Beggs - Brill model

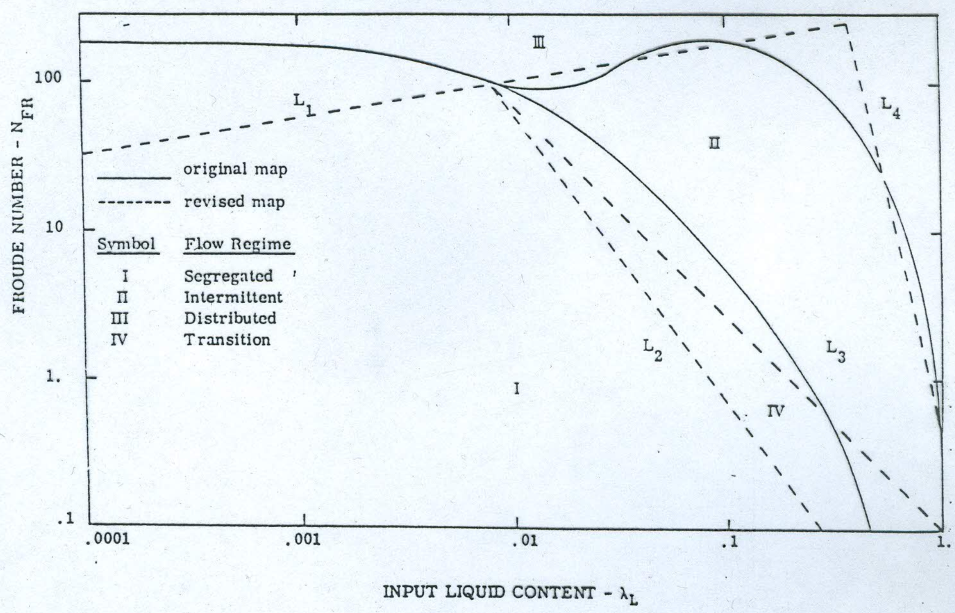
$$N_{FR} = \frac{V^2}{gD}$$

$$L_1 = 316\lambda_L^{0.302}$$

$$L_2 = 0.0009252\lambda_L^{-2.4684}$$

$$L_3 = 0.1\lambda_L^{-1.4516}$$

$$L_4 = 0.5\lambda_L^{-6.738}$$



HORIZONTAL FLOW PATTERN MAP

where:

$$\lambda_L = \frac{V_G}{V} \qquad V = V_G + V_L$$

* Segregated flow pattern

$\lambda_L < 0.01$ *and* $N_{FR} < L_1$

or

$\lambda_L > 0.01$ *and* $N_{FR} < L_2$

* Intermittent flow pattern

$0.01 < \lambda_L < 0.4$ *and* $L_3 < N_{FR} < L_1$

or

$\lambda_L > 0.4$ *and* $L_3 < N_{FR} < L_4$

* Distributed flow pattern

$\lambda_L < 0.4$ *and* $N_{FR} > L_1$

or

$\lambda_L > 0.4$ *and* $N_{FR} > L_4$

* Transition

$\lambda_L > 0.01$ *and* $L_2 < N_{FR} < L_3$

Taitel - Dukler model for vertical pipes

a. Criterion for existence of bubble flow

$$\left[\frac{\rho_L^{2} \cdot g \cdot D^2}{(\rho_L-\rho_G)\cdot\sigma}\right]^{1/4} \leq 4.36$$

b. Criterion for the transition from bubble to slug flow

$$V_L = 3.0V_G - 1.15\left[\frac{g(\rho_L-\rho_G)\cdot\sigma}{\rho_L^2}\right]^{1/4}$$

c. Transition from bubble to dispersed bubble flow

$$V = 4.0 \left\{ \frac{D^{0.429}\cdot(\sigma/\rho_L)^{0.089}}{\nu_L^{0.072}} \cdot \left[\frac{g(\rho_L-\rho_G)}{\rho_L}\right]^{0.446} \right\}$$

d. Transition from dispersed bubble to slug flow

$$V_L = \frac{0.48}{0.52}\cdot V_G - 0.48 * 1.53\left[\frac{g(\rho_L-\rho_G)\cdot\sigma}{\rho_L^2}\right]^{1/4}$$

e. Transition from slug to churn flow

$$\frac{l_e}{D} = 40.6\left(\frac{V}{\sqrt{g}}+0.22\right)$$

f. Transition to Annular flow

$$\frac{V_G \cdot \rho_G^{1/2}}{\left[\sigma g(\rho_L-\rho_G)\right]^{1/4}} = 3.1$$

Taitel - Dukler model for horizontal and inclined pipes

$$\tilde{h}_L = \frac{h_L}{D}$$

$$\tilde{A}_L = \frac{A_L}{D^2} = 0.25\left[\pi - \cos^{-1}(2\tilde{h}_L - 1) + (2\tilde{h}_L - 1)\sqrt{1-(2\tilde{h}_L - 1)^2}\right]$$

$$\tilde{A}_G = \frac{A_G}{D^2} = 0.25\left[\cos^{-1}(2\tilde{h}_L - 1) - (2\tilde{h}_L - 1)\sqrt{1-(2\tilde{h}_L - 1)^2}\right]$$

$$\tilde{P}_i = \frac{P_i}{D} = \sqrt{1-(2\tilde{h}_L - 1)^2}$$

$$\tilde{u}_G = \frac{u_G}{V_G} = \frac{\tilde{A}}{\tilde{A}_G} \qquad \tilde{u}_L = \frac{u_L}{V_L} = \frac{\tilde{A}}{\tilde{A}_L}$$

$$\tilde{A} = \frac{A}{D^2}$$

$$\tilde{P}_L = \frac{P_L}{D} = \pi - \cos^{-1}(2\tilde{h}_L - 1) \qquad \tilde{P}_G = \frac{P_G}{D} = \cos^{-1}(2\tilde{h}_L - 1)$$

$$\tilde{D}_L = \frac{4A_L}{P_L \cdot D} = \frac{4\tilde{A}_L}{\tilde{P}_L} \qquad \tilde{D}_G = \frac{4A_G}{D(P_G + P_i)} = \frac{4\tilde{A}_G}{\tilde{P}_G + \tilde{P}_i}$$

$$\tau_{OG}\cdot\frac{P_G}{A_G}-\tau_{OL}\cdot\frac{P_L}{A_L}+\tau_i P_i\left(\frac{1}{A_L}+\frac{1}{A_G}\right)-(\rho_L-\rho_G)g\sin\alpha=0$$

$$\tau_{OL} = \frac{f_L\rho_L u_L^2}{2} \qquad \tau_{OG} = \frac{f_G\cdot\rho_G\cdot u_G^2}{2}$$

$$\tau_i = \frac{f_i\rho_G\cdot(u_G-u_L)^2}{2}$$

$$f_L = C_L\left(\frac{D_L u_L\rho_L}{\mu_L}\right)^{-n} \qquad f_G = C_G\left(\frac{D_G u_G\rho_G}{\mu_G}\right)^{-m}$$

$$x^2\cdot(\tilde{u}_L\cdot\tilde{D}_L)^{-n}\cdot\tilde{u}_L^2\frac{\tilde{P}_L}{\tilde{A}_L}-(\tilde{u}_G\cdot\tilde{D}_G)^{-m}\cdot\tilde{u}_G^2\cdot\left(\frac{\tilde{P}_G}{\tilde{A}_G}+\frac{\tilde{P}_L}{\tilde{A}_L}+\frac{\tilde{P}_i}{\tilde{A}_G}\right)-4Y=0$$

$$x^2 = \frac{(dP_F/dz)_L}{(dP_F/dz)_G} \qquad Y = \frac{-(\rho_L-\rho_G)g\sin\alpha}{(dP_F/dz)_G}$$

$$k^2 = F^2\cdot Re_L = \frac{\rho_G\cdot V_G^2}{(\rho_L-\rho_G)Dg\cos\alpha}\cdot\frac{DV_L}{\nu_L}$$

$$F^2\cdot\frac{1}{C_2^2}\cdot\frac{\tilde{u}_G\cdot d\tilde{A}_L/d\tilde{h}_L}{\tilde{A}_G}\geq 1$$

$$F = \sqrt{\frac{\rho_G}{\rho_L-\rho_G}}\cdot\frac{u_G}{\sqrt{Dg\cos\alpha}}$$

$$k > \frac{20}{\tilde{u}_G\cdot\sqrt{\tilde{u}_L}}$$

$$\frac{d\tilde{A}_L}{d\tilde{h}_L} = \sqrt{1-(2\tilde{h}_L-1)^2}$$

$$T^2 < \frac{8\tilde{A}_G}{\tilde{P}_i\cdot\tilde{u}_L^2\cdot(\tilde{u}_L\cdot\tilde{D}_L)^{-n}}$$

$$C_2 = 1-\tilde{h}_L$$

$$T = \left[\frac{(dP_F/dz)_L}{(\rho_L-\rho_G)g\cos\alpha}\right]^{1/2}$$

Wallis model

$$u_G = C_o V + u_s \quad where \quad C_o = 1.2 \quad for \quad slug \quad flow$$

$$V_i = \frac{[D^3 g(\rho_L - \rho_G)\rho_L]^{1/2}}{\mu_L}$$

$$E_o = \frac{gD^2(\rho_L - \rho_G)}{\sigma}$$

$$\frac{u_S}{\sqrt{gD}} = 0.345[1-\exp(0.029V_i)]\cdot\left(1-\exp\frac{3.37-E_o}{m}\right)$$

$$m = 10 \quad for \quad V_i > 250$$

$$m = 69\cdot V_i^{-0.35} \quad for \quad 18 < V_i < 250$$

$$m = 25 \quad for \quad V_i < 18$$

$$u_s = 0.345\sqrt{gD}$$

$$e_G = \frac{V_G}{u_G}$$

Mukherjee - Brill model

$$H_L = \exp[(C_1 + C_2 \sin\alpha + C_3 \sin^2\alpha + C_4 N_L^2)] \cdot \frac{N_{gV}^{C_5}}{N_{LV}^{C_6}}$$

$$N_L = \mu_L \cdot \left(\frac{g}{\rho_L \cdot \sigma^3}\right)^{0.25}$$

$$N_{gv} = V_G \cdot \left(\frac{\rho_L}{g \cdot \sigma}\right)^{0.25}$$

$$N_{LV} = V_L \cdot \left(\frac{\rho_L}{g \cdot \sigma}\right)^{0.25}$$

C_1, C_2, C_3, C_4, C_5, C_6 : constants

Taitel - Dukler model

$$\epsilon_G = \frac{1}{\pi}\left[\cos^{-1}(2\tilde{h}_L - 1) - (2\tilde{h}_L - 1) \cdot \sqrt{1 - (2\tilde{h}_L - 1)^2}\right]$$

$$H_L = 1 - \epsilon_G$$