ΑΣΚΗΣΗ

$$\int \frac{1}{x^2-1^2}\,dx = \int \frac{1}{(x-1)(x+1)}\,dx \overset{(*)}{=}$$

$$= \int \frac{1}{x-1}\cdot\frac{1}{x+1}\,dx = \int \frac{1}{x-1}\,dx - \int \frac{1}{x+1}\,dx$$

$$\boxed{\frac{1}{(x-1)(x+1)} = \frac{1/2}{x-1} - \frac{1/2}{x+1}}$$

$$\overset{(*)}{=} \int \frac{1/2}{x-1}\,dx - \int \frac{1/2}{x+1}\,dx =$$

$$= \frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x+1) + c' =$$

$$= \frac{1}{2}\left[\ln(x-1) - \ln(x+1)\right] + c' =$$

$$= \boxed{\frac{1}{2}\ln\left(\frac{x-1}{x+1}\right) + c'}$$

ΑΝΑΛΥΣΗ ΣΕ ΑΠΛΑ ΚΛΑΣΜΑΤΑ.

$$\frac{1}{(x-1)\cdot(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$\Rightarrow 1 = \frac{A\cdot(x-1)(x+1)}{x-1} + \frac{B\cdot(x-1)(x+1)}{x+1}$$

$$\Rightarrow 1 = A\cdot(x+1) + B\cdot(x-1) \Rightarrow$$

$$1 = Ax + A + Bx - B \Rightarrow$$

$$\Rightarrow 1 = (A+B)x + (A-B) \Rightarrow$$

$$\Rightarrow \begin{cases} A+B=0 \\ A-B=1 \end{cases} \Rightarrow \boxed{A=-B}$$

$$-2B=1 \Rightarrow \boxed{B=-\frac{1}{2}}$$

$$\boxed{A=\frac{1}{2}}$$

ΠΑΡΑΤΗΡΗΣΗ

$$a_1x^2 + b_1x + c_1 = a_2x^2 + b_2x + c_2$$

$$a_1 = a_2,\quad b_1 = b_2,\quad c_1 = c_2$$

ΑΣΚΗΣΗ

$$\int \frac{x^2-6x+8}{x^2+6x+8}\,dx = \int \frac{x^2-6x+8 \overbrace{+6x-6x}^{-12x}}{x^2+6x+8}\,dx =$$

$$= \int \frac{x^2+6x+8-12x}{x^2+6x+8}\,dx = \int \underbrace{1}_{\smile} - \underbrace{\frac{12x}{x^2+6x+8}}_{\frown}\,dx =$$

$$= \int 1\,dx - \int \frac{12x}{x^2+6x+8}\,dx =$$

$$= x - 12\int \frac{x}{x^2+6x+8}\,dx =$$

$$= x - 12\int \frac{-1}{x+2} + \frac{2}{x+4}\,dx =$$

$$= x - 12\left[-\int \frac{1}{x+2}\,dx + \int \frac{2}{x+4}\,dx\right] =$$

$$= x - 12\left[-\ln(x+2) + 2\cdot\ln(x+4)\right] + C_1 =$$

$$= \boxed{x + 12\ln(x+2) - 24\ln(x+4) + C_1}$$

ΑΝΑΛΥΣΗ ΣΕ ΑΠΛΑ ΚΛΑΣΜΑΤΑ

$$\frac{x}{x^2+6x+8} = \frac{x}{(x+2)(x+4)} =$$

$$= \frac{A}{x+2} + \frac{B}{x+4} \Rightarrow$$

$$x = A\cdot(x+4) + B\cdot(x+2) \Rightarrow$$

$$\Rightarrow x = (A+B)x + 4A + 2B.$$

$$\begin{cases} A+B=1 \Rightarrow -\frac{B}{2}+B=1 \Rightarrow \\ \Rightarrow \boxed{B=2} \\ 4A+2B=0 \Rightarrow \cancel{4}A = -\cancel{2}B \Rightarrow \\ \Rightarrow 2A=-B \Rightarrow \\ \Rightarrow \boxed{A=-\frac{B}{2}} \\ \Rightarrow \boxed{A=-1} \end{cases}$$

ΑΣΚΗΣΗ

$$\int \frac{5x+3}{x^2+2x-3}\,dx =$$

$$= \int \frac{2}{x-1} + \frac{3}{x+3}\,dx =$$

$$= \int \frac{2}{x-1}\,dx + \int \frac{3}{x+3}\,dx =$$

$$= 2\int \frac{1}{x-1}\,dx + 3\int \frac{1}{x+3}\,dx =$$

$$= 2\cdot \ln(x-1) + 3\cdot \ln(x+3) + C_1$$

$$= \ln(x-1)^2 + \ln(x+3)^3 + C' =$$

$$= \boxed{\ln\left((x-1)^2(x+3)^3\right) + C}$$

$$\frac{5x+3}{x^2+2x-3} = \frac{5x+3}{(x-1)\cdot(x+3)} =$$

$$= \frac{A}{x-1} + \frac{B}{x+3} \Rightarrow$$

$$\Rightarrow 5x+3 = A\cdot(x+3) + B\cdot(x-1)$$

$$\Rightarrow 5x+3 = (A+B)\cdot x + 3A - B \Rightarrow$$

$$\begin{cases} A+B=5 \\ 3A-B=3 \end{cases} \quad \boxed{B = 5-A} \Rightarrow \boxed{B=3}$$

$$3A-(5-A)=3 \Rightarrow$$

$$\Rightarrow 3A-5+A=3 \Rightarrow$$

$$\Rightarrow 4A=8 \Rightarrow \boxed{A=2}$$

$$\ln A + \ln B = \ln AB$$

$$\int_7^{11} \frac{x+1}{x^2-5x+6}\,dx$$

$$\int_1^4 \frac{x+1}{x^2-5x+6}\,dx \rightarrow \boxed{\begin{matrix} x_1=2 \\ x_2=3 \end{matrix}}$$

ΓΕΝΙΚΕΥΜΕΝΑ ΟΛΟΚΛΗΡΩΜΑΤΑ.

Πόσο είναι το Γ.Ο.?

Συγχύση το Γ.Ο.

$$\int_7^{11} \frac{x+1}{x^2-5x+6}\,dx =$$

$$= \int_7^{11} \frac{-3}{x-2} + \frac{4}{x-3}\,dx =$$

$$= -3\int_7^{11} \frac{1}{x-2}\,dx + 4\int_7^{11} \frac{1}{x-3}\,dx =$$

$$= -3\,\ln(x-2)\Big|_7^{11} + 4\cdot \ln(x-3)\Big|_7^{11} =$$

$$= -3\left[\ln(11-2) - \ln(7-2)\right] +$$

$$+ 4\cdot\left[\ln(11-3) - \ln(7-3)\right] =$$

$$= -3\cdot\left[\ln 9 - \ln 5\right] + 4\left[\ln 8 - \ln 4\right]$$

$$= \boxed{-3\cdot \ln\left(\frac{9}{5}\right) + 4\ln 2} =$$

$$= \ln\left(\frac{9}{5}\right)^{-3} + \ln 2^4 = \ln\left[\left(\frac{9}{5}\right)^{-3}\cdot 2^4\right]$$

$$= \ln\left[\left(\frac{5}{9}\right)^{3}\cdot 2^4\right] \Leftarrow = \text{.....}$$

$$\frac{x+1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3} \Rightarrow$$

$$\Rightarrow x+1 = A(x-3) + B(x-2)$$

$$\Rightarrow x+1 = (A+B)x - 3A - 2B.$$

$$\Rightarrow \begin{cases} A+B=1 \\ -3A-2B=1 \end{cases} \Rightarrow \boxed{A=1-B}$$

$$\boxed{A=-3}$$

$$-3(1-B) - 2B = 1 \Rightarrow$$

$$\Rightarrow -3 + 3B - 2B = 1 \Rightarrow$$

$$\Rightarrow \boxed{B=4}$$

$$\boxed{ax^2+bx+y=0} \longrightarrow x_1$$

$$\longrightarrow x_2$$

$$\longrightarrow \boxed{a\cdot(x-x_1)\cdot(x-x_2)}$$