



ΠΑΝΕΠΙΣΤΗΜΙΟ
ΠΑΤΡΩΝ
UNIVERSITY OF PATRAS

ΑΝΟΙΚΤΑ ακαδημαϊκά
μαθήματα ΠΠ

Αριθμητικός Έλεγχος Εργαλειομηχανών

Ενότητα 8: Mathematics for Numerical Control
Programming

Δημήτρης Μούρτζης, Επίκουρος Καθηγητής
Πολυτεχνική Σχολή

Τμήμα Μηχανολόγων & Αεροναυπηγών Μηχανικών



COMPUTER NUMERICAL CONTROL OF MACHINE TOOLS

Laboratory for Manufacturing Systems and Automation
Department of Mechanical Engineering and Aeronautics
University of Patras, Greece



Dr. Dimitris Mourtzis
Assistant Professor

Patras, 2015

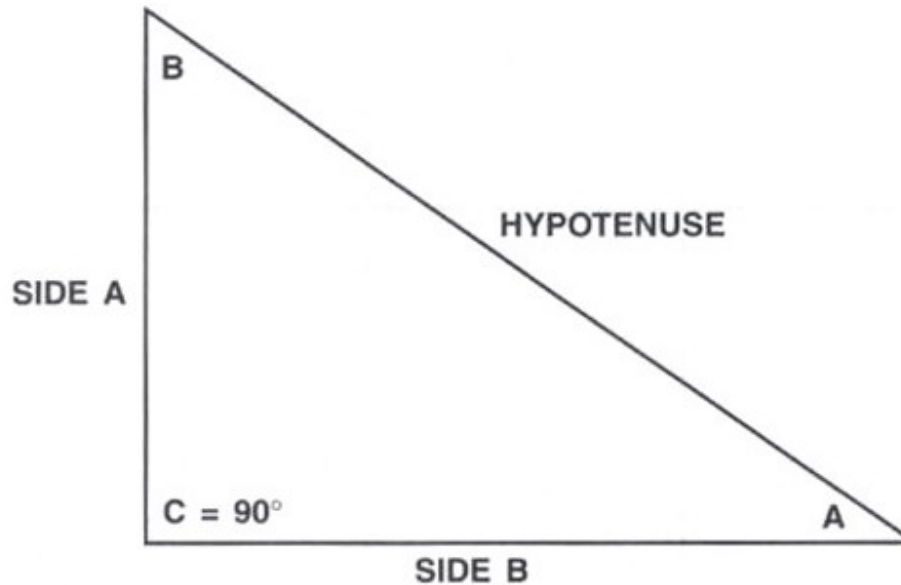


Objectives of section 8

- Use right-angle trigonometry to determine programming coordinates from part drawings



Basic Trigonometry



$$\text{SINE} = \frac{\text{OPPOSITE SIDE}}{\text{HYPOTENUSE}}$$

$$\text{COSINE} = \frac{\text{SIDE ADJACENT}}{\text{HYPOTENUSE}}$$

$$\text{TANGENT} = \frac{\text{SIDE OPPOSITE}}{\text{SIDE ADJACENT}}$$

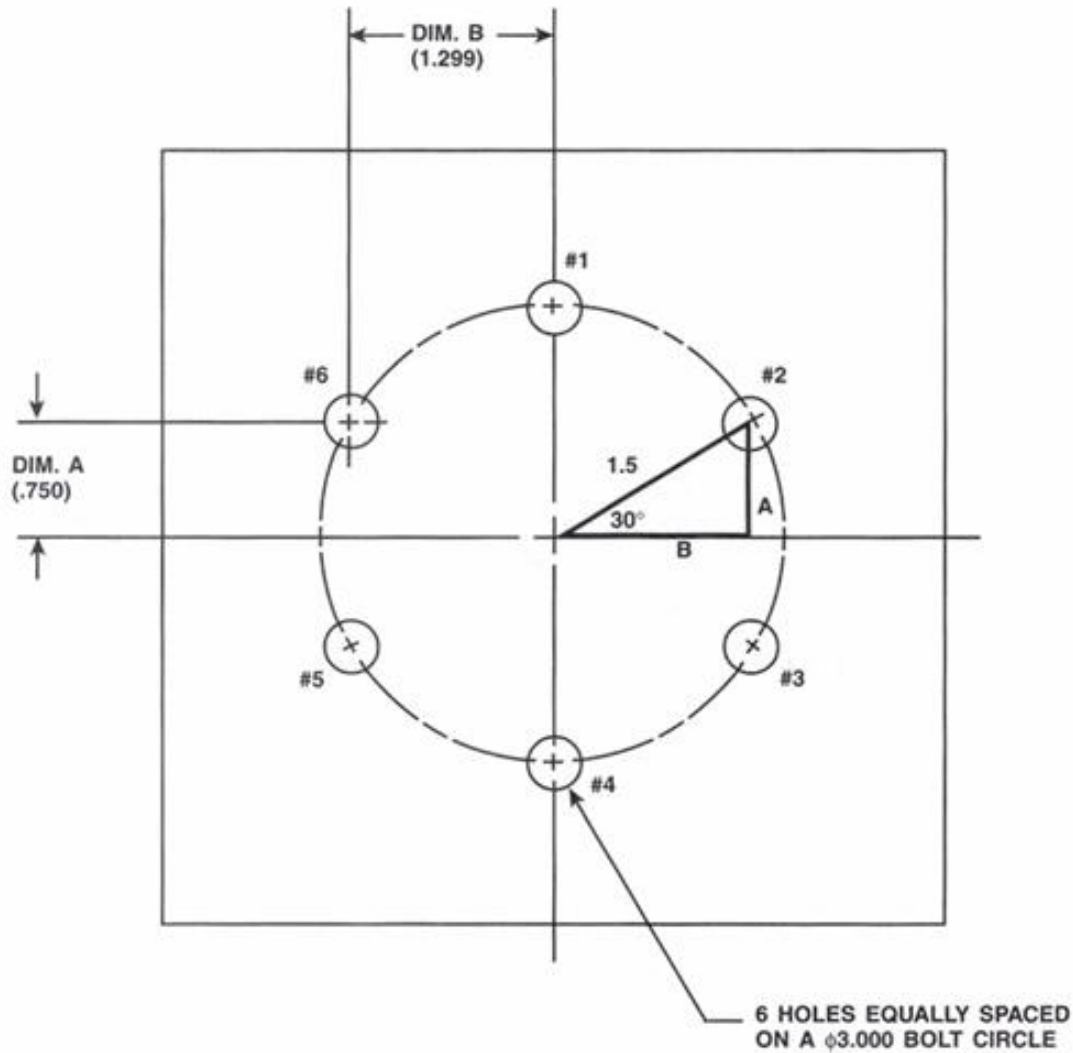
$$\text{COSECANT} = \frac{\text{HYPOTENUSE}}{\text{SIDE OPPOSITE}}$$

$$\text{SECANT} = \frac{\text{HYPOTENUSE}}{\text{SIDE ADJACENT}}$$

$$\text{COTANGENT} = \frac{\text{SIDE ADJACENT}}{\text{SIDE OPPOSITE}}$$



Basic Trigonometry



$$\text{SINE } A = \frac{\text{OPPOSITE SIDE}}{\text{HYPOTENUSE}}$$

$$\text{SINE } 30 = \frac{a}{1.500}$$

$$0.500 = \frac{a}{1.500}$$

$$a = 0.750$$

FIGURE 1 Basic Trigonometry



Basic Trigonometry

$$\text{COSINE } A = \frac{\text{ADJACENT SIDE}}{\text{HYPOTENUSE}}$$

$$\text{TAN } 40 = \frac{\text{OPPOSITE SIDE}}{\text{ADJACENT SIDE}}$$

$$\text{COS } A = \frac{b}{1.500}$$

$$0.866 = \frac{b}{1.500}$$

$$b = 1.299$$

$$\text{TAN } 40 = \frac{1.000}{b}$$

$$0.839 = \frac{1.000}{b}$$

$$b = 1.191$$



Basic Trigonometry

- Dimension X equals $3.000 - 1.191$, or 1.809

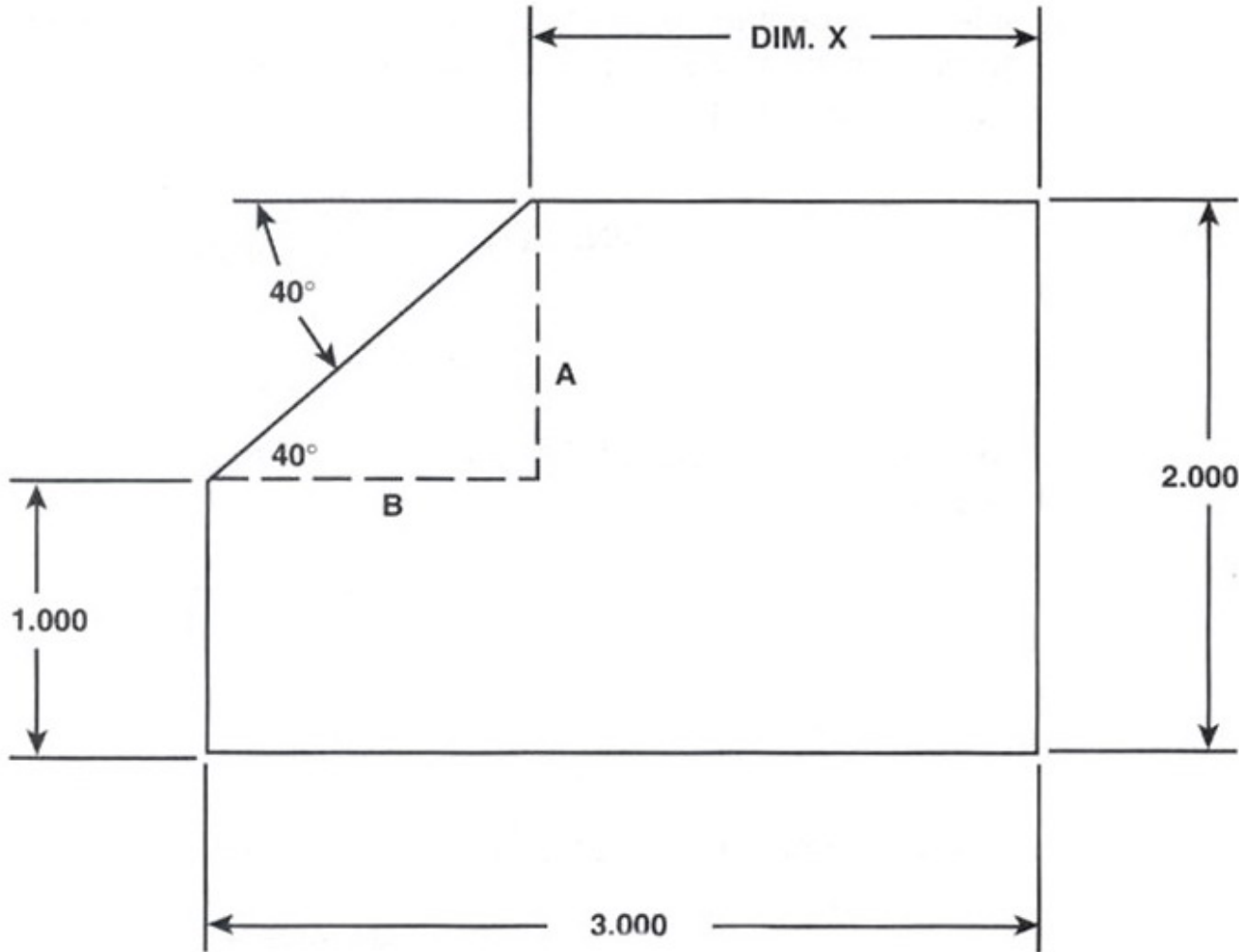
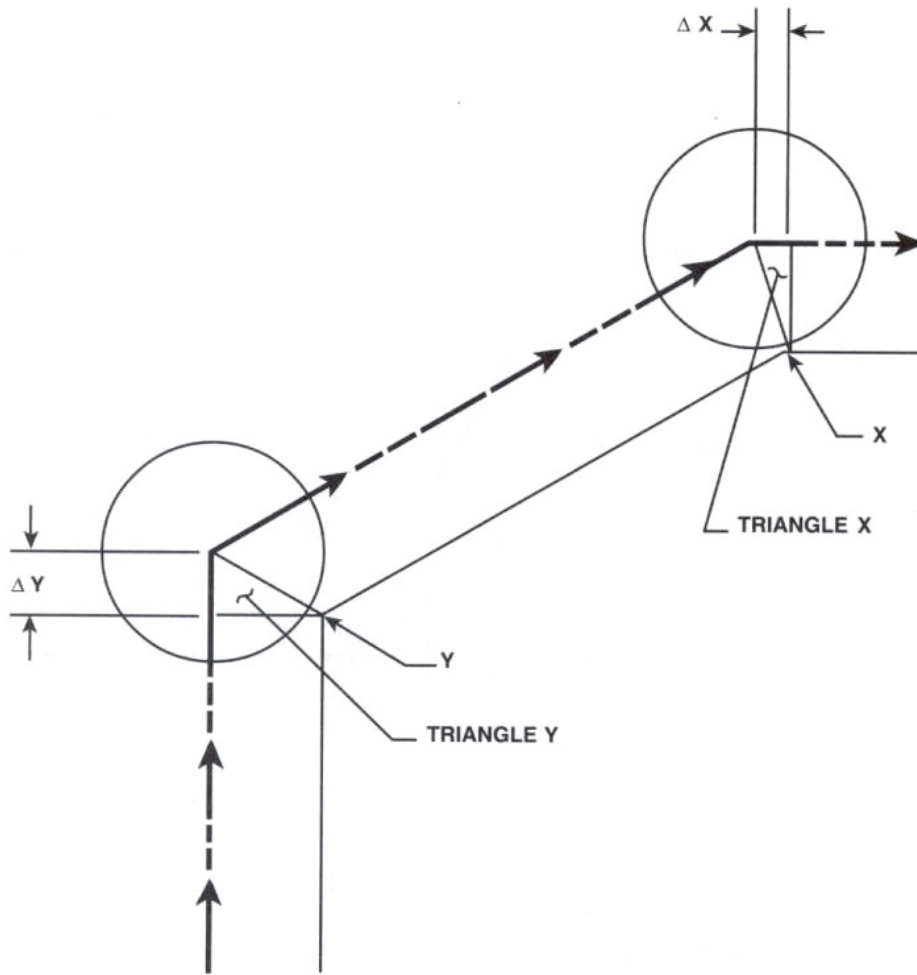


FIGURE 2 Basic Trigonometry

Using Trigonometry for Cutter Offsets

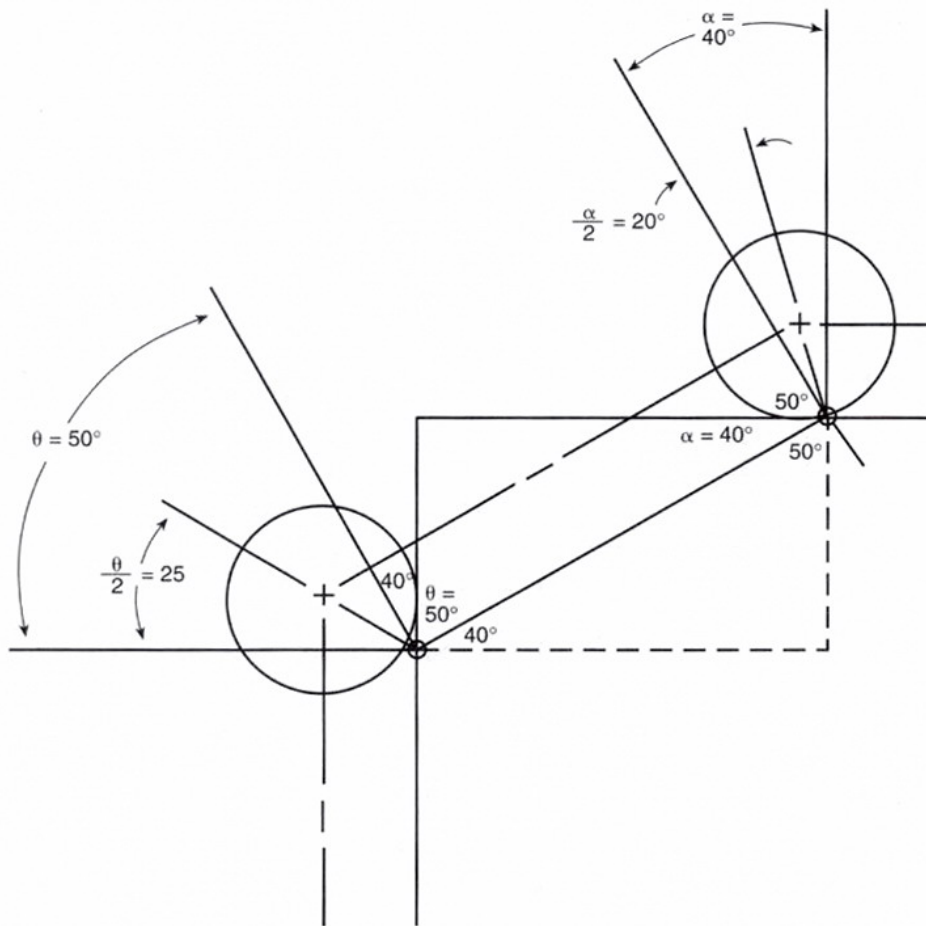


1. The total number of degrees in a circle is 360
2. The sum of the angles in a triangle is 180 degrees
3. The complement of an angle is 90 minus the angle

FIGURE 3 Trigonometry for Cutter Offsets



Using Trigonometry for Cutter Offsets



Solving triangle Y for ΔY :

$$\frac{\Delta Y}{0.250} = \text{TAN } 25$$

$$\Delta Y = \text{TAN } 25 (0.250)$$

$$\Delta Y = 0.11658 \text{ or } 0.117$$

Solving triangle X for ΔX :

$$\frac{\Delta X}{0.250} = \text{TAN } 20$$

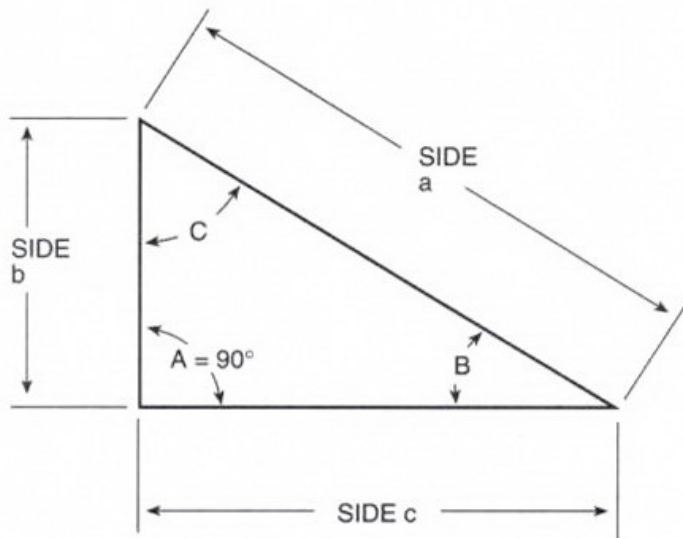
$$\Delta X = \text{TAN } 20 (0.250)$$

$$\Delta X = 0.09099 \text{ or } 0.091$$

FIGURE 4 Trigonometry for Cutter Offsets - II



Using Trigonometry for Cutter Offsets

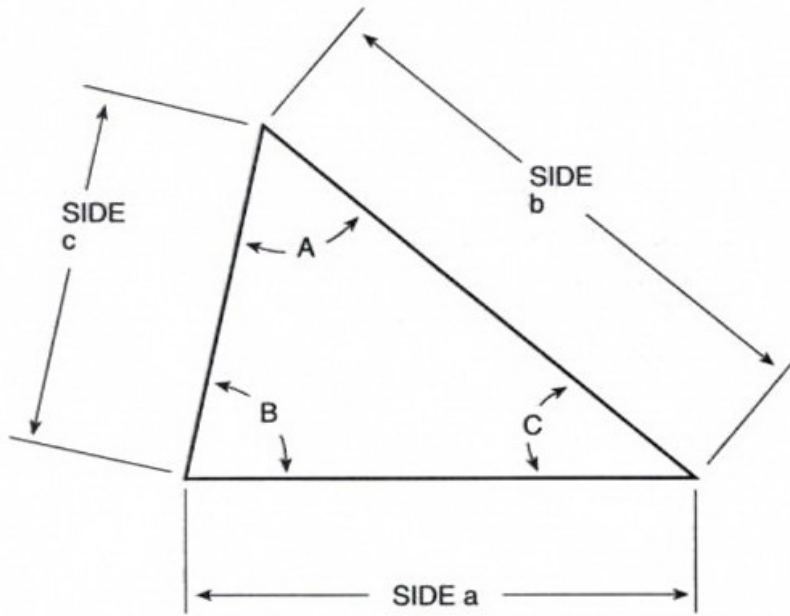


KNOWN VARIABLES	SOLUTION FORMULAS		
SIDE a, ANGLE B	$b = a \times \sin B$	$c = a \times \cos B$	$C = 90^\circ - B$
SIDE a, ANGLE C	$b = a \times \cos C$	$c = a \times \sin C$	$B = 90^\circ - C$
SIDE b, ANGLE B	$a = \frac{b}{\sin B}$	$c = b \times \cot B$	$C = 90^\circ - B$
SIDE b, ANGLE C	$a = \frac{b}{\cos C}$	$c = b \times \tan C$	$B = 90^\circ - C$
SIDE c, ANGLE B	$a = \frac{c}{\cos B}$	$b = c \times \tan B$	$C = 90^\circ - B$
SIDE c, ANGLE C	$a = \frac{c}{\sin C}$	$b = c \times \cot C$	$B = 90^\circ - C$
SIDES a AND b	$c = \sqrt{a^2 - b^2}$	$\sin B = \frac{b}{a}$	$C = 90^\circ - B$
SIDES a AND c	$b = \sqrt{a^2 - c^2}$	$\sin C = \frac{c}{a}$	$B = 90^\circ - C$
SIDES b AND c	$a = \sqrt{b^2 + c^2}$	$\tan B = \frac{b}{c}$	$C = 90^\circ - B$

FIGURE 5 Solutions of right triangles



Using Trigonometry for Cutter Offsets



ONE SIDE AND TWO ANGLES KNOWN: GIVEN: SIDE a, OPPOSITE ANGLE A, AND OTHER ANGLE B		
$C = 180^\circ - (A + B)$	$b = \frac{a \times \sin B}{\sin A}$	$c = \frac{a \times \sin C}{\sin A}$
TWO SIDES AND THE ANGLE BETWEEN THEM KNOWN: GIVEN: SIDES a, b, AND ANGLE C		
$\tan A = \frac{a \times \sin C}{b - (a \times \cos C)}$	$B = 180^\circ - (A + C)$	$c = \frac{a \times \sin C}{\sin A}$
$c = \sqrt{a^2 + b^2 - (2ab \times \cos C)}$		
TWO SIDES AND ANGLE OPPOSITE ONE SIDE KNOWN: GIVEN: SIDE a, OPPOSITE ANGLE A, AND SIDE B		
$\sin B = \frac{b \times \sin A}{a}$	$C = 180^\circ - (A + B)$	$c = \frac{a \times \sin C}{\sin A}$
ALL THREE SIDES KNOWN:		
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$	$\sin B = \frac{b \times \sin A}{a}$	$C = 180^\circ - (A + B)$

FIGURE 6 Solutions of oblique-angled triangles



Summary

- Right – angle trigonometry is the mathematical science of solving right triangles
- The sine of an angle equals the side opposite the angle divided by the hypotenuse of the triangle
- The cosine of an angle equals the side adjacent to the angle divided by the hypotenuse of the triangle
- The tangent of an angle equals the side opposite the angle divided by the side adjacent to the angle
- The use of trigonometry is necessary for determining cutter offsets for linear and circular interpolation and for determining other part information from a blueprint



Vocabulary Introduced in this section

- Cosine
- Cutter offsets
- Sine
- Tangent
- Trigonometry



End of Section



Funding

- This educational material has been developed in the teaching duties of the respective educator.
- The Project “**Open Academic Courses at the University of Patras**” has funded only the reformation of the educational material.
- The Project is implemented within the context of the Operational Programme "Education and Lifelong Learning" (EdLL) and is co-funded by the European Union (European Social Fund) and national resources.



Reference Note

Copyright University of Patras, School of Engineering, Dept. of Mechanical Engineering & Aeronautics, Dimitris Mourtzis. Dimitris Mourtzis. «Computer Numerical Control of Machine Tools. Mathematics for Numerical Control Programming». Version: 1.0. Patras 2015. Available at: <https://eclass.upatras.gr/courses/MECH1213/>



License Note

This material is provided under the license terms of Creative Commons Attribution-NonCommercial-NoDerivatives (CC BY-NC-ND 4.0) [1] or newer, International Version. Works of Third Parties (photographs, diagrams etc) are excluded from this license and are referenced in the respective “Third Parties’ works Note”



[1] <https://creativecommons.org/licenses/by-nc-nd/4.0/>

As **NonComercial** is denoted the use that:

does not involve directed or indirect financial profit for the use of this content, for the distributor and the licensee

does not involve any financial transaction as a prerequisite of the using or accessing this content

does not offer to the distributor and licensee indirect financial profit (e.g. ads) from websites

The owner can provide the licensee a separate license for commercial use upon request.



Notes Preservation

Any reproduction or modification of this material must include:

- the Reference Note
- the License Note
- the Notes Preservation statement
- the Third Parties' Works Note (if exists)

as well as the accompanying hyperlinks.

