

## ΟΡΙΖΟΥΣΕΣ

### ΟΡΙΖΟΥΣΑ 2x2

$$A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

### ΟΡΙΖΟΥΣΑ 3x3 ΑΝΑΠΤΥΓΜΑ ΩΣ ΠΡΟΣ ΤΑ ΣΤΟΙΧΕΙΑ ΤΗΣ 1ης ΓΡΑΜΜΗΣ

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

## ΜΕΘΟΔΟΣ CRAMER

$$a_1x + b_1y + c_1z = d_{11}$$

$$a_2x + b_2y + c_2z = d_{21}$$

$$a_3x + b_3y + c_3z = d_{31}$$

$$D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$$

## ΠΑΡΑΓΩΓΟΙ

### ΠΑΡΑΓΩΓΟΙ ΒΑΣΙΚΩΝ ΣΥΝΑΡΤΗΣΕΩΝ

**f(x)**

**f'(x)**

c (σταθερά)

0

x

1

$x^a, a \in \mathbb{R}^*, x > 0$

$\alpha x^{a-1}$

sin x

cos x

cos x

-sin x

tan x

$\frac{1}{\cos^2 x}$

tagx

$-\frac{1}{\sin^2 x}$

$e^x$

$e^x$

ln x

$\frac{1}{x}, x \neq 0$

$\sqrt{x}, x > 0$

$\frac{1}{2\sqrt{x}}$

$\frac{1}{x}, x \neq 0$

$-\frac{1}{x^2}$

arctan x

$\frac{1}{1+x^2}$

### ΠΑΡΑΓΩΓΟΙ ΣΥΝΘΕΤΩΝ ΣΥΝΑΡΤΗΣΕΩΝ

**f(g(x))**

**f'(g(x))'**

cf(x)

cf'(x)

$f(x)^a, a \in \mathbb{R}^*, x > 0$

$\alpha f(x)^{a-1} f'(x)$

sin f(x)

cos f(x) f'(x)

cos f(x)

-sin f(x) f'(x)

tan f(x)

$\frac{1}{\cos^2 f(x)} f'(x)$

tagf(x)

$-\frac{1}{\sin^2 f(x)} f'(x)$

$e^{f(x)}$

$e^{f(x)} f'(x)$

lnf(x)

$\frac{1}{f(x)} f'(x)$

$\sqrt{f(x)}, x > 0$

$\frac{1}{2\sqrt{f(x)}} f'(x)$

$\frac{1}{f(x)}, x \neq 0$

$-\frac{1}{f(x)^2} f'(x)$

arctan f(x)

$\frac{1}{1+f(x)^2} f'(x)$

## ΚΑΝΟΝΕΣ ΠΑΡΑΓΩΓΙΣΗΣ

$$(c \cdot f)'(x) = cf'(x)$$

$$(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$$

$$(f + g)'(x) = f'(x) + g'(x)$$

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$(f - g)'(x) = f'(x) - g'(x)$$

## ΟΛΟΚΛΗΡΩΜΑΤΑ

### ΠΑΡΑΓΟΥΣΕΣ ΒΑΣΙΚΩΝ ΣΥΝΑΡΤΗΣΕΩΝ

### ΠΑΡΑΓΟΥΣΕΣ ΣΥΝΘΕΤΩΝ ΣΥΝΑΡΤΗΣΕΩΝ

$$\int_{\alpha}^{\beta} 1 dx = [x]_{\alpha}^{\beta} = \beta - \alpha$$

$$\int_{\alpha}^{\beta} x^{\kappa} dx = \left[ \frac{x^{\kappa+1}}{\kappa+1} \right]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \frac{1}{x} dx = [\ln x]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} e^x dx = [e^x]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \sigma \upsilon \nu x dx = [\eta \mu x]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \eta \mu x dx = [-\sigma \upsilon \nu x]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \frac{1}{\sigma \upsilon \nu^2 x} dx = [\varepsilon \varphi x]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \frac{1}{\eta \mu^2 x} dx = [-\sigma \varphi x]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \frac{1}{2\sqrt{x}} dx = [\sqrt{x}]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \frac{1}{x^2} dx = \left[ -\frac{1}{x} \right]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \frac{1}{1+x^2} dx = [\arctan x]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} f(x)^{\kappa} f'(x) dx = \left[ \frac{(f(x))^{\kappa+1}}{\kappa+1} \right]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \frac{1}{f(x)} f'(x) dx = [\ln f(x)]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} e^{f(x)} f'(x) dx = [e^{f(x)}]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \sigma \upsilon \nu f(x) f'(x) dx = [\eta \mu f(x)]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \eta \mu f(x) f'(x) dx = [-\sigma \upsilon \nu f(x)]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \frac{1}{\sigma \upsilon \nu^2 f(x)} f'(x) dx = [\varepsilon \varphi f(x)]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \frac{1}{\eta \mu^2 f(x)} f'(x) dx = [-\sigma \varphi f(x)]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \frac{1}{2\sqrt{f(x)}} f'(x) dx = [\sqrt{f(x)}]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \frac{1}{f(x)^2} f'(x) dx = \left[ -\frac{1}{f(x)} \right]_{\alpha}^{\beta}$$

$$\int_{\alpha}^{\beta} \frac{1}{1+f(x)^2} f'(x) dx = [\arctan f(x)]_{\alpha}^{\beta}$$

### ΟΡΙΣΜΟΣ

$$\int_{\alpha}^{\beta} f(x) dx = [F(x)]_{\alpha}^{\beta} = F(\beta) - F(\alpha)$$

### ΙΔΙΟΤΗΤΕΣ

$$\int_{\alpha}^{\beta} c dx = c(\beta - \alpha)$$

$$\int_{\alpha}^{\alpha} f(x) dx = 0$$

### ΠΑΡΑΓΟΝΤΙΚΗ ΟΛΟΚΛΗΡΩΣΗ

$$\int_{\alpha}^{\beta} f(x) dx = -\int_{\beta}^{\alpha} f(x) dx$$

$$\int_{\alpha}^{\beta} f(x) dx = \int_{\alpha}^{\gamma} f(x) dx + \int_{\gamma}^{\beta} f(x) dx$$

$$\int_{\alpha}^{\beta} f'(x) g(x) dx = [f(x) g(x)]_{\alpha}^{\beta} - \int_{\alpha}^{\beta} f(x) g'(x) dx$$

$$\int_{\alpha}^{\beta} [\lambda f(x) + \mu g(x)] dx = \lambda \int_{\alpha}^{\beta} f(x) dx + \mu \int_{\alpha}^{\beta} g(x) dx$$

### ΜΙΓΑΔΙΚΟΙ

**ΚΑΡΤΕΣΙΑΝΗ ΜΟΡΦΗ**  $z_1 = x_1 + y_1 i$     **ΣΥΖΥΓΤΗΣ**  $\bar{z} = x - y i$

**ΠΡΟΣΘΕΣΗ**     $z_1 + z_2 = (x_1 + y_1 i) + (x_2 + y_2 i)$

**ΑΦΑΙΡΕΣΗ**     $z_1 - z_2 = (x_1 + y_1 i) - (x_2 + y_2 i)$

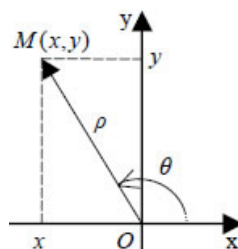
**ΠΟΛΙΚΗ ΜΟΡΦΗ**     $z = \rho | \theta$     **ΣΥΖΥΓΤΗΣ**     $z = \rho | -\theta$

**ΠΟΛΛΑΠΛΑΣΙΑΜΟΣ**     $z_1 z_2 = (\rho_1 \cdot \rho_2) | \theta_1 + \theta_2$

**ΔΙΑΙΡΕΣΗ**     $z_1 / z_2 = (\rho_1 / \rho_2) | \theta_1 - \theta_2$

**ΔΥΝΑΜΕΙΣ  
ΦΑΝΤΑΣΤΙΚΗΣ  
ΜΟΝΑΔΑΣ**

$$i^n = \begin{cases} 1 & , \alpha \nu \nu = 4\kappa \\ i & , \alpha \nu \nu = 4\kappa + 1 \\ -1 & , \alpha \nu \nu = 4\kappa + 2 \\ -i & , \alpha \nu \nu = 4\kappa + 3 \end{cases}$$



$$\rho = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \left( \frac{y}{x} \right)$$