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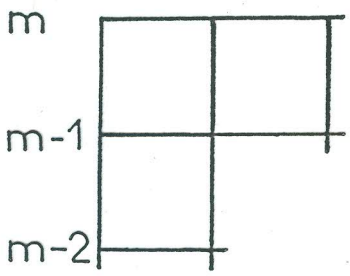
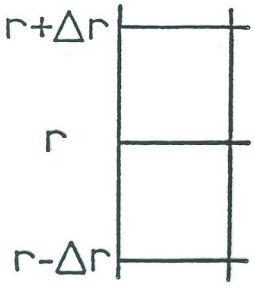
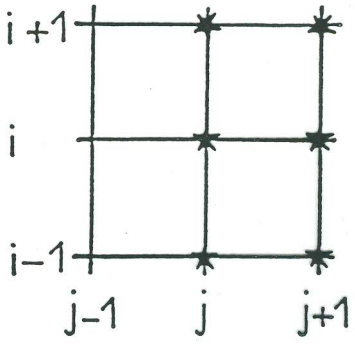
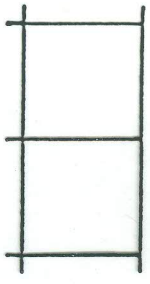


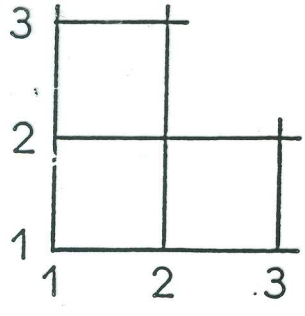
$r=(i-1)\Delta r \qquad x=(j-1)\Delta x$

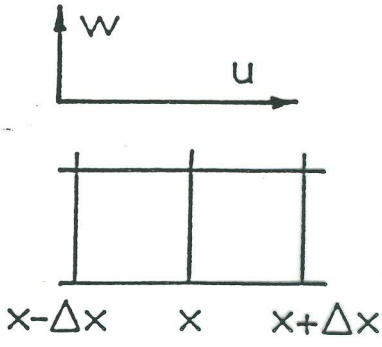
$H=(m-1)\Delta r \qquad L=(n-1)\Delta x$

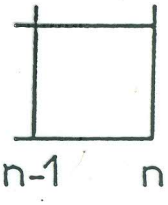












• Εκφράσεις πεπερασμένων διαφορών

$$\left(\frac{\partial M}{\partial x}\right)_{i,j+1} = \frac{M_{i,j+1}-M_{i,j}}{\Delta x}$$

$$\left(\frac{\partial M}{\partial r}\right)_{i,j+1} = \frac{\theta(M_{i+1,j+1}-M_{i-1,j+1})}{2\Delta r} + \frac{(1-\theta)(M_{i+1,j}-M_{i-1,j})}{2\Delta r}$$

$$\left(\frac{\partial^2 M}{\partial r^2}\right)_{i,j+1} = \frac{\theta(M_{i+1,j+1}-2M_{i,j+1}+M_{i-1,j+1})}{(\Delta r)^2} + \frac{(1-\theta)(M_{i+1,j}-2M_{i,j}+M_{i-1,j})}{(\Delta r)^2}$$

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$$Ar_{i,j}\,ur_{i+1,j+1} + Br_{i,j}\,ur_{i,j+1} + Cr_{i,j}\,ur_{i-1,j+1} = Dr_{i,j}$$

$$Ar_{i,j} = \left[\frac{\lambda\Delta r\bar{\rho}_r w r_{i,j}}{2} - \frac{\lambda\mu r_{i,j}}{Re_R} - \frac{\lambda}{4Re_R}\left(\mu r_{i+1,j} - \mu r_{i-1,j}\right) - \right.$$

$$\left. - \frac{\lambda k\,\mu r_{i,j}}{2(i-1)Re_R} + M_1\,\frac{J_y Z\rho_g (w_g - w_p)_{i,j}\lambda\Delta r}{2}\right]\theta = A^{*}_{r_{i,j}}\theta$$

$$Br_{i,j} = \bar{\rho}_r\, ur_{i,j} + \frac{2\lambda\mu r_{i,j}\theta}{Re_R}$$

$$Cr_{i,j} = \left[-\frac{\lambda\Delta r\bar{\rho}_r w r_{i,j}}{2} - \frac{\lambda\mu r_{i,j}}{Re_R} + \frac{\lambda}{4Re_R}\left(\mu r_{i+1,j} - \mu r_{i-1,j}\right) + \right.$$

$$\left. + \frac{\lambda\, k\,\mu r_{i,j}}{2(i-1)Re_R} - M_1\,\frac{J_y\, Z\rho_g (w_g - w_p)\lambda\,\Delta\, r}{2}\right]\theta = C^{*}_{r_{i,j}}\theta$$

$$Dr_{i,j} = -p r_{i,j+1} + p r_{i,j} - A^{*}_{r_{i,j}}\,ur_{i+1,j}\,(1-\theta) +$$

$$+ \left[Br_{i,j} - \frac{2\mu r_{i,j}\lambda}{Re_R}\right] ur_{i,j} - C^{*}_{r_{i,j}}\,ur_{i-1,j}\,(1-\theta) +$$

$$+ M_3\,\frac{3c_{Dx}}{8r_p}\,Z\rho_g\,(u_g - u_p)^2_{i,j}\,\Delta x +$$

$$+ M_2\,\frac{J_y Z\rho_g (w_g - w_p)_{i,j}\lambda\Delta r}{2}\left(ug_{i+1,j+1} - ug_{i-1,j+1}\right)\theta +$$

$$+ M_2\,\frac{J_y Z\rho_g (w_g - w_p)_{i,j}\lambda\Delta r}{2}\left(ug_{i+1,j} - ug_{i-1,j}\right)(1-\theta)$$

● Σύστημα Γραμμικών Αλγεβρικών Εξισώσεων

$$i = 1 \quad Rr_{1,j}\,ur_{2,j+1} + Sr_{1,j}\,ur_{1,j+1} = Tr_{1,j}$$

$$i = 2 \quad Ar_{2,j}\,ur_{3,j+1} + Br_{2,j}\,ur_{2,j+1} + Cr_{2,j}\,ur_{1,j+1} = Dr_{2,j}$$

$$i = 3 \quad Ar_{3,j}\,ur_{4,j+1} + Br_{3,j}\,ur_{3,j+1} + Cr_{3,j}\,ur_{2,j+1} = Dr_{3,j}$$

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$$i=m-2 \quad Ar_{m-2,j}\,ur_{m-1,j+1} + Br_{m-2,j}\,ur_{m-2,j+1} + Cr_{m-2,j}\,ur_{m-3,j+1} = Dr_{m-2,j}$$

$$i=m-1 \quad Br_{m-1,j}\,ur_{m-1,j+1} + Cr_{m-1,j}\,ur_{m-2,j+1} = Dr_{m-1,j}$$

● Αλγοριθμος Richtmyer

$$\boxed{ur_{i,j+1} = Er_{i,j}\,ur_{i+1,j+1} + Fr_{i,j}}$$

$$Er_{i,j} = \frac{-Ar_{i,j}}{Br_{i,j} + Cr_{i,j}\,Er_{i-1,j}} \qquad Er_{1,j} = -\frac{Rr_{1,j}}{Sr_{1,j}}$$

$$Fr_{i,j} = \frac{Dr_{i,j} - Cr_{i,j}\,Fr_{i-1,j}}{Br_{i,j} + Cr_{i,j}\,Er_{i-1,j}} \qquad Fr_{1,j} = \frac{Tr_{1,j}}{Sr_{1,j}}$$

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● εξίσωση ορμης r -διεύθυνση

$$\frac{\partial p}{\partial r} = -\frac{1}{Fr_R} Z_p(\rho_{sp}-\rho_g) - \frac{1}{Fr_R}\rho_g$$

$$p_{i,j+1} = p_{i-1,j+1} - p_{i,j} + p_{i-1,j} - \frac{\Delta r}{Fr_R}\left[Zp_{i,j}(\rho_{sp}-\rho_g)+\rho_g\right]$$

$$Zg_{i,j+1} = Zg_{i+1,j+1} + Zg_{i+1,j} - Zg_{i,j} +$$

$$+ \left(\frac{Z_g}{p}\right)_{i,j}(p_{i+1,j+1} - p_{i,j+1} + p_{i+1,j} - p_{i,j}) +$$

$$+ \frac{2\Delta r}{p_{i,j}}\left(\frac{3}{8r_p} c_{D_r} Z_p \rho_g (w_g-w_p)^2\right)_{i,j} + \frac{2\Delta r}{p_{i,j}}\frac{\rho_g}{Fr_R} +$$

$$+ \frac{J_y Z_p \rho_g}{p_{i,j}}(u_g-u_p)_{i,j}(ug_{i+1,j+1} - ug_{i,j+1} + ug_{i+1,j} - ug_{i,j})$$

• εξίσωση συνέχειας

$$\frac{\partial(\bar{\rho}_r u_r)}{\partial x} + \frac{\partial(\bar{\rho}_r w_r)}{\partial r} + \frac{k\bar{\rho}_r w_r}{r} = 0$$

Υπολογιστικο σχημα Paskonov

$$\left(\frac{\partial M}{\partial x}\right)_p = \frac{M_{i,j+1} - M_{i,j} + M_{i-1,j+1} - M_{i-1,j}}{2\,\Delta x}$$

$$\left(\frac{\partial M}{\partial r}\right)_p = \frac{M_{i,j+1} - M_{i-1,j+1} + M_{i,j} - M_{i-1,j}}{2\,\Delta r}$$

$$\left(\frac{kM}{r}\right)_p = \frac{\left(\frac{kM}{r}\right)_{i,j} + \left(\frac{kM}{r}\right)_{i-1,j} + \left(\frac{kM}{r}\right)_{i,j+1} + \left(\frac{kM}{r}\right)_{i-1,j+1}}{4}$$

Εξίσωση πεπερασμενων διαφορων

$$(\bar{\rho}_r w_r)_{i,j+1} = -(\bar{\rho}_r w_r)_{i,j} + \frac{(i-1)\left[2(i-2)-k\right]}{(i-2)\left[2(i-1)+k\right]}\left[(\bar{\rho}_r w_r)_{i-1,j+1} + (\bar{\rho}_r w_r)_{i-1,j}\right] - \frac{2(i-1)\,\Delta r}{(2(i-1)+k)\Delta x}\left[(\bar{\rho}_r u_r)_{i,j+1} - (\bar{\rho}_r u_r)_{i,j} + (\bar{\rho}_r u_r)_{i-1,j+1} - (\bar{\rho}_r u_r)_{i-1,j}\right]$$

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