ΣΥΝΕΧΕΙΑ ΟΛΟΚΛΗΡΩΣΗΣ ΚΑΤΑ ΠΑΡΑΓΟΝΤΕΣ

$$\int x\cdot\sqrt{1+x}\,dx = \int x\cdot\underbrace{(1+x)^{1/2}}\,dx = \int \overset{f(x)}{x}\cdot\Big(\overbrace{\tfrac{2}{3}(1+x)^{3/2}}^{g(x)}\Big)'\,dx =$$

$$= x\cdot\left(\frac{2}{3}(1+x)^{3/2}\right) - \int \cancel{(x)'}^{\,1}\left(\frac{2}{3}\cdot(1+x)^{3/2}\right)dx =$$

$$= \frac{2}{3}x(1+x)^{3/2} - \frac{2}{3}\underbrace{\int(1+x)^{3/2}dx}_{= I_1} \overset{(*)}{=}$$

ΔΟΚΙΜΗ

$$\left(\frac{2}{3}\cdot\underline{(1+x)^{1/2 \to 3/2}}\right)' = \frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}}\cdot(1+x)^{-1/2}\cdot\cancel{(1+x)'}^{\,1} = \frac{1}{3\sqrt{1+x}}$$

Για την επίλυση του I_1

Θέτω: $1+x=u \Rightarrow \boxed{dx=du}$

$$I_1 = \int(1+x)^{3/2}dx = \int u^{3/2}du =$$

$$= \frac{2}{5}u^{5/2} = \frac{2}{5}(1+x)^{5/2}$$

$$\left(u^{5/2}\right)' = \frac{5}{2}u^{3/2} \Rightarrow$$

$$\Rightarrow\left(\frac{2}{5}u^{5/2}\right)' = \frac{\cancel{2}}{\cancel{5}}\cdot\frac{\cancel{5}}{\cancel{2}}\cdot u^{3/2} = u^{3/2}$$

$$n-1=3/2 \Rightarrow n=5/2$$

$$\overset{(*)}{=} \frac{2}{3}x(1+x)^{3/2} - \frac{2}{3}\cdot\frac{2}{5}(1+x)^{5/2} + C =$$

$$= \boxed{\frac{2}{3}x\cdot(1+x)^{3/2} - \frac{4}{15}(1+x)^{5/2} + C_1}$$

$$\left(f(x)^n\right)' = n\cdot\left(f(x)\right)^{n-1}\cdot f'(x) \qquad , \qquad n-1=\tfrac{1}{2}$$

$$n = 1+\tfrac{1}{2}$$

$$\boxed{n=\tfrac{3}{2}}$$

$$\left[(1+x)^{3/2}\right]' = \frac{3}{2}\boxed{(1+x)^{1/2}}\,\cancel{(1+x)'}^{\,1}$$

$$\left(\frac{2}{3}\cdot(1+x)^{3/2}\right)' = (1+x)^{1/2}$$

$\boxed{\text{ΕΞΗΓΗΣΗ}}$

$$\bullet \int \sin^2 x \, dx = \int \sin x \cdot \sin x \, dx =$$

$$\boxed{\begin{aligned} (\cos x)' &= -\sin x. \\ -(\cos x)' &= \sin x \end{aligned}}$$

$$= \int \sin x \cdot (-\cos x)'\, dx = -\int \sin x\cdot (\cos x)'\, dx =$$

$$\boxed{\begin{aligned} \sin^2 x + \cos^2 x = 1 &\Rightarrow \\ \cos^2 x = 1-\sin^2 x \end{aligned}}$$

$$= -\left[\sin x\cdot\cos x - \int (\sin x)'\cdot \cos x\, dx\right] = -\sin x\cdot\cos x + \int \cos x\cdot\cos x\, dx =$$

$$= -\sin x\cdot\cos x + \int \cos^2 x\, dx = -\sin x\cdot\cos x + \int 1-\sin^2 x\, dx \iff$$

$$\int \sin^2 x\,dx = -\sin x\cdot\cos x + \int \underline{1 - \sin^2 x\,dx} \iff$$

$$\int f(x)+g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int \sin^2 x\,dx = -\sin x\cdot\cos x + \boxed{\int 1\,dx} - \int \sin^2 x\,dx \Rightarrow$$

$$\int 1\,dx = x$$

$$(x)' = 1$$

$$\Rightarrow \int \sin^2 x\,dx = -\sin x\cdot\cos x + x - \int \sin^2 x\,dx \Rightarrow$$

$$\Rightarrow 2\cdot\int \sin^2 x\,dx = -\sin x\cdot\cos x + x \Rightarrow \boxed{\int \sin^2 x\,dx = \frac{1}{2}\left(-\sin x\cdot\cos x + x\right) + c}$$

ΠΑΡΑΤΗΡΗΣΗ 1 — ΑΠΟΔΕΙΞΗ

$$\int \cos^2 x\,dx = +\frac{\sin^3 x}{3}$$

$$\left(\frac{\sin^3(x)}{3}\right)' = \frac{1}{\cancel{3}}\cdot\cancel{3}\cdot\sin^2 x\cdot(\sin x)' = \sin^2 x\cdot\cos x$$

$$\left(f^n(x)\right)' = n\,f^{n-1}(x)\cdot f'(x)$$

$$(f(x))^n = f^n(x)$$

$$(\sin x)^3 = \sin^3(x) \neq \boxed{\sin(x^3)}$$

$$\frac{d}{dx}(\sin x)^3 = 3\cdot(\sin x)^2 \cdot \frac{d}{dx}(\sin x) = 3\cdot \sin^2 x \cdot \cos x.$$

$$\frac{d}{dx}\left(\sin(x^3)\right) = \cos(x^3)\cdot\frac{d}{dx}(x^3) =$$

$$= 3\cdot x^2 \cdot \cos(x^3)$$

$$\boxed{\begin{aligned}&(\sin(f(x)))' = \cos(f(x))\cdot f'(x)\\ &((f(x))^n)'\end{aligned}}$$

$$= \frac{1}{2}x^3 e^{2x} - \frac{3}{4}x^2\cdot e^{2x} + \frac{3}{4}x\cdot e^{2x} - \frac{3}{8}e^{2x} + c$$

$$\bullet \int x^3 \cdot e^{2x}\, dx = \int x^3 \cdot \left(\frac{e^{2x}}{2}\right)' dx = \frac{x^3 \cdot e^{2x}}{2} - \int 3x^2 \cdot \frac{e^{2x}}{2}\, dx = \frac{x^3 \cdot e^{2x}}{2} - \frac{3}{2}\int x^2 \cdot \left(\frac{e^{2x}}{2}\right)' dx =$$

$$= \frac{x^3 \cdot e^{2x}}{2} - \frac{3}{2}\left[\frac{x^2 \cdot e^{2x}}{2} - \int \cancel{2}x \cdot \frac{e^{2x}}{\cancel{2}}\, dx\right] = \underbrace{\frac{x^3 \cdot e^{2x}}{2} - \frac{3}{4}x^2 \cdot e^{2x}} + \frac{3}{2}\int x\, e^{2x}\, dx =$$

$$= \frac{x^3 \cdot e^{2x}}{2} - \frac{3}{4}x^2 \cdot e^{2x} + \frac{3}{2}\int x \cdot \left(\frac{e^{2x}}{2}\right)' dx = \frac{x^3 \cdot e^{2x}}{2} - \frac{3}{4}x^2 \cdot e^{2x} + \frac{3}{2}\left[\frac{x e^{2x}}{2} - \int \frac{e^{2x}}{2}\, dx\right] =$$

$$= \frac{x^3 \cdot e^{2x}}{2} - \frac{3}{4}x^2 \cdot e^{2x} + \frac{3}{4}x \cdot e^{2x} - \frac{3}{4}\int e^{2x}\, dx =$$

$$= \boxed{\frac{x^3 \cdot e^{2x}}{2} - \frac{3}{4}x^2 \cdot e^{2x} + \frac{3}{4}x \cdot e^{2x} - \frac{3}{8}e^{2x} + C}$$

|Β' ΤΡΟΠΟΣ|

$$\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} dx =$$

$$= \frac{1}{2} \cdot \left[\int dx - \int \cos 2x \, dx \right] =$$

$$= \frac{1}{2} x - \frac{1}{2} \int \cos 2x \, dx =$$

$$= \frac{x}{2} - \frac{1}{2} \int \cos(u) \cdot \frac{du}{2} = \frac{x}{2} - \frac{1}{4} \int \cos u \, du$$

$$= \frac{x}{2} - \frac{1}{4} \sin u + c_1 = \boxed{\frac{x}{2} - \frac{1}{4} \sin(2x) + c_1}$$

$$= \frac{x}{2} - \frac{1}{2 \cancel{4}} \cancel{2} \cdot \sin x \cdot \cos x + c_1 = \frac{x}{2} - \frac{1}{2} \sin x \cdot \cos x + c_1$$

(1) $\Rightarrow$ $\sin^2 x = \dfrac{1 - \cos 2x}{2}$ ✓

(2) $\Rightarrow$ $\cos^2 x = \dfrac{1 + \cos 2x}{2}$. ✓

(3) $\Rightarrow$ $\boxed{\sin^2 x + \cos^2 x = 1}$ ✓

$\cos(f(x))$

Θέτω

$2x = u. \Rightarrow$

$\Rightarrow \dfrac{du}{dx} = 2 \Rightarrow$

$\Rightarrow \boxed{du = 2\,dx}$

$\boxed{du/2 = dx}$

$\sin 2x = 2 \sin x \cdot \cos x$ (4)

$\cos 2x = 2\cos^2 x - 1$ (5)

- $\int \cos^2 x \, dx = ???$