess their depth of understanding through a series of structured probes that explore students' understanding of magnitude, the concept of "bigger than," and the strategies they use in counting. Table 1 presents sample items from several levels of the *Number Knowledge Test*.

Baker, Gersten, Flojo, et al. (2002) also included a series of measures of specific skills and proficiencies developed by Geary et al. (2000). They included measures of quantity discrimination (magnitude comparison), counting knowledge, number identification, and working memory. Brief descriptions of these measures are provided in Table 2.

The *Number Knowledge Test*, the measure with the most breadth, was the best predictor in the set of measures of SAT-9 Procedures and Problem Solving. The zero-order predictive validity correlations were .73 for Total Mathematics on the SAT-9, .64 for the Procedures subtest, and .69 for Problem Solving. All correlations were moderately strong and significant, $p < .01$.

Table 3 presents the predictive validity correlations for each of the measures administered in kindergarten for SAT-9 Total Mathematics and the *Number Knowledge Test*, which was readministered in first grade. Note that the predictive validity correlations are significant for each of the screening measures at $p < .01$ and in the moderate range, with the exception of rapid automatized naming and letter naming fluency, which are smaller in magnitude and significant at the .05 level. As might be expected, the measures that involve mathematics tend to predict better than those that are unrelated to mathematics—Phoneme Segmentation (a measure of phonemic awareness), letter naming fluency, and ability to name pictures and colors. In particular, the magnitude comparison task and the digit span backward task seem particularly promising for a screening battery. The reader should note that both of these measures were untimed.

We next attempted to examine which combination of measures appeared to be the best predictor of the most reliable criterion measure—the SAT-9 Total Mathematics score 1 year later. The number of predictor variables that can be included in the multiple regressions is limited by sample sizes (Frick, Lahey, Christ, Loeber, & Green, 1991). Because of the small sample sizes, two predictors were chosen to be included in the regression analysis. The second best predictor was Digit Span Backward, a measure of working memory. Working memory (for abstract information, such as a sequence of numbers) seems to be related to many arithmetic operations. Several other researchers have also found that problems with numerical digit span have been identified with MD (Geary & Brown, 1991; Siegel & Ryan, 1988; Swanson & Beebe-Frankenberger, 2004). From a pragmatic perspective, the predictive ability of the set of two measures over a 12-month period is impressive, $R = .74$; $F(2, 64) = 38.50$, $p < .01$.

TABLE 1
Sample Items From the *Number Knowledge Test*

Sample item	Level
I'm going to show you some counting chips. Would you count these for me?	0
Here are some circles and triangles. Count just the triangles and tell me how many there are.	0
How much is 8 take away 6?	1
If you had 4 chocolates and someone gave you 3 more, how many chocolates would you have altogether?	1
Which is bigger: 69 or 71?	2
Which is smaller: 27 or 32?	2
What number comes 9 numbers after 999?	3
Which difference is smaller: the difference between 48 and 36 or the difference between 84 and 73?	3

TABLE 2
Description of Selected Early Math Measures

Subtest	Description
Geary (2003)	
Digit Span	Student repeats a string of numbers either forwards or backwards.
Magnitude comparison	From a choice of four visually or verbally presented numbers, student chooses the largest.
Numbers from Dictation	From oral dictation, student writes numbers.
Clarke & Shinn (2004)	
Missing Number	Student names a missing number from a sequence of numbers between 0 and 20.
Number Identification	Student identifies numbers between 0 and 20 from printed numbers.
Quantity Discrimination	Given two printed numbers, student identifies which is larger.

Use of Rate Measures as Early Predictors of MD

In 1999, Gersten and Chard noted that two good indicators of number sense in young children were (a) quantity discrimination and (b) identifying a missing number in a sequence. A group of researchers explored the predictive validity of timed assessments of these two components. Both studies examined predictive validity from fall to spring, Clarke and Shinn (2004) only for first grade, and Chard et al. (in press) for both kindergarten and first grade. All predictive measures used were timed measures and were easy to administer. Brief descriptions of some of the measures are in Table 2. The researchers also included a measure of rapid automatized number naming, analogous to the rapid letter naming measures that are effective screening measures for beginning readers (Baker, Gersten, & Keating, 2000).

The screening measures used by Baker et al. (2000) were *untimed* measures. These included instruments developed by Okamoto and Case (1996) and Geary (2003) in their lines of research. In the area of reading, researchers have found that timed measures of letter naming and phoneme segmentation administered to kindergartners and children entering first grade are solid predictors of reading achievement in Grades 1 and 2 (Dynamic Indicators of Basic Early Literacy Skills [DIBELS]; available at http://idea.uoregon.edu/~dibels). In particular, the ability to rapidly name letters of the alphabet seems to consistently predict whether a student will experience difficulty in learning to read in the primary grades (Adams, 1990; Schatschneider, Fletcher, Francis, Carlson, & Foorman, 2004). The three measures developed by Clarke and Shinn (2004), although brief, proved to be reasonably reliable, with test–retest reliabilities ranging from .76 to .86. The predictive validity of these three measures for first grade (Chard et al., in press; Clarke & Shinn, 2004) and kindergarten (Chard et al., in press) are presented in Table 4. For both studies, number knowledge was used as a criterion measure. Clarke and Shinn used the *Woodcock-Johnson Tests of Achievement* (Woodcock & Mather, 1989) Applied Problems test as a second criterion measure. Each of the three measures appears to be a reasonably valid predictor of future performance. They are presented in Table 4, in the right column.

Chard et al. (in press) explored the predictive validity of the set of three screening measures developed by Clarke and Shinn (2004) using a larger sample. Their study differed from that of Clarke and Shinn in that (a) they correlated fall and spring performance for both kindergarten and first grade, and (b) they used the *Number Knowledge Test* as a criterion measure rather than the Woodcock-Johnson. *R*s were significant at $p < .01$ and moderately large (.66 for kindergarten and .68 for first grade).

When we attempt to put together the information gained from these studies, it appears that the strongest screening measure is the relatively comprehensive *Number Knowledge Test*

TABLE 3
Predictive Validity of Number Sense Measures

Spring of Kindergarten Predictor	Spring of First Grade	
	SAT-9[a]	NKT[b]
Mathematical Measures		
NKT	.72**	.72**
Digit Span Backward	.47**	.60**
Numbers From Dictation	.47**	.48**
Magnitude Comparison	.54**	.45**
Nonmathematical measures		
Phonemic Segmentation	.42**	.34**
Letter Naming Fluency	.43**	.27*
Rapid Automatized Naming (Colors and Pictures)	.34**	.31*

Note. SAT-9 = *Stanford Achievement Test*, ninth edition (Harcourt Educational Measurement, 2001); NKT = *Number Knowledge Test* (Okamoto & Case, 1996).
[a]$n = 65$. [b]$n = 64$.
*$p < .05$. **$p < .01$.

TABLE 4
Predictive Validity Correlations of Early Math Measures

Fall pretest	Spring Scores[a]	
	Chard et al. (in press)	Clarke & Shinn (2004)
Kindergarten sample		
n	436	
Number Identification	.58	
Quantity Discrimination	.53	
Missing Number	.61	
First-grade sample		
n	483	52
Number Identification	.58	.72
Quantity Discrimination	.53	.79
Missing Number	.61	.72

Note. All correlations significant at $p < .05$.
[a]Criterion measure for Chard et al. (in press) was the *Number Knowledge Test* (Okamoto & Case, 1996); criterion measure for Clarke and Shinn (2004) was the *Woodcock-Johnson Tests of Achievement* (Woodcock & Mather, 1989) Applied Problems subtest.

(Okamoto & Case, 1996). However, three relatively brief measures appear to be quite promising: (a) of *quantity discrimination* or *magnitude comparison;* (b) identifying the *missing number* in a sequence, a measure of counting knowledge; (c) some measure of *number identification*. It also appears that a measure of *rapid automatized naming* (such as Clarke & Shinn's Number Identification) and a measure of *working memory* for mathematical information (such as reverse digit span) also seem to be valid predictors. Note that with the exception of working memory, all of these measures can be linked to instructional objectives that are appropriate at these grade levels.

We envision several critical next steps in this line of research. These steps can and should be concurrent. From a technical point of view, we need to continue to study the long- and short-term predictive validity of these types of number sense measures. Equally important would be research that determines whether measures administered to kindergartners and first graders should be timed or untimed. Although there is good evidence to suggest that the *rate* of retrieval of basic arithmetic combinations is a critical correlate of mathematical proficiency, to date there is no evidence suggesting, for example, that the speed with which students can identify which number of two is the biggest, or count backwards from a given number, is an important variable to measure.

Instructional Implications

There is a paucity of research on early interventions to prevent MD in struggling students. Only two researchers have described early intervention research (Fuchs, Fuchs, Karns, Hamlett, & Katzaroff, 1999; Griffin, Case, & Siegler, 1994), and these have been studies of whole-class instruction. Therefore, in discussing possible interventions for children indicating a potential for MD in kindergarten or first grade, we primarily rely on interpolations from longitudinal and developmental research and on a recent synthesis of the intervention research for older students with LD or low achievement in mathematics (Baker, Gersten, & Lee, 2002; Gersten et al., in press) and, to some extent, on the thinking about early intervention of Fuchs, Fuchs, and Karns (2001) and Robinson, Menchetti, and Torgesen (2002).

We have a good sense of some of the goals of early intensive intervention. Certainly, one goal is increased fluency and accuracy with basic arithmetic combinations. Other, related goals are the development of more mature and efficient counting strategies and the development of some of the foundational principles of number sense—in particular, magnitude comparison and ability to use some type of number line. It is quite likely that other aspects of number sense are equally important goals.

The evidence does not suggest one best way to build these proficiencies in young children; in fact, the consistent finding that there are at least two different types of MD (i.e., MD only vs. MD + RD) indicates that certain approaches might work better for some subgroups of students (Fuchs et al., 2004). The need for differentiated intervention is also highlighted by the finding (Okamoto & Case, 1996) that in young children, counting ability and number sense develop relatively independently. Thus, there is no clear "best way" to reach this goal, but there are some promising directions.

Early Intervention Research on Arithmetic Combinations

Pellegrino and Goldman (1987) and Hasselbring et al. (1988) provided intensive interventions to older elementary students with MD. However, we believe that their research raises important implications for intervention in earlier grades. Both groups of researchers directly targeted fluency and accuracy with basic arithmetic combinations. They attempted to create situations that helped students with MD store arithmetic combinations in memory in such a manner that the children could retrieve the fact "quickly, effortlessly and without error" (Hasselbring et al., 1988, p. 2). Hasselbring et al. designed a computer program that created individually designed practice sets consisting of a mix of combinations that the student could fluently recall and those that were difficult for that student. The program used controlled response times to force students to rely on retrieval rather than counting. This was a clear attempt to somewhat forcefully restructure students' strategies toward retrieval rather than inefficient finger counting. Practice continued until the student consistently used retrieval. This procedure was effective for the majority but not all of the students diagnosed as having MD. Students who relied only on finger counting did not benefit at all. It appears that as students begin to mature in counting strategy use, some type of individualized practice on combinations may be useful in enhancing fluency. The use of technology to create sensible, individualized practice sets also seems a viable alternative to worksheets or whole-class practice.

Other Options for Building Fluency

Because fluency and accuracy with arithmetic combinations require the use of mature strategies, instruction and guidance in strategy use seems critical. It seems logical that some students will need direct instruction in strategy use that may be unnecessary for their peers. Siegler (1988) described the intuitive benefits of this type of differential instruction as follows: "Teaching children to execute backup strategies more accurately affords them more opportunities to learn the correct answer [and] . . . reduces the likelihood of associating incorrect answers, produced by faulty execution of backup strategies, with the problem" (p. 850).

Shrager and Siegler (1998) demonstrated that the generalization of strategy use proceeds slowly in young children. Adults often underestimate the time it takes a child to use a newly learned mathematical strategy consistently. This observation is important to keep in mind as preventative interventions are designed. Shrager and Siegler also found that at least for very basic arithmetic strategies, generalization increases with the presentation of challenging problems (i.e., problems that are very difficult to solve without the use of a strategy and fairly easy to solve with a strategy). This finding, too, would seem to have important instructional implications for the design of interventions.

Recently, some researchers (Jordan et al., 2003; Robinson et al., 2002) have argued that children can derive answers quickly and with minimal cognitive effort by employing calculation principles or "shortcuts," such as using a known number combination to derive an answer (2 + 2 = 4, so 2 + 3 = 5), relations among operations (6 + 4 = 10, so 10 – 4 = 6), $n + 1$, commutativity, and so forth. This approach to instruction is consonant with recommendations by the National Research Council (2001). Instruction along these lines may be much more productive than rote drill without linkage to counting strategy use. Rote drill places heavy demands on associative memory, an area of weakness in many children with LD in mathematics (Geary, 1994). As children become more proficient in applying calculation principles, they rely less on their fingers and eventually master many combinations (Jordan, Levine, & Huttenlocher, 1994). Mental calculation shortcuts have the advantage of developing children's number sense as well as their fluency. Research has shown that even adults who are competent in math use mental shortcuts instead of automatic retrieval on some number combinations (LeFevre et al., 1996).

Interventions to Build Number Sense and Conceptual Understanding

Robinson et al. (2002) proposed that interventions for students with poor mastery of arithmetic combinations should include two aspects: (a) interventions to help build more rapid retrieval of information, and (b) concerted instruction in any and all areas of number sense or arithmetic concepts that are underdeveloped in a child (e.g., principles of commutativity, or "counting on" from the larger addend.) This seems a logical framework for tailoring interventions to ensure that students develop increasingly mature and efficient counting strategy use that is linked to fluent retrieval of combinations, yet continues to solidify their knowledge of mathematics concepts and algorithms. Tailoring can and should be linked to the assessments discussed previously in this article.

In the view of Robinson et al. (2002), number sense is "a skill or kind of knowledge about numbers rather than an intrinsic process" (p. 86). Thus number sense, like phonemic awareness, should be teachable. This is supported by Case and Griffin's (1990) finding that number sense is linked, in part, to the amount of informal or formal instruction on number concepts provided at home.

In terms of developing interventions to develop number sense, we draw inferences from existing research. One seemingly promising approach is the *Number Worlds* curriculum, developed by Griffin (2004) and evaluated as *Rightstart.* However, based on implementation research (Gersten, Chard, Griffiths, Katz, & Bryant, 2003), we would only recommend the whole-class activities, such as practice in "counting on," practice in listening to coins being dropped in a box and counting, practice in counting backwards, practice in linking adding and subtracting to the manipulation of objects. These could easily be done with small groups of children and appeared to be helpful in building a sense of number in students who needed work in this area. In contrast, the games that composed much of the curriculum proved extraordinarily difficult to implement in a typical classroom. We therefore envision the use of the conceptual framework developed by Griffin and Case (1997) serving as a basis for structured small-group learning activities. We also think that the goals of number sense games—to encourage young students in kindergarten and first grade to learn how to talk about numbers and relationships—can be better realized in the context of activities that accompany explicit, small-group instruction. As Milgram (2004) noted, students can learn to use sophisticated strategies by viewing models of proficient performance and being provided with steps that help them solve a problem. This practice is strongly supported by meta-analyses of research on effective approaches for teaching students with MD (Baker et al., 2002; Gersten et al., in press). Students will need to learn the very abstract language of mathematics before they can express their ideas; thus, learning the vocabulary of mathematics seems another critical piece of early intervention.

Siegler (1988) precisely demonstrated that procedural and conceptual knowledge in mathematics are frequently integrated, and parsing them out is difficult. Therefore, embedding the teaching of concepts in work on procedures seems the ideal route. Siegler demonstrated that for young children, very simple arithmetic computations are, at some point in their development, extremely difficult problems to solve, requiring a good deal of effort and orchestration of several knowledge bases (counting knowledge, number–symbol correspondence, order irrelevance). Thus, the creation of learning situations where students are explicitly shown how to orchestrate all these concepts when doing arithmetic would seem ideal. Exactly how to reach that goal is much less clear. Interventions should follow the finding of Rittle-Johnson, Siegler, and Alibali (2001), who noted that procedural and conceptual abilities in mathematics "lie on a continuum and cannot always be separated" (p. 346). They also found that conceptual knowledge is "implicit or explicit understanding of the principles that govern a domain. . . . This knowledge is flexible and not tied to specific problem types and is therefore generalizable. . . . Furthermore it may or may not be verbalizable" (p. 346). It seems that we need to develop a far better understanding of when it makes sense for students to verbalize their approach and when this is not a helpful instructional procedure. It is possible that students with MD only may be much more amenable to this commonly advocated approach than students with MD + RD, who may lack the verbal skill to articulate either their understandings or their confusions.

The meta-analyses (Baker, Gersten, & Lee, 2002; Gersten et al., in press) have indicated several promising directions: using structured peer work, using visuals and multiple representations, providing students with strategies that could then serve as a "hook" for them as they solve problems. Many of these studies have dealt only with arithmetic computations or simple arithmetic word problems. We have a good deal to learn about how to use visuals—how best to model and think aloud so that students can internalize and truly understand some of the concepts that they only partially understand.

Summary and Conclusions

We see a major goal of early mathematics interventions to be the development of fluency and proficiency with basic arithmetic combinations and the increasingly accurate and efficient use of counting strategies. We believe that one reason why the fast retrieval of arithmetic combinations is critical is that students cannot really comprehend any type of dialogue about number concepts or various problem-solving approaches unless they automatically know that 6 + 4 is 10, that doubling 8 makes 16, and so forth. In other words, students who are still slowly using their fingers to count a combination such as 7 + 8 or 3 × 2 are likely to be totally lost when teachers assume they can effortlessly retrieve this information and use this assumption as a basis for explaining concepts essential for problem solving or for understanding what division means.

It thus appears that lack of fluency with arithmetic combinations remains a critical *correlate* of MD and needs to be a goal of intervention efforts for many children. Furthermore, teachers need to be aware which students have not mastered basic combinations and note that these students may well need additional time to understand the concepts and operations that are explained or discussed.

From a conceptual point of view, we need to work more on linking specific measures that can be used for early screening and identification of MD with the theories that have been developed about MD. For example, in early elementary school, the shift from concrete to mental representations seems critical for developing calculation fluency (Jordan & Hanich, 2003; Jordan et al., 2003). Early screening measures should examine children's calculation strategies on different types of problems to determine whether children are making this transition. We believe that further advances in developing valid measures for screening and early detection will need to be based on more refined, better operationalized definitions of number sense. In order to accomplish this goal, we need to understand in depth the specific skills, strategies, and understandings that predict subsequent problems in becoming proficient in mathematics. This understanding will help the field to refine the nature of measures developed for early identification and help to shape the nature of effective early intervention programs.

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AUTHORS' NOTE

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