ΜΑΘΗΜΑΤΙΚΑ II

Μάθημα 8º

ΟΛΟΚΛΗΡΩΣΗ ΚΑΤΑ ΠΑΡΑΓΟΝΤΕΣ

ΣΚΕΨΗ – ΔΟΚΙΜΗ

$(f^n(x))' = n \cdot f^{n-1}(x) \cdot f'(x)$

$n-1=-2$

$\boxed{n=-1}$

$[(x+1)^{-1}]' = -1 \cdot (x+1)^{-2} \cdot 1$

$[-(x+1)^{-1}]' = \cancel{-}(x+1)^{-2}$

$(x^2+x)' = 2x+1$

$\int f(x)+g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

$(x+1)' = 1$

$\ln(a+b) \neq \ln a + \ln b$

$\ln a + \ln b = \ln(ab)$

$\ln a - \ln b = \ln(a/b)$

- $\displaystyle\int \frac{\ln x}{(x+1)^2}\,dx =$

$$= \int \overset{f(x)}{\ln x} \cdot \frac{1}{(x+1)^2}\,dx = \int \ln x\,(x+1)^{-2}\,dx =$$

$$= \int \ln x\,\left(-(x+1)^{-1}\right)'\,dx = \text{ΕΦΑΡΜΟΓΗ ΟΛΟΚΛΗΡ ΚΑΤΑ ΠΑΡΑΓ.}$$

$$= \ln x\left(-(x+1)^{-1}\right) - \int (\ln x)' \cdot \left(-(x+1)^{-1}\right)dx =$$

$$= -\frac{\ln x}{x+1} + \int \frac{1}{x}\cdot(x+1)^{-1}\,dx = -\frac{\ln x}{x+1} + \int \frac{1}{x}\cdot\frac{1}{x+1}\,dx =$$

$$= -\frac{\ln x}{x+1} + \int \frac{1}{x(x+1)}\,dx =$$

$$= -\frac{\ln x}{x+1} + \int \frac{1+x-x}{x(x+1)}\,dx =$$

$$= -\frac{\ln x}{x+1} + \int \frac{\cancel{x+1}}{x\cancel{(x+1)}} - \frac{\cancel{x}}{\cancel{x}(x+1)}\,dx =$$

$$= -\frac{\ln x}{x+1} + \int \frac{1}{x}\,dx - \int \frac{1}{x+1}\,dx = -\frac{\ln x}{x+1} + \ln x - \int \frac{(x+1)'}{x+1}\,dx$$

$$= -\frac{\ln x}{x+1} + \boxed{\ln x - \ln(x+1)} + c_1 =$$

$$= -\frac{\ln x}{x+1} + \ln\left(\frac{x}{x+1}\right) + c_1$$

$$\int \frac{\ln(x+1)}{\sqrt{x+1}}\,dx = \int \frac{\ln u^2}{\cancel{u}}\cdot 2\cancel{u}\,du =$$

$$= 2\int \ln u^2\,du = 2\int 2\cdot \ln u\,du =$$

$$= 4\int \ln u\,du = 4\int 1\cdot \ln u\,du =$$

$$= 4\int \underbrace{u'}_{g'}\cdot \overset{=f}{\ln u}\,du = 4\left[u\ln u - \int \cancel{u}\,\frac{1}{\cancel{u}}\,du\right]$$

$$= 4\left[u\ln u - u\right] + c_1 =$$

$$= 4u\ln u - 4u + c_1 =$$

$$= 4\cdot\sqrt{x+1}\cdot \ln\sqrt{x+1} - 4\sqrt{x+1} + C$$

$$= 4\sqrt{x+1}\left(\ln\sqrt{x+1} - 1\right) + c_1$$

$$= 4\cdot\sqrt{x+1}\cdot\left(\ln\sqrt{x+1} - \ln e\right) + c_1$$

$$= 4\cdot\sqrt{x+1}\,\ln\frac{\sqrt{x+1}}{e} + c_1$$

Θέτω

$$\sqrt{x+1} = u \Rightarrow$$

$$x+1 = u^2$$

$$1\cdot dx = 2u\,du$$

$$\ln x^a = a\cdot\ln x$$

$$(\ln u)' = \frac{1}{u}$$

$$(\ldots)' = \ln u$$

λύση

$$(u)' = 1$$

$$\ln e = 1$$

ΠΑΡΑΤΗΡΗΣΗ

$$\int \ln x\,dx = \int 1\cdot \ln x\,dx =$$

$$= \int x'\cdot \ln x\,dx =$$

$$((\ldots))' = \ln x$$

λύση

$$\int f\cdot g'\,dx = f\cdot g - \int f'\cdot g\,dx$$

$$\bullet \int e^{-x}\cdot\cos(2x)\,dx = \int (-e^{-x})'\cos 2x\,dx \overset{(*)}{=}$$

$$\boxed{\begin{aligned}(e^{x})' &= e^{x}\\ (e^{-x})' &= e^{-x}\cdot(-1)\end{aligned}}$$

$$(\cos 2x)' = -2\sin 2x$$

$$\overset{(*)}{=} -e^{-x}\cos(2x) - \int(-e^{-x})\cdot(-2\sin 2x)\,dx$$

$$= -e^{-x}\cos(2x) - 2\int e^{-x}\cdot\sin 2x\,dx =$$

$$= -e^{-x}\cos(2x) - 2\int(-e^{-x})'\cdot\sin 2x\,dx =$$

$$= -e^{-x}\cos(2x) - 2\left[-e^{-x}\cdot\sin 2x - \int(-e^{-x})\cdot 2\cos 2x\,dx\right]$$

$$= -e^{-x}\cos(2x) + 2e^{-x}\sin 2x + 2\int(-e^{-x})\cdot 2\cos 2x\,dx$$

$$= -e^{-x}\cos(2x) + 2e^{-x}\sin 2x - 4\int e^{-x}\cos 2x\,dx$$

$$\Longleftrightarrow$$

$$\int e^{-x}\cos 2x\,dx + 4\int e^{-x}\cos 2x\,dx =$$

$$= -e^{-x}\cos(2x) + 2\cdot e^{-x}\sin 2x$$

$$\Longleftrightarrow 5\int e^{-x}\cos 2x\,dx =$$

$$= -e^{-x}\cos(2x) + 2\cdot e^{-x}\sin 2x$$

$$\Rightarrow \boxed{\int e^{-x}\cos 2x\,dx = \frac{1}{5}\left(-e^{-x}\cos(2x) + 2\cdot e^{-x}\sin 2x\right) + C}$$

$$\Sigma F = \boxed{\frac{dp}{dt}} = \frac{d}{dt}(m\cdot u) =$$

$$\Sigma F = m\frac{du}{dt} \quad a$$

$$\boxed{\Sigma F = m\cdot a}$$

$$\Sigma F = m\frac{du}{dt} \Rightarrow u(t)$$

$$\frac{du}{dt} = \frac{du}{dx}\cdot\frac{dx}{dt}, \quad \frac{dx}{dt} = u$$

$$\boxed{\Sigma F = m\cdot u\cdot\frac{du}{dx}} \Rightarrow u(x)$$

$$\frac{\Sigma F}{m} = u\cdot\frac{du}{dx} \Rightarrow$$

$$\Rightarrow \int_{\dots}^{\dots}\frac{\Sigma F}{m}\,dx = \int_{\dots}^{\dots}u\cdot du$$

Με μεταβλητή μάζα

$$\Sigma F = \frac{dp}{dt} = \frac{d}{dt}(m\cdot u) =$$

$$= \frac{d}{dt}\left(m(t)\cdot u(t)\right) =$$

$$= \frac{dm(t)}{dt}u(t) + m(t)\cdot\frac{du}{dt}$$

ρυθμός
ροής μάζας

- $\int (\sqrt{x}+1)\cdot(x+\sqrt{x}+1)\,dx.$ $\Longrightarrow$ ΠΑΡΑΤΗΡΗΣΗΣ: ΠΡΩΤΑ ΠΡΑΞΕΙΣ

- $\int \frac{1}{e^x+1}\,dx$ $\Longrightarrow$ ΠΕΡΙΕΡΓΗ ΣΚΕΨΗ

- $\int \frac{1}{\sin x}\,dx$ $\Longrightarrow$ ΧΡΗΣΙΜΗ ΤΡΙΓΩΝ. ΤΑΥΤΟΤΗΤΑ

$$\sin x = 2\cdot \sin\left(\frac{x}{2}\right)\cdot \cos\left(\frac{x}{2}\right)$$