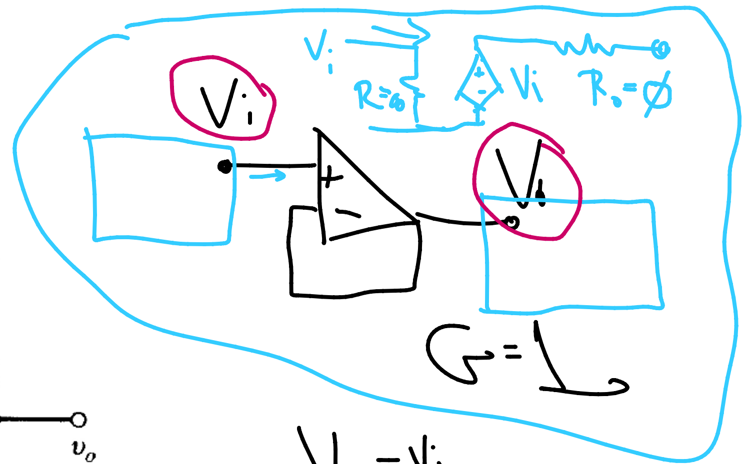
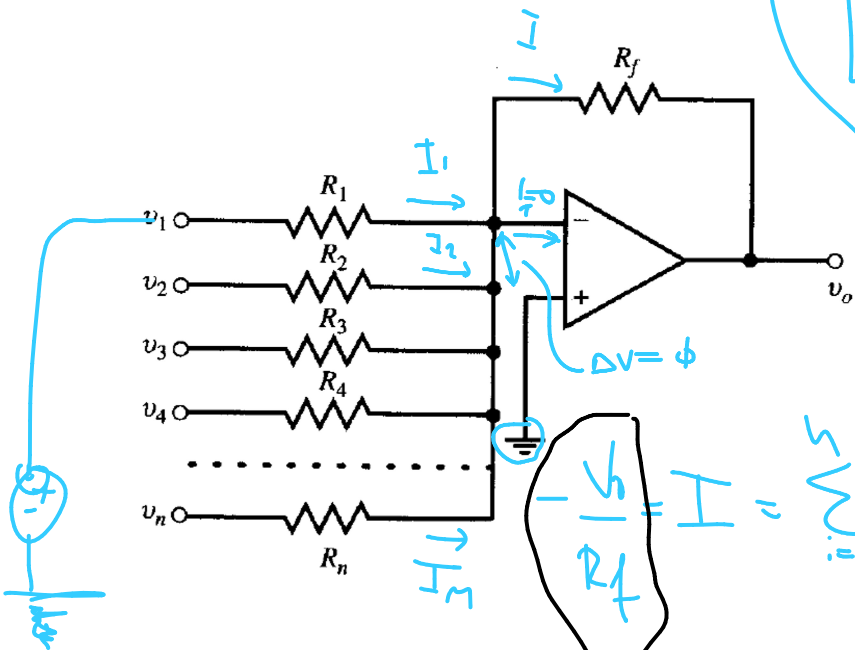
# Παθητικά και Ενεργά Αναλογικά φίλτρα

Monday, June 8, 2020 9:33 PM



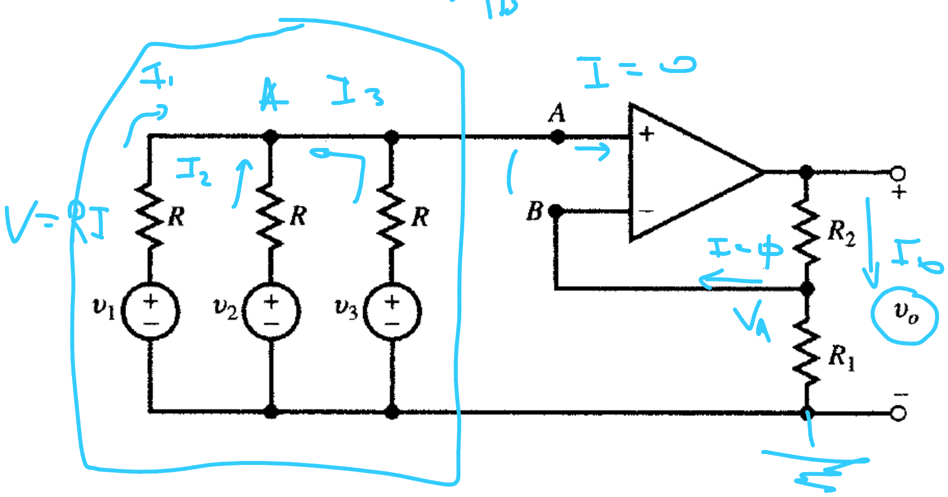

$V_0 = V_i$




$$-\frac{V_0}{R_f} = I = \sum_{i=1}^{n} I_i = \sum_{i=1}^{n} \frac{V_i}{R_i} \Longrightarrow$$

$$\Rightarrow \boxed{V_0 = -\sum_{i=1}^{n} \frac{R_f}{R_i} V_i}$$

$V_{AB} = \emptyset$




$$V_o = \frac{R_2 + R_1}{R_1} V_A \Rightarrow$$

$$\Rightarrow V_0 = \left(\frac{R_2}{R_1} + 1\right)\left(\frac{V_1 + V_2 + V_3}{3}\right)$$

$V_A$

$$I_1 + I_2 + I_3 = \emptyset \longrightarrow$$

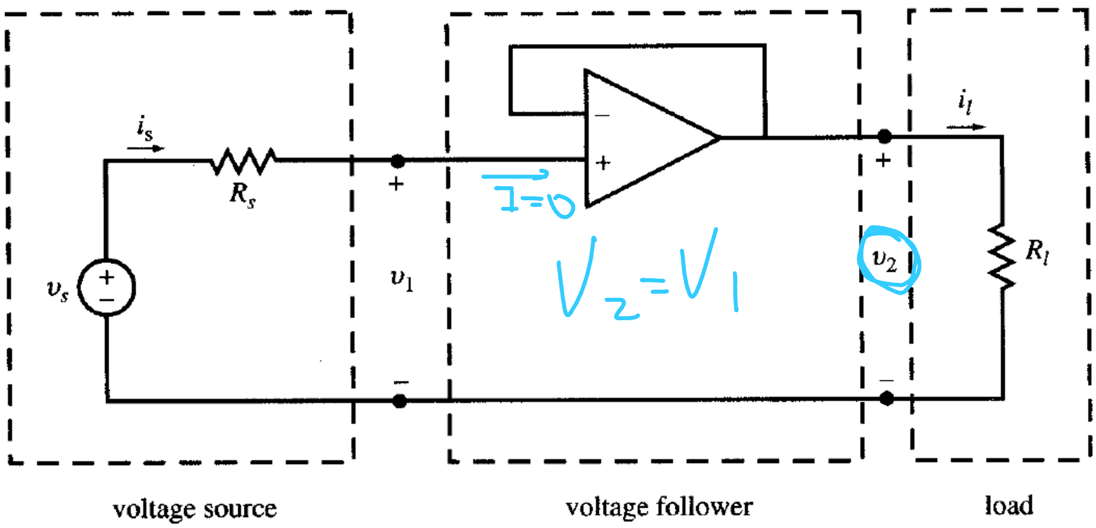
$V_A$ $\quad I_1+I_2+I_3=\emptyset \Longrightarrow$

$\boxed{G>1}$

$$\Rightarrow \frac{\cancel{R}(V_1-V_A)}{\cancel{R}} + \frac{\cancel{R}(V_2-V_A)}{\cancel{R}} + \frac{\cancel{R}(V_3-V_A)}{\cancel{R}} = \emptyset \Rightarrow$$

$$\Rightarrow 3V_A = V_1+V_2+V_3 \Rightarrow \boxed{V_A = \frac{V_1+V_2+V_3}{3}}$$

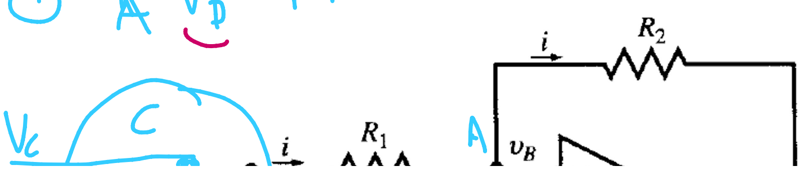
$G = 1$

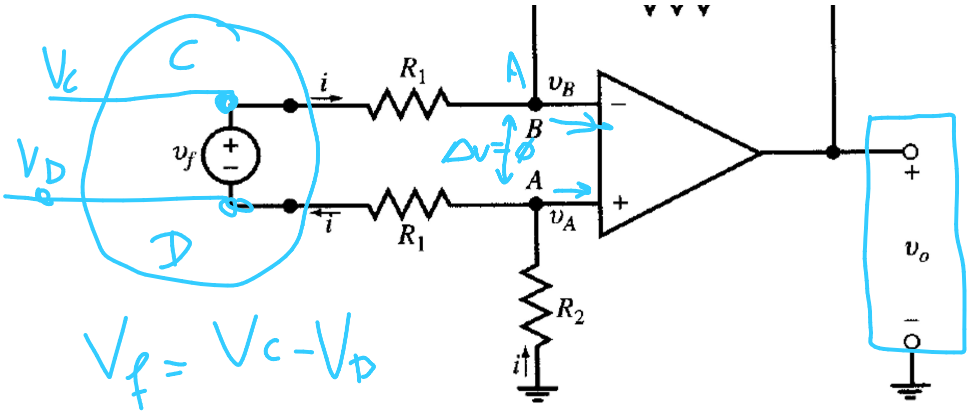



③ $V_A = -R_2 \cdot I$ $\qquad V_C - V_A = R_1 \cdot I$ ②

① $V_A - V_D = R_1 \cdot I$ $\qquad V_A - V_O = R_2 \cdot I \Rightarrow$

$V_o = V_A - R_2 \cdot I$






$V_o = v_A \quad R_2 \cdot I$

$\Downarrow$ ③

$V_o = -R_2 I - R_2 I$

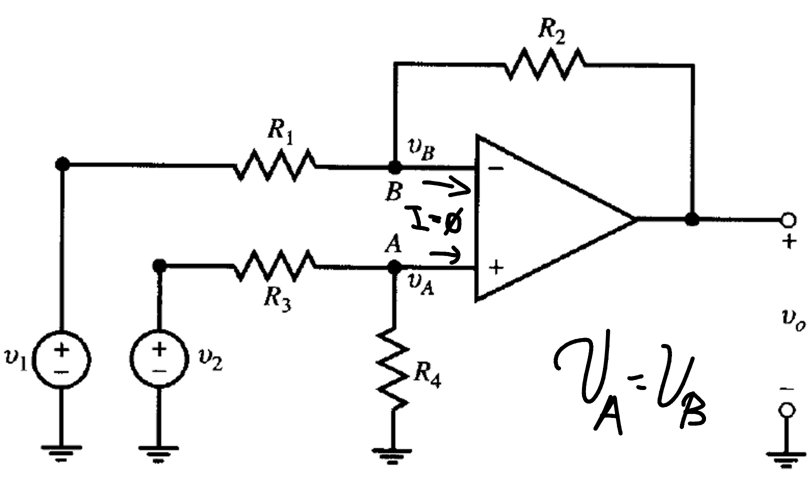
$= -2R_2 I$ ④

$V_f = V_C - V_D$

①+② $\Rightarrow V_C - V_D = 2R_1 \cdot I$ ④ $\Rightarrow V_C - V_D = -\cancel{2} R_1 \dfrac{V_o}{\cancel{2} R_2} \Rightarrow$

$$\Rightarrow V_o = -\frac{R_2}{R_1}(V_C - V_D) = \boxed{-\frac{R_2}{R_1} V_f = V_o}$$

$$V_o(t) = G\, V_f(t)$$




$$\frac{v_A - v_2}{R_3} + \frac{v_A}{R_4} = 0$$
$$\frac{v_B - v_1}{R_1} + \frac{v_B - v_o}{R_2} = 0$$

$R_3 = 0\,k\Omega$

$R_1 = 1\,k\Omega$

$R_2 = 2\,k\Omega$

$R_4 = 0\,k\Omega$

$$\boxed{v_o = \overbrace{\frac{R_4(R_1 + R_2)}{R_1(R_3 + R_4)}}^{3} v_2 - \overbrace{\frac{R_2}{R_1}}^{2} v_1}$$

$V_o = 3V_2 - 2V_1$

$R_1 = R$

$\boxed{R_2 = 2R_1}$

$\Rightarrow \dfrac{3R_4 \cancel{R_1}}{\cancel{R_1}(R_3 + R_4)} = \cancel{3}$

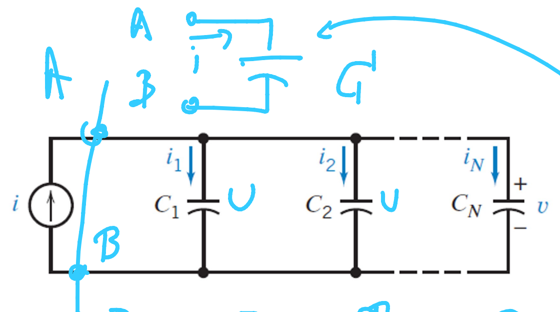
$R_4 = R_3 + R_4 \Rightarrow R_3 = 0 - D$

$R_4 = R_3 + R_4 \Rightarrow R_3 = 0$ — $R_4$ ότι θέλω $\Rightarrow \frac{1}{R(R_3+R_4)}$

# ΠΥΚΝΩΤΗΣ - CAPACITOR

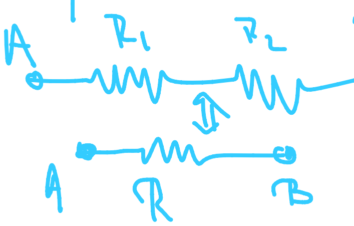
$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau \Rightarrow \boxed{i(t) = C\frac{dv(t)}{dt}}$$

The voltage across a **capacitor** cannot change instantaneously.

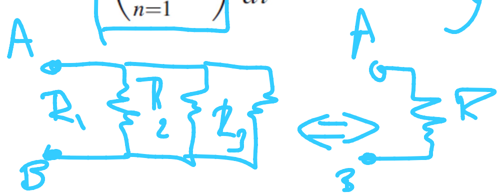



$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

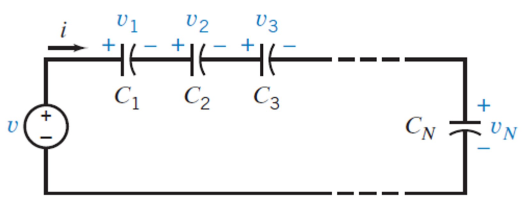
$$\begin{aligned} i &= C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt} \\ &= (C_1 + C_2 + C_3 + \cdots + C_N)\frac{dv}{dt} \\ i &= \boxed{\left(\sum_{n=1}^{N} C_n\right)}\frac{dv}{dt} \end{aligned}$$




$$\underline{R = R_1 + R_2 + R_3}$$




$$y = \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$




$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

$$\begin{aligned} v &= \boxed{\frac{1}{C_1}\int_{t_0}^{t} i\,d\tau + v_1(t_0)} + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i\,d\tau + v_N(t_0) \\ &= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i\,d\tau + \sum_{n=1}^{N} v_n(t_0) \\ &= \underbrace{\left(\sum_{n=1}^{N}\frac{1}{C_n}\right)\int_{t_0}^{t} i\,d\tau + \sum_{n=1}^{N} v_n(t_0)} \end{aligned}$$

$$\frac{1}{C_s} = \sum_{n=1}^{N}\frac{1}{C_n}$$

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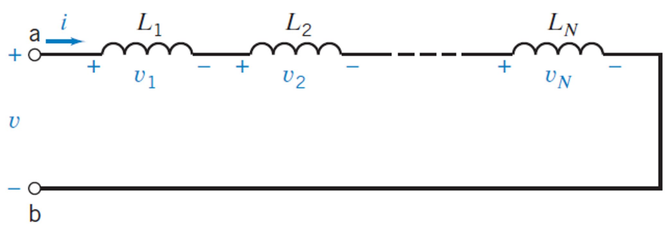
# ΕΠΑΓΩΓΗ - INDUCTOR

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau$$

$$\Rightarrow \boxed{V = L\frac{di}{dt}}$$

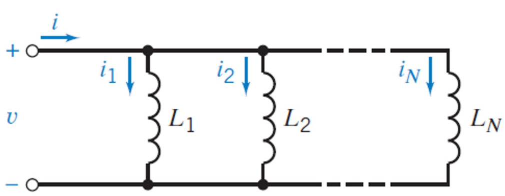
The current in an inductance cannot change instantaneously.




$$\begin{aligned} v &= v_1 + v_2 + \cdots + v_N \\ &= L_1\frac{di}{dt} + L_2\frac{di}{dt} + \cdots + L_N\frac{di}{dt} \\ &= \left(\sum_{n=1}^{N} L_N\right)\frac{di}{dt} \end{aligned}$$

$$L_s = \sum_{n=1}^{N} L_n$$

$$R = \sum_{i=1}^{N} R_i$$




$$i = \sum_{n=1}^{N} i_n$$

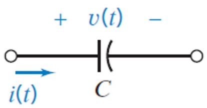
$$i = \left(\sum_{n=1}^{N}\frac{1}{L_n}\right)\int_{t_0}^{t} v\,d\tau + \sum_{n=1}^{N} i_n(t_0)$$

$$\frac{1}{L_p} = \sum_{n=1}^{N}\frac{1}{L_n}$$

CAPACITOR INDUCTOR

$$V = A\angle\varphi$$

## CAPACITOR

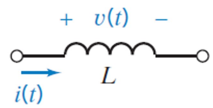



$$i(t) = C\frac{d}{dt}v(t)$$

$A\cos(\omega t+\varphi)$

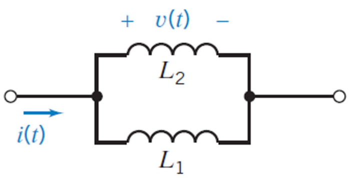
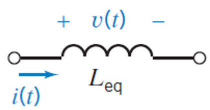
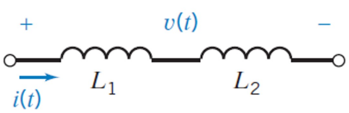
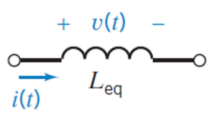
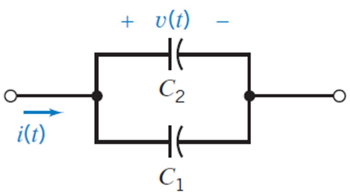
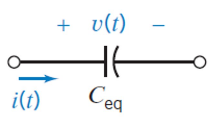
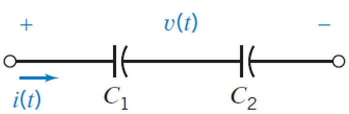
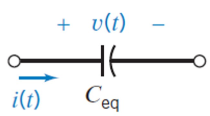
$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

## INDUCTOR




$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$$

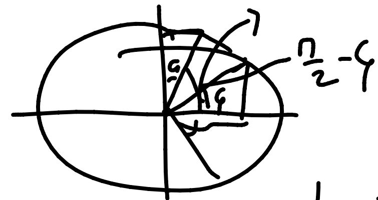
$$v(t) = L\frac{d}{dt}i(t)$$

| SERIES OR PARALLEL CIRCUIT | EQUIVALENT CIRCUIT | EQUATION |
|---|---|---|
|  |  | $L_{eq} = \dfrac{1}{\dfrac{1}{L_1}+\dfrac{1}{L_2}}$ |
|  |  | $L_{eq} = L_1 + L_2$ |
|  |  | $C_{eq} = C_1 + C_2$ |
|  |  | $C_{eq} = \dfrac{1}{\dfrac{1}{C_1}+\dfrac{1}{C_2}}$ |

$$V = A\angle\varphi$$

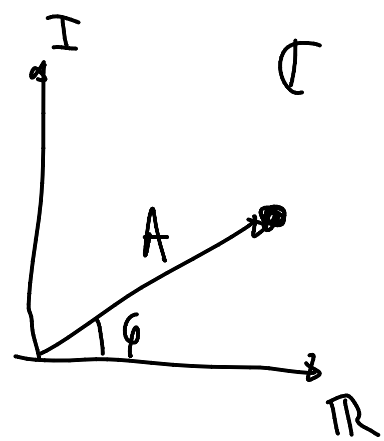
$$I(t) = CA\sin(\omega t+\varphi)\cdot\omega$$

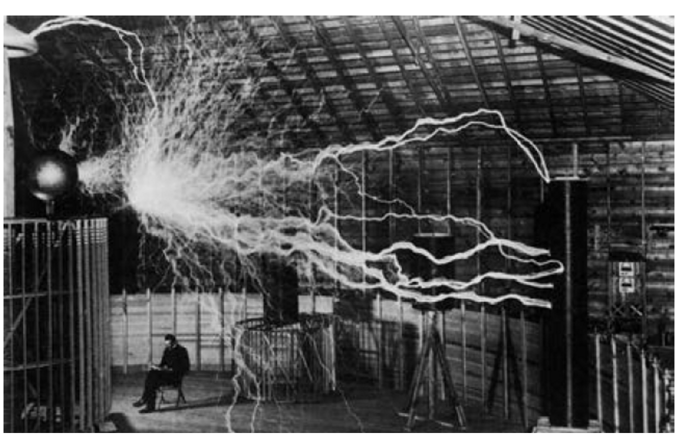
$$= \omega CA\cos\left(-\frac{\pi}{2}+\omega t+\varphi\right)$$




$$I(t) = \omega CA\angle\varphi-\frac{\pi}{2}$$

$$\frac{V}{I} = \frac{A\angle\varphi}{\omega CA\angle\varphi-\frac{\pi}{2}} = Z$$



$\boxed{i_s = 100 \sin 400t \text{ A}}$

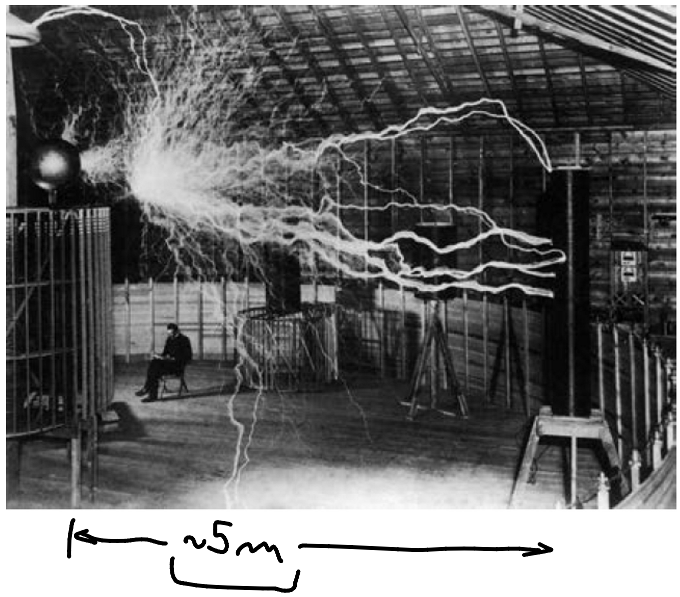
$L = ?$ H

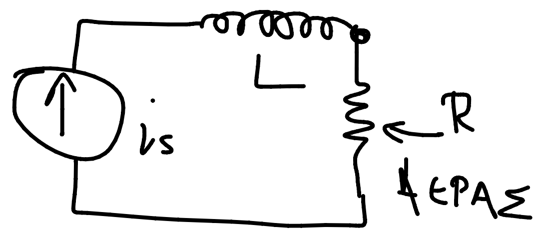
ΔΙΗΛΕΚΤΡΙΚΗ ΑΝΤΙΣΤΑΣΗ ΑΕΡΑ: $3\cdot 10^6 \frac{V}{m}$

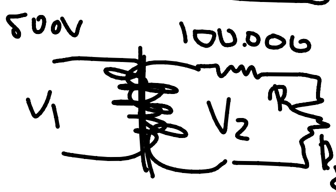
$\boxed{i_s = 100 \sin 400t \text{ A}}$

$L = \,?\;\; H$

ΔΙΗΛΕΚΤΡΙΚΗ ΑΝΤΙΣΤΑΣΗ ΑΕΡΑ: $3 \cdot 10^6 \frac{V}{m}$

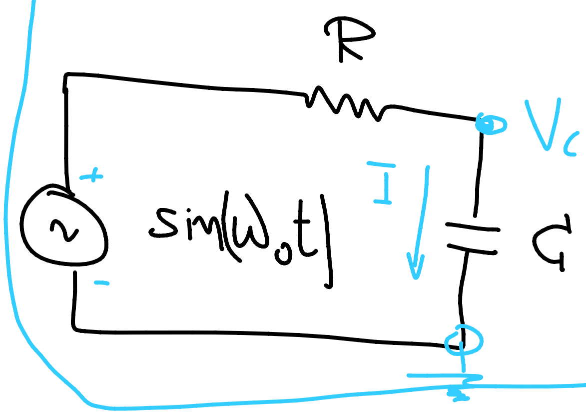



$\frac{V_2}{V_1} = \frac{n_2}{n_1}$

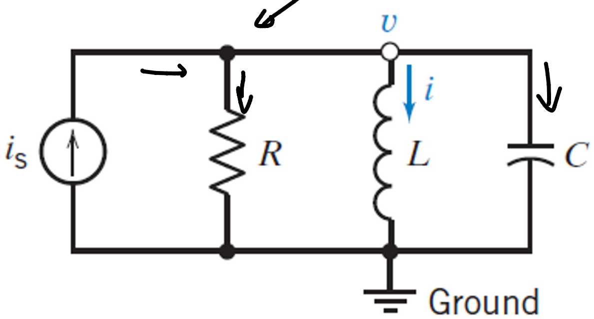
ΠΟΣΗ Η ΚΑΤΑΝΑΛΩΣΗ ΕΝΕΡΓΕΙΑΣ ΣΤΟΝ ΠΥΚΝΩΤΗ??




$$i(t) = C\frac{d}{dt}v(t)$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

$P = V \cdot I =$




$$\frac{v}{R} + i + C\frac{dv}{dt} = i_s$$

$$v = L\frac{di}{dt} \Rightarrow v = L \cdot \frac{di}{dt}$$

$$\boxed{\frac{L}{R}\frac{di}{dt} + i + CL\frac{d^2 i}{dt^2} = i_s}$$

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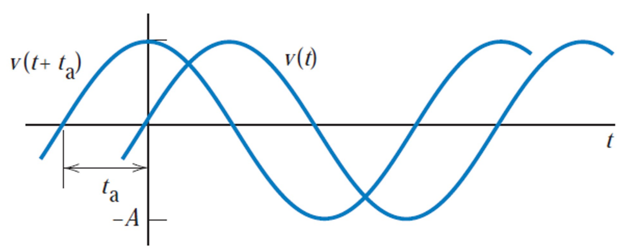
H. Hertz.

ΗΜΙΤΟΝΙΚΑ ΣΗΜΑΤΑ

$$\omega = 2\pi f = \frac{2\pi}{T}$$




A phasor is a complex number that is used to represent the amplitude and phase angle of a sinusoid. The relationship between the sinusoid and the phasor is described by

$$A\cos(\omega t + \theta) \quad \leftrightarrow \quad A \angle \theta \tag{10.3-1}$$

ΠΛΑΤΟΣ

ΣΥΧΝΟΤΗΤΑ

ΦΑΣΗ

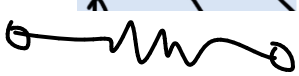
ΜΙΓΑΔΙΚΟΣ ΑΡΙΘΜΟΣ

ΣΥΧΝΟΤΗΤΑ ΦΑΣΗ

The impedance of an element of an ac circuit is defined to be the ratio of the voltage phasor to the current phasor. The impedance is denoted as $\mathbf{Z}(\omega)$ so

$$\mathbf{Z}(\omega) = \frac{\mathbf{V}(\omega)}{\mathbf{I}(\omega)} = \frac{V_m \angle \theta}{I_m \angle \phi} = \frac{V_m}{I_m} \angle (\theta - \phi) \ \Omega \qquad (10.4\text{-}2)$$

$A\cos(\omega t + \theta)$

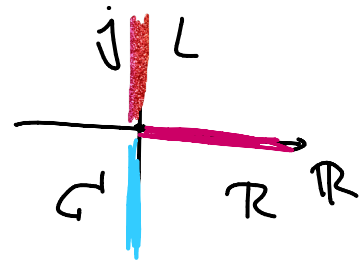


$$\frac{V}{I} = \frac{V_m \cos(\omega t + \theta)}{\frac{1}{R} V_m \cos(\omega t + \theta)} = R$$

Αντίσταση: $Z_R(\omega) = \dfrac{V_m \angle \theta}{I_m \angle \theta} = R$

Χωρητικότητα:

$v_C(t) = A\cos(\omega t + \theta)$ V $\Rightarrow V(\omega) = A \angle \theta$

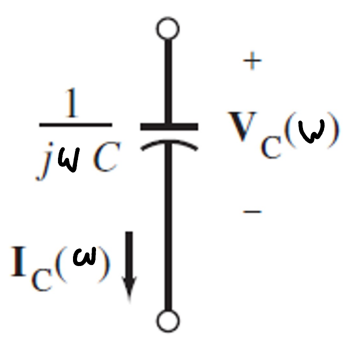



$$i_C(t) = C\frac{d}{dt}v_C(t) = -C\omega A \sin(\omega t + \theta) = C\omega A \cos(\omega t + \theta + 90^\circ) \text{ A}$$

$$\mathbf{V}_C(\omega) = A\angle\theta \text{ V and } \mathbf{I}_C(\omega) = C\omega A \angle(\theta + 90^\circ) = (C\omega \angle 90^\circ)(A\angle\theta) = j\omega CA \angle\theta \text{ A}$$

$\boxed{C\omega \cos(\omega t + 90) \cdot A\cos(\omega t + \theta)}$

$$\mathbf{Z}_C(\omega) = \frac{\mathbf{V}_C(\omega)}{\mathbf{I}_C(\omega)} = \frac{A\angle\theta}{j\omega CA\angle\theta} = \frac{1}{j\omega C} \ \Omega$$



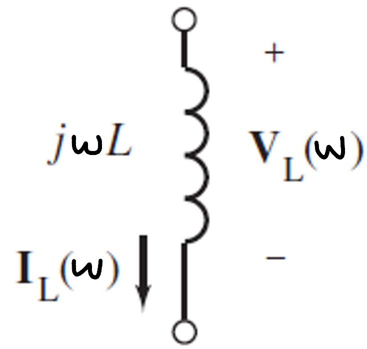

$$Z_C(\omega) = -\frac{j}{\omega C}$$

# ΕΠΑΓΩΓΗ

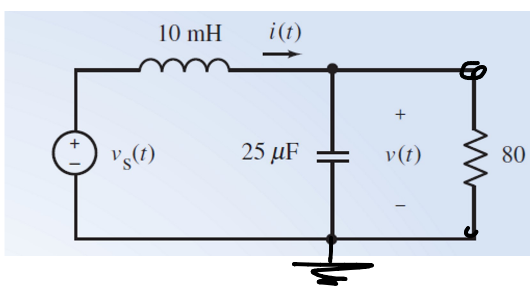
$$v_L(t) = L\frac{d}{dt}i_L(t) = -L\omega A\sin(\omega t+\theta) = L\omega A\cos(\omega t+\theta+90°)\ \text{V}$$

$$\mathbf{I}_L(\omega) = A\angle\theta\ \text{A and } \mathbf{V}_L(\omega) = L\omega A\angle(\theta+90°) = j\omega LA\angle\theta\ \text{V}$$

$$\mathbf{Z}_L(\omega) = \frac{\mathbf{V}_L(\omega)}{\mathbf{I}_L(\omega)} = \frac{j\omega LA\angle\theta}{A\angle\theta} = j\omega L\ \Omega$$




# Ασκηση:




$$v_S(t) = 48\cos(500t+75°)\ \text{V}$$

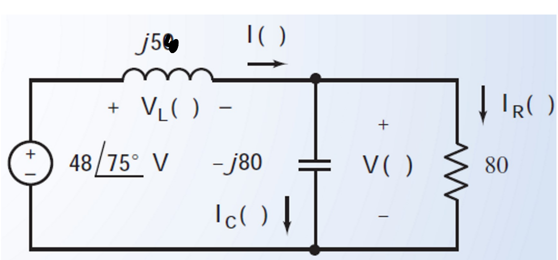
Υπολογίστε το V(t)

$$\mathbf{Z}_C(\omega) = \frac{1}{j\omega C} = \frac{1}{j500\,(25\times10^{-6})} = \frac{80}{j} = -j80\ \Omega,$$

$$\mathbf{Z}_L(\omega) = j\omega L = j500\,(0.1) = j50\ \Omega$$

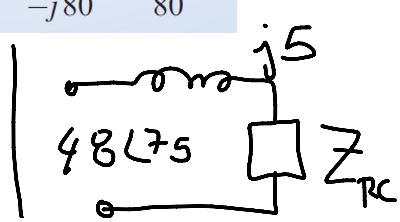
0.01 j5Ω

$$Z_{CR} = \frac{\cancel{8}0(-j80)}{\cancel{8}0-j\cancel{80}} = \frac{-j80}{1-j} = \frac{80}{1+j}$$



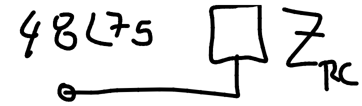

$$\mathbf{V}_L(\omega) = j50\,\mathbf{I}(\omega),\quad \mathbf{I}_C(\omega) = \frac{\mathbf{V}(\omega)}{-j80}\ \text{and}\ \mathbf{I}_R(\omega) = \frac{\mathbf{V}(\omega)}{80}$$

$$\mathbf{I}(\omega) = \mathbf{I}_C(\omega) + \mathbf{I}_R(\omega) = \frac{\mathbf{V}(\omega)}{-j80} + \frac{\mathbf{V}(\omega)}{80}$$



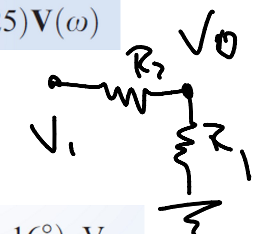

$$48\angle75° = \mathbf{V}_L(\omega) + \mathbf{V}(\omega) = j50\,\mathbf{I}(\omega) + \mathbf{V}(\omega)$$

$$48\angle 75^\circ = \mathbf{V_L}(\omega)+\mathbf{V}(\omega) = j50\,\mathbf{I}(\omega)+\mathbf{V}(\omega)$$




$$\Downarrow$$

$$48\angle 75^\circ = j50\left[\frac{\mathbf{V}(\omega)}{-j80}+\frac{\mathbf{V}(\omega)}{80}\right]+\mathbf{V}(\omega) = \left[\frac{j50}{-j80}+\frac{j50}{80}+1\right]\mathbf{V}(\omega)$$
$$= [-0.625+j0.625+1]\mathbf{V}(\omega) = (0.375+j0.625)\mathbf{V}(\omega)$$

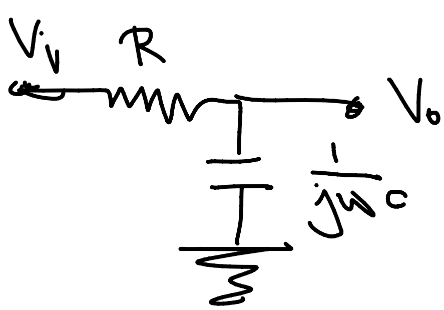



$$\mathbf{V}(\omega) = \frac{48\angle 75^\circ}{0.375+j0.625} = \frac{48\angle 75^\circ}{0.7289\angle 59^\circ} = 65.9\angle 16^\circ \text{ V} \Rightarrow v(t) = 65.9\cos(500t+16^\circ)\text{ V}$$

$$\frac{V(\omega)}{48\angle 75^\circ} = \frac{\frac{80}{1+j}}{\frac{80}{1+j}+\frac{5j(1+j)}{1+j}} \Rightarrow V(\omega) = 48\angle 75$$

$Z_R = R \quad Z_c = \frac{1}{j\omega c} \quad Z_L = j\omega L$

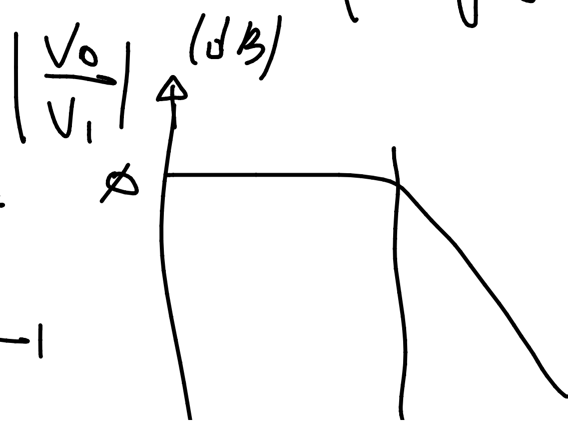
# ΠΑΘΗΤΙΚΑ ΦΙΛΤΡΑ ΜΕ ΦΑΣΟΡΕΣ




$$\left|\frac{V_0}{V_i}\right| = \left|\frac{\frac{1}{j\omega c}}{R+\frac{1}{j\omega c}}\right| =$$

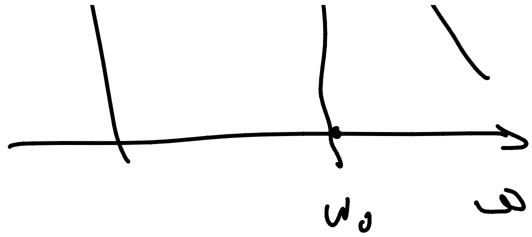
$$= \left|\frac{1}{1+j\omega RC}\right| =$$

$$= |1+j\omega RC|^{-1}$$

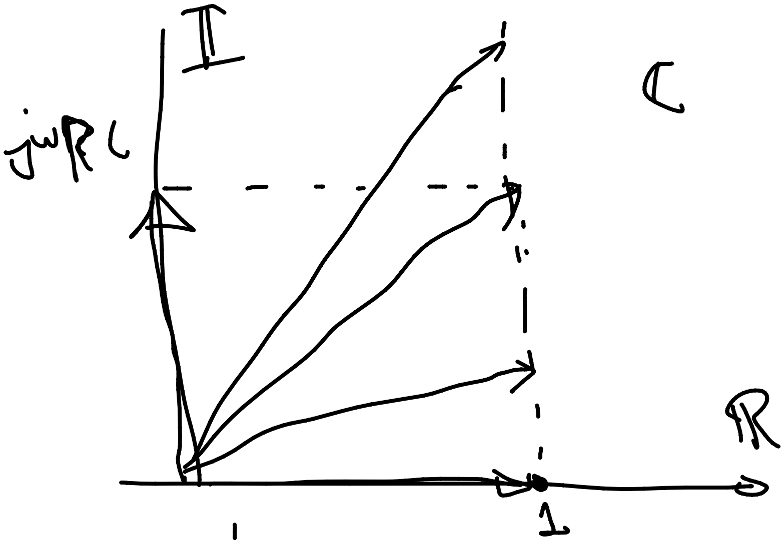



$\omega = 2\pi f$

$$= \left|1 + j\omega RC\right|^{-1}$$




$$f = \frac{1}{2\pi RC}$$

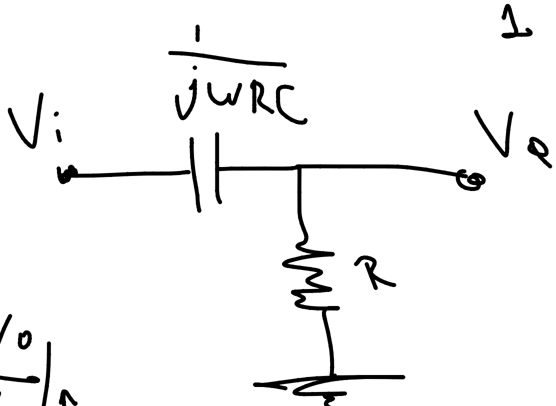



C

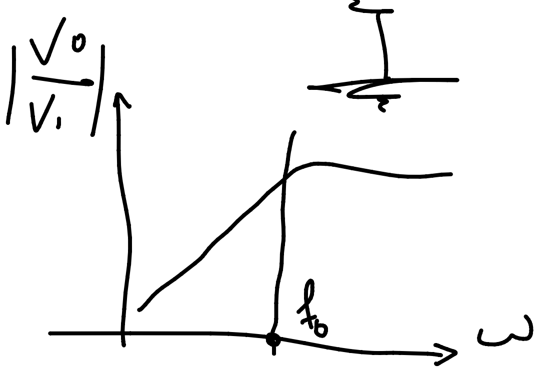
$$\boxed{\omega_0 = \frac{1}{RC}} \Rightarrow$$

$$= \frac{1}{2\pi \cdot 10^4 \cdot 10^{-9}} = \frac{1}{6.28} \cdot 10^5 .$$

$$= 1.59 \cdot 10^4 \text{ Hz}$$




$$\left|\frac{V_o}{V_i}\right| = \left|\frac{R}{R + \frac{1}{j\omega C}}\right| =$$




$$= \left|\frac{1}{1 + \frac{1}{j\omega RC}}\right| \qquad f_0 = \frac{1}{2\pi RC}$$

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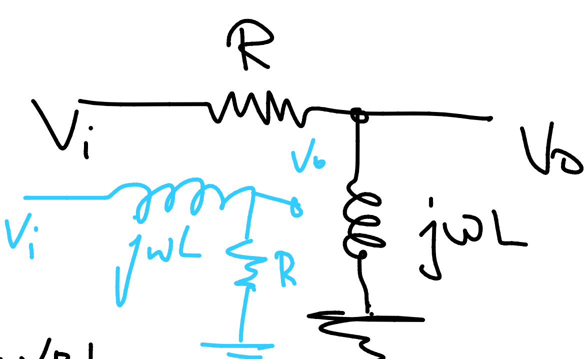
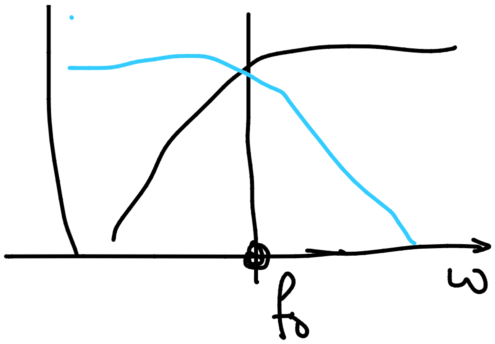



$$\left|\frac{V_o}{V_i}\right| = \left|\frac{j\omega L}{R + j\omega L}\right| =$$

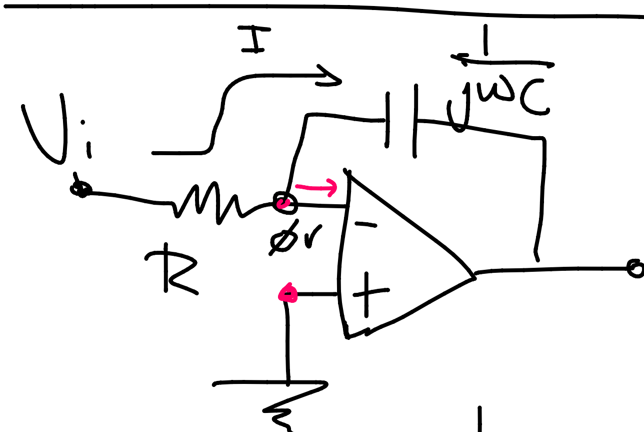
$$= \left|\frac{1}{1 + \frac{R}{j\omega L}}\right| =$$




$$= \left| 1 + \frac{R}{j\omega L} \right|^{-1}$$

$$f_0 = \frac{R}{2\pi \omega L}$$

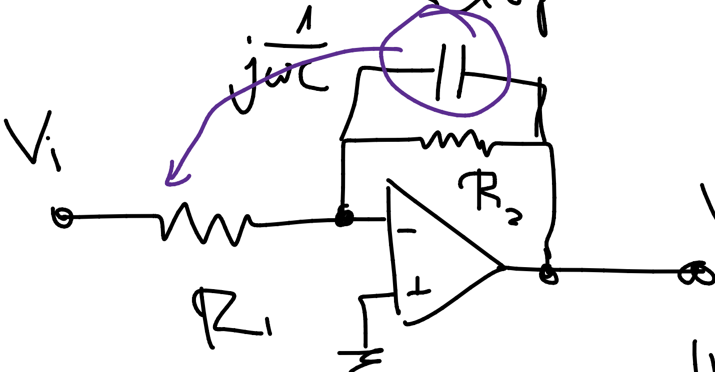
ΕΝΕΡΓΑ




$$\left| \frac{V_0}{V_i} \right| = \left| \frac{-\frac{1}{j\omega C}}{\frac{R}{1}} \right| = \left| \frac{1}{j\omega RC} \right| = \frac{1}{\omega RC}$$


$$20 \log_{10} \left| \frac{V_0}{V_i} \right| =$$

$$= 20 \log \frac{1}{\omega RC} =$$



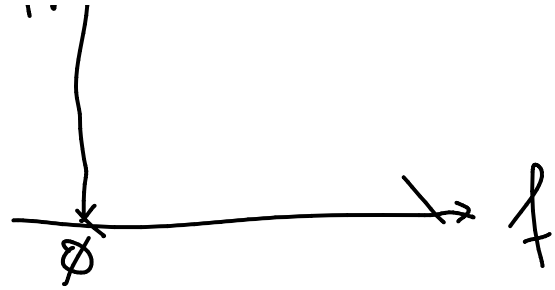

$$\omega \to \phi \Rightarrow G = -\frac{R_2}{R_1}$$

$$\omega \to +\infty \Rightarrow G = \phi$$

$$\left| \frac{V_0}{V_i} \right| = \cancel{R_2} R_1$$

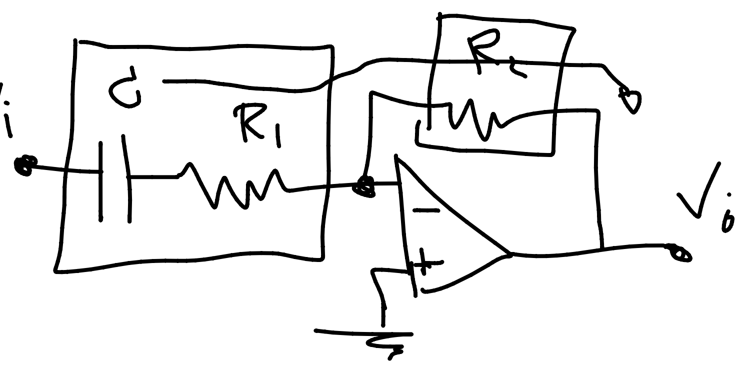
$$V \quad \frac{R_2 \cdot \frac{1}{j\omega C}}{\ldots}$$

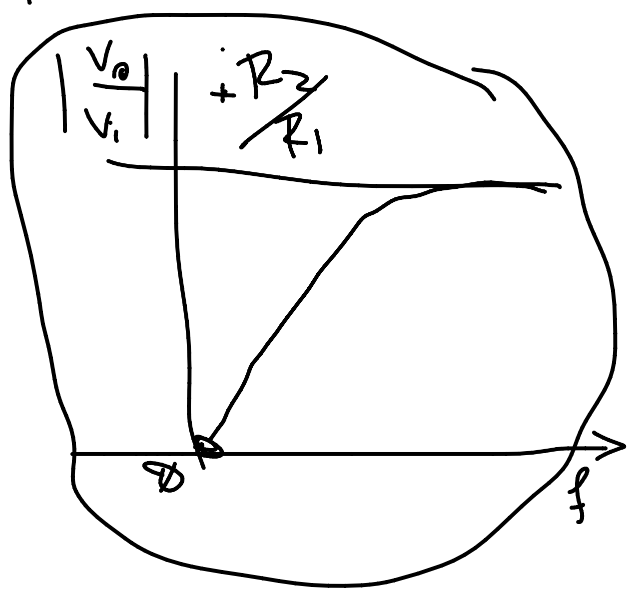
$$\frac{V_0}{V_i} = -\frac{\dfrac{R_2 \cdot j\omega C}{R_2 + \frac{1}{j\omega C}}}{\dfrac{R_1}{1}} =$$




$$\Rightarrow \left|\frac{V_0}{V_i}\right| = \frac{\left|R_2 \frac{1}{j\omega C}\right|}{\left|R_1R_2 + \frac{R_1}{j\omega C}\right|} = \frac{\frac{R_2}{\omega C}}{\left|R_1R_2 + \frac{R_1}{j\omega C}\right|} =$$

$$= \frac{R_2}{R_1\left|\omega C R_2 + \frac{1}{j}\frac{j}{j}\right|} = \frac{R_2}{R_1\left|\omega C R_2 - j\right|}$$



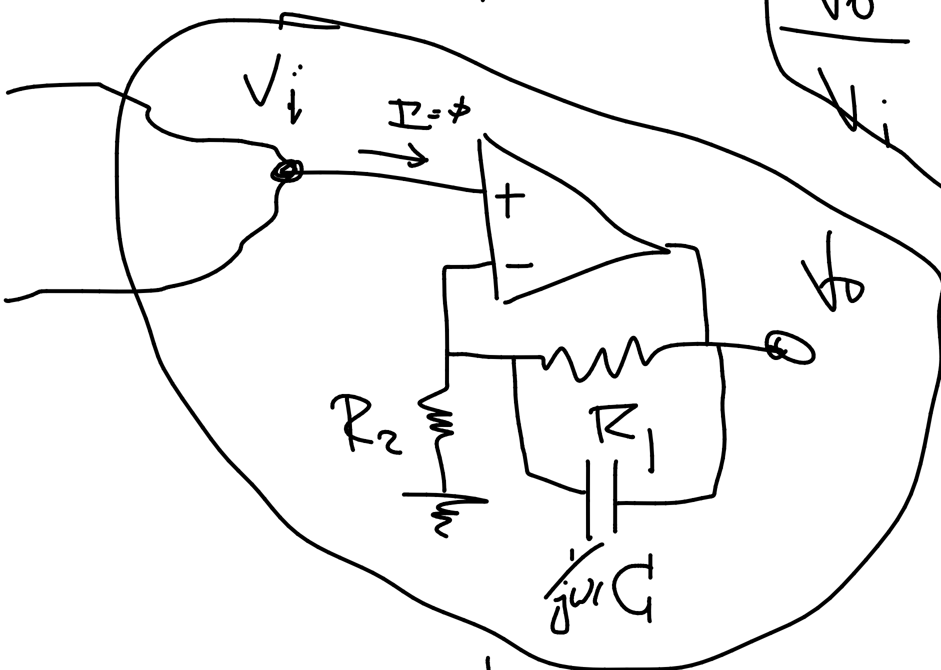




$$\left|\frac{V_0}{V_i}\right| = \left|-\frac{R_2}{R_1 + \frac{1}{j\omega C}}\right| =$$

$$= \left|\frac{R_2}{R_1 + \frac{1}{j\omega C}}\right|$$

$$\frac{V_0}{\;} = 1 + \frac{R_1}{\;}$$

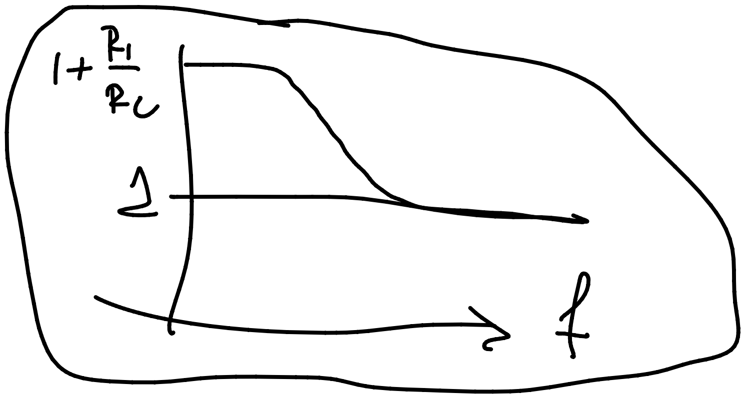
$$\boxed{\frac{V_0}{V_i} = 1 + \frac{R_1}{R_2}}$$

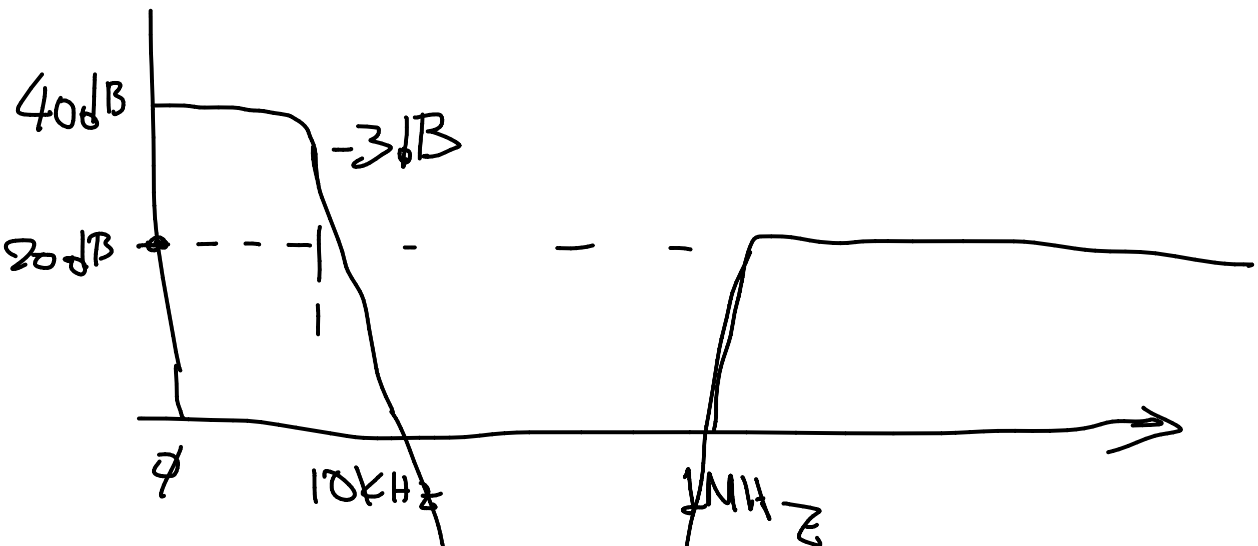



$$\frac{V_0}{V_i} = 1 + \frac{\dfrac{R_1 \cdot \frac{1}{j\omega C}}{R_1 + \frac{1}{j\omega C}}}{R_2} = 1 + \frac{\dfrac{R_1}{1 + j\omega R_1 C}}{\dfrac{R_2}{1}} =$$

$$= 1 + \frac{R_1}{R_2 + j\omega R_1 R_2 C} = 1 + \frac{R_1}{R_2} \frac{1}{1 + j\omega R_1}$$

$$\left|\frac{V_0}{V_i}\right| = \left|1 + \frac{R_1}{R_2} \cdot \frac{1}{1 + j\omega R_1}\right|$$



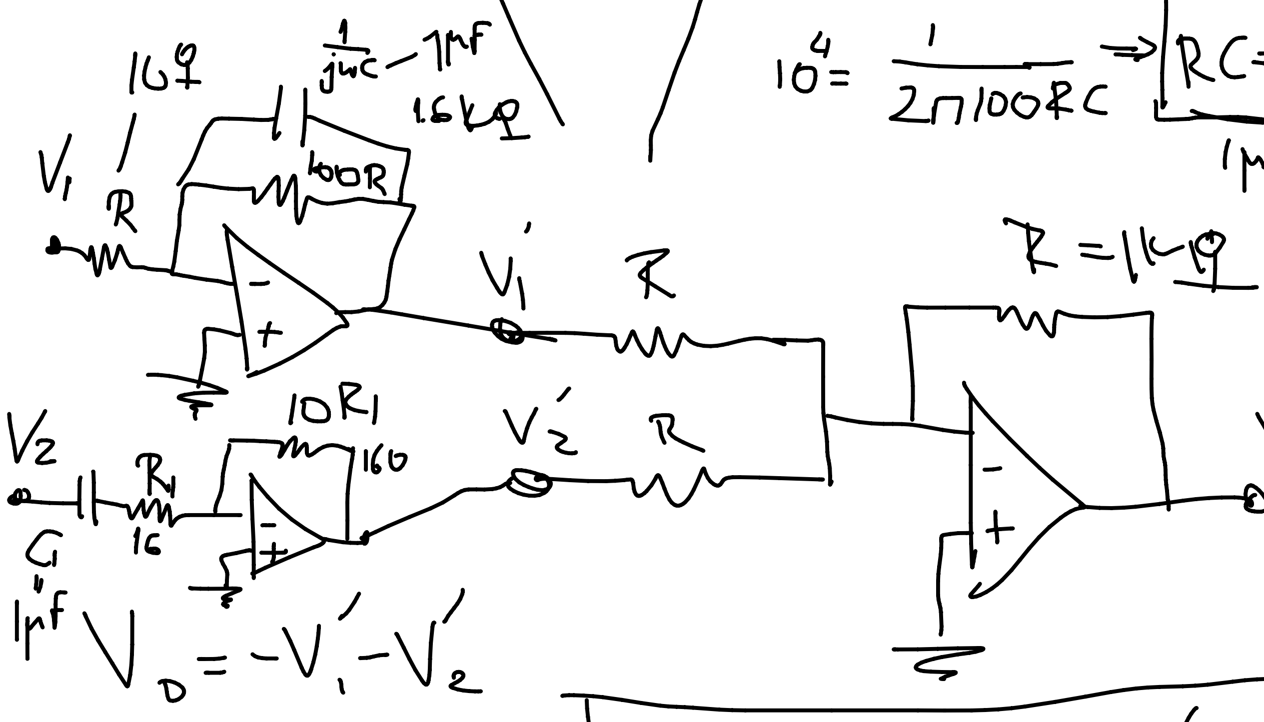

$10^4 = \frac{1}{2\pi 100RC} \Rightarrow \boxed{RC = \frac{1}{2\pi}\cdot 10^{-6}}$

1μF

⇓

$R = 1k\Omega \quad R = \frac{100}{2\pi}$

$\approx 16\Omega$




$V_0 = -V_1' - V_2'$

6 1 1 $10^{-6}$

$$f = 10^6 = \frac{1}{2\pi R_1 C_1} \Rightarrow \boxed{R_1 C_1 = \frac{1}{2\pi} \cdot 10^{-6}}$$

$$\frac{10 R_1}{R_1 + \frac{1}{j\omega C}} = 10 \frac{1}{1 + \frac{1}{j\omega R C}}$$

$$f = \frac{1}{2\pi RC} = \frac{1}{2\pi \cdot 10^{-6} \cdot R} = 10^6 \Rightarrow$$

$$R = \frac{1}{2\pi} 10^{12} = 1.5 \cdot 10^{11}$$

$C = 1mF$

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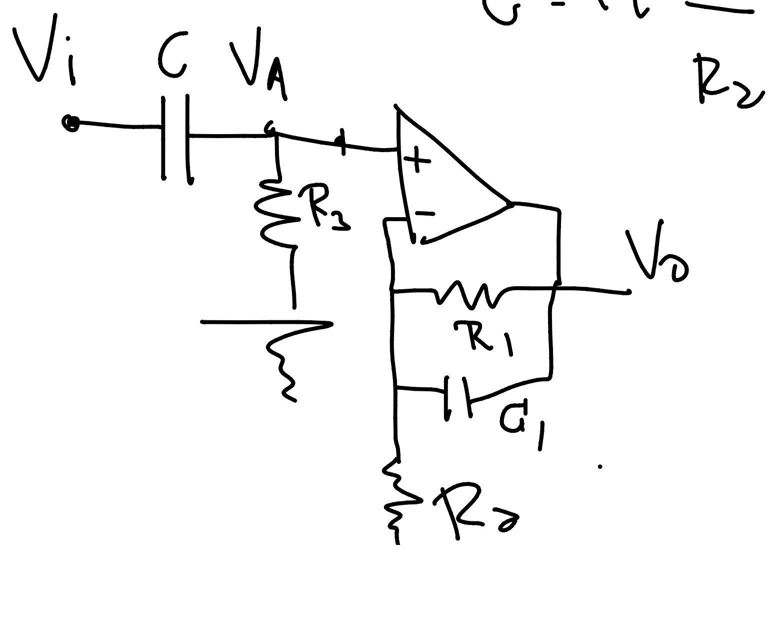
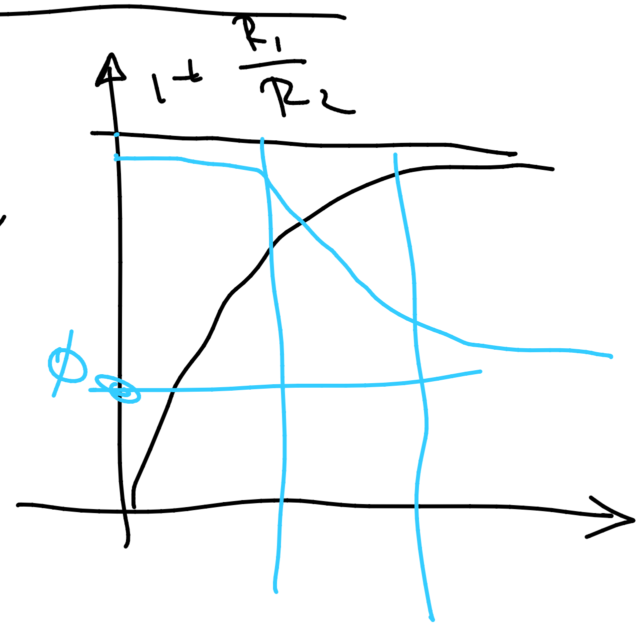
$$G = 1 + \frac{R_1}{R_2}$$



$R_2$

ΕΝ.Κ. ΠΑΘ.

$$\left|\frac{V_0}{V_i}\right| = \left|\left(1 + \frac{\dfrac{R_1 \cdot \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}}}{R_0}\right)\right| \cdot \left|\frac{R_3}{R_3 + \frac{1}{j\omega C}}\right| = \left|\frac{V_0}{V_A}\right| \cdot \left|\frac{V_A}{V_i}\right|$$

$$10^6$$

$$\frac{1}{2\pi RC} = 0.15 \cdot 10^{-3} \cdot 10^{+9} = 15\,kHz$$