Subject	Date	Project
ΤΥΠΟΛΟΓΙΟ ΜΗΧΑΝΙΚΗ ΡΕΥΣΤΩΝ ΙΙ	Author	Report

ΔΥΝΑΜΙΚΗ ΡΟΗ

Ἡ ἐξίσωση δυναμικοῦ ταχύτητας

$$\vec{w} = \text{grad}\ \Phi$$

μέ συνιστῶσες

$$u = \frac{\partial \Phi}{\partial x}, \quad \upsilon = \frac{\partial \Phi}{\partial y} \quad \text{καί} \quad w = \frac{\partial \Phi}{\partial z}$$

Ροϊκή Συνάρτηση

$$w_r = \frac{1}{r}\frac{\partial \Psi}{\partial \theta}$$

$$w_\theta = -\frac{\partial \Psi}{\partial r}$$

ἀστρόβιλη ροή

$$\frac{\partial w}{\partial y} - \frac{\partial \upsilon}{\partial z} = 0$$

$$\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$$

$$\frac{\partial \upsilon}{\partial x} - \frac{\partial u}{\partial y} = 0$$

Ἡ ροϊκή συνάρτηση $u = \frac{\partial \Psi}{\partial y}$, $\upsilon = -\frac{\partial \Psi}{\partial x}$

Laplace $\text{div}\ \vec{w} = \text{div}\ (\text{grad}\ \Phi) = 0$ $\qquad \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$

ΟΡΙΑΚΟ ΣΤΡΩΜΑ

	Λύση Blasius	Λύση Pohlhausen
πάχος οριακού στρώματος	$\delta = \frac{4{,}92\,x}{\sqrt{Re_x}} \approx \frac{5x}{\sqrt{Re_x}}$	$\delta = \sqrt{\frac{2\beta_1}{\alpha_1}}\ \frac{x}{\sqrt{Re_x}}$
Τοπικός συντελεστής αντίστασης	$c_f = \frac{\tau_o}{\rho u_\infty^2/2} = \frac{0{,}664}{\sqrt{Re_x}}$	$c_f = \frac{\tau_o(x)}{\rho\ u_\infty^2/2}$
Διατμητική τάση	$\tau_o(x) = 0.332\ \mu\ u_\infty \frac{\sqrt{Re_x}}{x}$	$\tau_o(x) = \sqrt{\frac{\alpha_1\beta_1}{2}}\ \mu\ \frac{u_\infty}{x}\ \sqrt{Re_x}$
Ολικός συντελεστής αντίστασης και για τις δυο πλευρές	$c_f = \frac{2D}{A\rho u_\infty^2/2} = \frac{1.328}{\sqrt{Re_L}}$	
Αντίσταση της μιας πλευράς	$D = 0.664\ \sqrt{u_\infty^3\, \mu\, \rho\, L}\ b$	$D = b\sqrt{2\alpha_1\beta_1}\ \sqrt{\mu\,\rho\, L\, u_\infty^3}$
Πάχος μετάθεσης	$\delta_1 = \frac{1.7208\,x}{\sqrt{Re_x}}$	$\delta_1 = \alpha_2\ \delta$
Πάχος απώλειας ορμής	$\delta_2 = \frac{0.664\,x}{\sqrt{Re_x}}$	$\delta_2 = \alpha_1\ \delta$

ὅπου:

$$\alpha_1 = \int_0^1 f(\eta)\,(1 - f(\eta))\,d\eta \qquad \alpha_2 = \int_0^1 (1 - f(\eta))\,d\eta \qquad \beta_1 = f'(\eta)\,|_{\eta=0}$$

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ΕΞΙΣΩΣΕΙΣ ΣΥΝΕΧΕΙΑΣ ΚΑΙ ΟΡΜΗΣ (Navier-Stokes)

Καρτεσιανές συντεταγμένες

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

x:
$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x$$

y:
$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) + \rho g_y$$

z:
$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \rho g_z$$

Κυλινδρικές συντεταγμένες

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

r:
$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho g_r$$

θ:
$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta$$

z:
$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Νόμος τριβής Newton

$$\tau = \mu \frac{du}{dy}$$

ΔΥΝΑΜΕΙΣ ΣΕ ΣΩΜΑΤΑ

άνωση

$$L = c_L A q_\infty = c_L A \frac{\rho u_\infty^2}{2}$$

αντίσταση

$$D = c_D A \frac{\rho u_\infty^2}{2}$$

στατική - άνωση

$$F_A = \int_V \rho g \, dV = \rho g V$$