$f(x,y) \rightarrow$ Συνάρτηση 2 μεταβλητών.

$$f_x \equiv \frac{\partial f(x,y)}{\partial x} \quad , \quad f_y \equiv \frac{\partial f(x,y)}{\partial y}$$

1ες μερικές παράγωγοι

$$f_{xx} \equiv \frac{\partial^2 f(x,y)}{\partial x^2} \quad , \quad f_{yy} \equiv \frac{\partial^2 f(x,y)}{\partial y^2}$$

2ες μερικές παράγωγοι.

$$\cancel{\frac{d f(x,y)}{dx}} \Rightarrow \frac{\partial f(x,y)}{\partial x}$$

$$f_{xy} \equiv \frac{\partial}{\partial x}\left(\frac{\partial f(x,y)}{\partial y}\right) \quad , \quad f_{yx} \equiv \frac{\partial}{\partial y}\left(\frac{\partial f(x,y)}{\partial x}\right)$$

ΑΣΚΗΣΗ

Έστω $f(x,y) = 2xy^2$, να βρεθούν οι πρώτες μερικές παράγωγοι?

$$f_x \equiv \frac{\partial f(x,y)}{\partial x} = \frac{\partial(2xy^2)}{\partial x} = 2y^2\frac{dx}{dx} = \boxed{2y^2}$$

$$f_y \equiv \frac{\partial f(x,y)}{\partial y} = \frac{\partial(2xy^2)}{\partial y} = 2x\,2y = \boxed{4xy}$$

$$f_{xx} \equiv \frac{\partial}{\partial x}\left(\frac{\partial f(x,y)}{\partial x}\right) = \frac{\partial}{\partial x}(2y^2) = \boxed{0}$$

$$f_{yy} \equiv \frac{\partial}{\partial y}\left(\frac{\partial f(x,y)}{\partial y}\right) = \frac{\partial}{\partial y}(4xy) = 4x\frac{\partial(y)}{\partial y} = \boxed{4x}$$

$$(a \cdot f(x))' = a \cdot f'(x)$$

$$(a + f(x))' = \cancel{a'}^{\,0} + f'(x)$$

$$(c)' = 0$$

ΑΣΚΗΣΗ

$$f(x,y) = x^2 + xy + y^2$$

$$\begin{cases} f_x \longrightarrow \\ f_y \longrightarrow \\ f_{xx} \longrightarrow \\ f_{yy} \longrightarrow \end{cases}$$

$$f_x = \frac{\partial f(x,y)}{\partial x} = \frac{\partial(x^2+xy+y^2)}{\partial x} = \frac{\partial}{\partial x}(x^2) + \frac{\partial(xy)}{\partial x} + \frac{\partial(y^2)}{\partial x} =$$

$$= \boxed{2x + y} \Leftarrow$$

$$f_y = \frac{\partial f(x,y)}{\partial y} = \boxed{x + 2y} \quad \checkmark$$

$$\left\{\begin{array}{l} f_{xx} = \frac{\partial^2 f(x,y)}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f(x,y)}{\partial x}\right) = \boxed{2} \\ f_{yy} = \ldots\ldots\ldots = \boxed{2} \end{array}\right\}$$

$$\boxed{f_{xx} = f_{yy}}$$

τυχαίο γεγονός

$$\left.\begin{array}{l} f_{xy} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}(x+2y) = 1 \\ f_{yx} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}(2x+y) = 1 \end{array}\right\}$$

$$\boxed{f_{xy} = f_{yx}}$$

Enw $\quad f(x,y) = \underbrace{e^x}\cdot\underbrace{\sin(xy)}$

$$\left(f(x)\cdot g(x)\right)' = f'(x)\cdot g(x) + f(x)\,g'(x)$$

$$f_x \equiv \frac{\partial f(x,y)}{\partial x} = \frac{\partial (e^x)}{\partial x}\cdot \sin(xy) + e^x\cdot \frac{\partial(\sin(xy))}{\partial x} =$$

$$= e^x\cdot\sin(xy) + e^x \cos(xy)\cdot\frac{\partial(xy)}{\partial x} =$$

$$= e^x\cdot\sin(xy) + y\cdot e^x\cdot\cos(xy) = e^x\left(\sin(xy)+y\cdot\cos(xy)\right)$$

$$f_y \equiv \frac{\partial f(x,y)}{\partial y} = \frac{\partial}{\partial y}\left(e^x\cdot\sin(xy)\right) = e^x\,\frac{\partial}{\partial y}\left(\sin(xy)\right) =$$

$$= e^x\cdot\cos(xy)\cdot\frac{\partial}{\partial y}(xy) = \boxed{x\cdot e^x\cdot\cos(xy)}$$

$$\boxed{(ay)' = a\cdot y'}$$

$$f_{xx} = \frac{\partial}{\partial x}\left[e^x\left(\sin(xy)+y\cdot\cos(xy)\right)\right] =$$

$$= \frac{\partial(e^x)}{\partial x}\cdot\left[\sin(xy)+y\cdot\cos(xy)\right] + e^x\cdot\frac{\partial}{\partial x}\left(\sin(xy)+y\cos(xy)\right) =$$

$$= e^x\left[\sin(xy)+y\cdot\cos(xy)\right] + e^x\left(\frac{\partial}{\partial x}(\sin(xy)) + \frac{\partial}{\partial x}(y\cdot\cos(xy))\right) =$$

$$= e^x\left[\sin(xy)+y\cdot\cos(xy)\right] + e^x\left(\cos(xy)\,\frac{\partial}{\partial x}(xy) + y\,(-\sin(xy))\cdot\frac{\partial(xy)}{\partial x}\right)$$

$$= e^x\left[\sin(xy)+y\cdot\cos(xy)\right] + e^x\left(\cos(xy)\cdot y - y^2\sin(xy)\right)$$

$f_{yy} = \ldots\ldots, \quad f_{xy} = \ldots\ldots, \quad f_{yx} = \ldots\ldots$

$$f_{yy} = \frac{\partial}{\partial y}\left(xe^x \cdot \cos(xy)\right) = xe^x \frac{\partial}{\partial y}\left(\cos(xy)\right) = xe^x\left(-\sin(xy)\right) \cdot \frac{\partial}{\partial y}(xy) =$$

$$= -x^2 e^x \sin(xy)$$

$$f_{xy} = \frac{\partial}{\partial x}\left(\frac{\partial f(x,y)}{\partial y}\right) = \frac{\partial}{\partial x}\left(xe^x \cos(xy)\right) = \boxed{\frac{\partial}{\partial x}(x \cdot e^x)} \cdot \cos(xy) + xe^x \cdot \frac{\partial}{\partial x}(\cos(xy))$$

$(*)$ f g

$$= \left(1 \cdot e^x + x \cdot e^x\right) \cdot \cos(xy) + x \cdot e^x \left(-\sin(xy)\right) \cdot y =$$

$$= e^x(1+x) \cdot \cos(xy) - x \cdot y\, e^x \cdot \sin(xy) =$$

$$= e^x\left[(1+x)\cos(xy) - xy \cdot \sin(xy)\right].$$

$$f_{yx} = \frac{\partial}{\partial y}\left(\frac{\partial f(x,y)}{\partial x}\right) = \frac{\partial}{\partial y}\left[e^x\left(\sin(xy) + y \cdot \cos(xy)\right)\right] =$$

$$= e^x\left[\frac{\partial}{\partial y}(\sin(xy)) + \frac{\partial}{\partial y}(y \cdot \cos(xy))\right] =$$

$$= e^x\left[\cos(xy) \cdot x + 1 \cdot \cos(xy) + y \cdot (-\sin xy) \cdot x\right] =$$

$$= e^x\left[x \cdot \cos(xy) + \cos(xy) - xy \cdot \sin(xy)\right]$$

$(*)$ $\left(f(x)\, g(x)\right)' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$ $\Leftarrow$

Έστω $g(x,y,z) = x^2y - y^2z^3 + \sin(xy) \longrightarrow$ πρώτες μερικές παράγωγοι της $g(x,y,z)$.

g_x , g_y , $g_z \longrightarrow \dfrac{\partial g(x,y,z)}{\partial z}$.

$\dfrac{\partial g(x,y,z)}{\partial x}$ $\quad$ $\dfrac{\partial g(x,y,z)}{\partial y}$

$$g_x = \frac{\partial g(x,y,z)}{\partial x} = \frac{\partial}{\partial x}(x^2y) - \frac{\partial}{\partial x}(y^2z^3) + \frac{\partial}{\partial x}\left(\sin(xy)\right) =$$

$$= 2xy - 0 + y\cos(xy) = \boxed{2xy + y\cdot\cos(xy)}$$

$$g_y = \frac{\partial g(x,y,z)}{\partial y} = x^2 - 2yz^3 + x\cos(xy)$$

$$g_z = \frac{\partial g(x,y,z)}{\partial z} = 0 - 3y^2z^2 + 0 = \boxed{-3y^2z^2}$$

$$g_{xx} = \frac{\partial}{\partial x}\left(2xy + y\cdot\cos(xy)\right) = y\cdot\frac{\partial}{\partial x}\left(2x + \cos(xy)\right) =$$

$$= y\cdot\left(2 - \sin(xy)\,y\right) = \boxed{2y - y^2\sin(xy)}$$

$$g_{yy} = \frac{\partial}{\partial y}\left(x^2 - 2yz^3 + x\cdot\cos(xy)\right) = 0 - 2z^3 + x^2\left(-\sin(xy)\right) =$$

$$= \boxed{-2z^3 - x^2\sin(xy)}$$

$$g_{zz} = \frac{\partial}{\partial z}\left(-3y^2z^2\right) = -3y^2(2z) = \boxed{-6y^2z}$$

$$\bullet\ f(x,y) = \frac{xy\,(x^2+y^2)}{x^2+y^2+1} \quad , \quad \boxed{f_x \equiv \frac{\partial f(x,y)}{\partial x}} \ , \ \boxed{f_y = \frac{\partial f(x,y)}{\partial y}}$$

$$f_x = \frac{\partial f(x,y)}{\partial x} = \frac{\partial}{\partial x}\left[\frac{xy\,(x^2+y^2)}{x^2+y^2+1}\right] = \frac{\partial}{\partial x}\left[\frac{x^3y+xy^3}{x^2+y^2+1}\right] =$$

$$= \frac{(x^2+y^2+1)\cdot\frac{\partial}{\partial x}(x^3y+xy^3) - (x^3y+xy^3)\,\frac{\partial}{\partial x}(x^2+y^2+1)}{(x^2+y^2+1)^2} =$$

$$= \frac{(x^2+y^2+1)\cdot(3x^2y+y^3) - (x^3y+xy^3)\cdot(2x)}{(x^2+y^2+1)^2} = \ldots\ldots$$

<u>ΑΠΛΕΣ ΠΡΑΞΕΙΣ</u>

$$f_y = \frac{\partial f(x,y)}{\partial y} = \frac{(x^2+y^2+1)\,\frac{\partial}{\partial y}(x^3y+xy^3) - (x^3y+xy^3)\cdot\frac{\partial}{\partial y}(x^2+y^2+1)}{(x^2+y^2+1)^2} =$$

$$= \frac{(x^2+y^2+1)\cdot(x^3+3xy^2) - (x^3y+xy^3)\cdot(2y)}{(x^2+y^2+1)^2} = \ldots\ldots$$

… <u>ΑΠΛΕΣ ΠΡΑΞΕΙΣ</u>

ΑΝΑΔΕΛΤΑ (ΤΕΛΕΣΤΗΣ)

ΤΕΛΕΣΤΕΣ → [Μαθηματικές οντότητες] : Γενικά στα Μαθηματικά ως τελεστής ορίζεται μια "συνάρτηση" που δρα πάνω σε κάποια άλλη "συνάρτηση" μετασχηματίζοντάς την κατά ένα καθορισμένο τρόπο.

π.χ. Τελεστής 1ης παραγώγου

$$\boxed{\hat{D} = \frac{d}{dx}} \rightarrow \text{τελεστής}$$

$$\frac{d}{dx} f$$

[Μαθηματικός Τελεστής με Διανυσματικό Χαρακτήρα]

$\vec{\nabla}$: Διανυσματικός Διαφορικός Τελεστής των μερικών παραγώγων που εφαρμόζεται σε συνάρτηση πολλών ανεξάρτητων μεταβλητών.

$$\boxed{\bar{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}}$$

Συναρτήσεις :

- Βαθμωτές : $\theta(x,y,z)$
- Διανυσματικές : $\bar{F}, \bar{v}, \bar{E}$ → ηλεκτρικό πεδίο.

[Διεύθυνση και Φορά] + Μέτρο

[ΦΥΣΙΚΑ ΜΕΓΕΘΗ : m, ℓ, T, t]

1) Ανάδελτα (Βαθμωτό → f) ⟶ κλίση. $(\bar{\nabla} f)$ ✓

2) Ανάδελτα (Διανυσματικό → $\bar{f}$)
- $\cdot$ → Απόκλιση. $(\bar{\nabla} \cdot \bar{f})$ ✓
- $\times$ → Στροβιλισμός. $(\bar{\nabla} \times \bar{f})$ ✓