Εσωτερικός διαγωνισμός επιλογής για τον IMC 2023
Τμήμα Μαθηματικών Πατρών

26/05/23

Διάρκεια εξέτασης: 3 ώρες.

Problem 1. Let $\exp(x) = e^x$. Estimate the product

$$\prod_{n=3}^{\infty} \exp\left(\frac{n^2-1}{n^4-5n^2+4}\right).$$

Problem 2. Let n be a positive integer which is not divisible by 2 or 5. Prove that there is a multiple of n consisting entirely of ones.

Problem 3. For $n \geq 1$ calculate the integral

$$\int_{-n}^{n} \frac{\cos(nx)}{e^x+1} dx.$$

Problem 4. Let

$$A = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}.$$

Calculate the matrix A^n.

Problem 5. For every $n \geq 1$ prove that

$$\int_0^{\pi} (\sin x)^{2n} dx \geq \frac{3}{\pi} \left(\int_0^{\frac{\pi}{2}} (\sin x)^n dx \right)^2.$$