$$\int \frac{1}{\sqrt{x}} + \frac{2}{\sqrt[3]{x}} + \frac{4}{\sqrt[5]{x}}\,dx = \int \frac{1}{\sqrt{x}}\,dx + \int \frac{2}{\sqrt[3]{x}}\,dx + \int \frac{4}{\sqrt[5]{x}}\,dx =$$

$$= \underbrace{\int x^{-1/2}\,dx}_{2\sqrt{x}} + \int 2\cdot x^{-1/3}\,dx + \int 4\cdot x^{-1/5}\,dx =$$

$$= 2\sqrt{x} + 3\sqrt[3]{x^2} + 5\cdot\sqrt[5]{x^4} + C$$

$$\sqrt[3]{x^2} = x^{2/3}$$

$$\cancel{2}\,\frac{3}{\cancel{2}}\cdot x^{2/3} = 3\,x^{2/3} = \sqrt[3]{x^2} = (x^2)^{1/3}$$

ΔΟΚΙΜΗ

$$\left(\frac{2}{3}x^{3/2}\right)' = \frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}\,x^{3/2-1} = x^{1/2}$$

$$\sqrt{x} = x^{1/2}$$

$$\frac{1}{\sqrt{x}} = x^{-1/2}$$

$$\sqrt[3]{x} = x^{1/3}$$

$$\frac{1}{\sqrt[3]{x}} = x^{-1/3}$$

$$\sqrt[5]{x} = x^{1/5}$$

$$\frac{1}{\sqrt[5]{x}} = x^{-1/5}$$

$$\int x^2\sqrt{2-3x^3}\,dx =$$

ΘΕΤΩ

$$2-3x^3 = u \Rightarrow$$

$$\Rightarrow \frac{du}{dx} = -9x^2 \Rightarrow \boxed{du = -9x^2\,dx}$$

$$\Downarrow$$

$$\boxed{\frac{du}{-9} = x^2\,dx}$$

$$\int \sqrt{u}\cdot\frac{du}{-9} = -\frac{1}{9}\int \sqrt{u}\,du =$$

$$= -\frac{1}{9}\int u^{1/2}\,du = -\frac{1}{9}\cdot\frac{2}{3}\cdot u^{3/2} + C$$

$$= -\frac{2}{27}\left(2-3x^3\right)^{3/2} + C = -\frac{2}{27}\sqrt{\left(2-3x^3\right)^3} + C$$

ΔΟΚΙΜΗ

$$\int x^{-1/3}\,dx = \frac{3}{2}x^{2/3}$$

$$(\ldots)' = x^{-1/3}$$

$$(x^n)' = n\cdot x^{n-1}$$

$$n-1 = -1/3 \Rightarrow n = 1 - 1/3$$

$$n = 2/3$$

ΔΟΚΙΜΗ

$$(x^{2/3})' = \frac{2}{3}\cdot x^{-1/3}$$

$$\left(\frac{3}{2}x^{2/3}\right)' = \frac{\cancel{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}\,x^{-1/3}$$

$$\int x^2 \sqrt{x+1}\, dx =$$

Θέτω

$$\sqrt{x+1} = u \Rightarrow (x+1)^{1/2} = u.$$

$$\Rightarrow x+1 = u^2 \Longrightarrow \frac{du^2}{du} = 2u$$

$$\frac{d(x+1)}{dx} = 1 \qquad d(x+1) = dx$$

$$\Rightarrow \boxed{dx = 2\cdot u \cdot du.} \qquad d(u^2) = 2u\,du$$

$$x = u^2 - 1 \Rightarrow x^2 = (u^2-1)^2$$

$$= \int (u^2-1)^2 \cdot u \cdot 2 \cdot u\, du =$$

$$= \int (u^2-1)^2 \cdot 2\, u^2\, du =$$

$$= 2 \cdot \int u^2 \, (u^2-1)^2\, du = 2 \cdot \int u^2 (u^4 - 2u^2 + 1)\, du$$

$$= 2 \cdot \int u^6 - 2u^4 + u^2 \, du =$$

$$= 2 \cdot \left[\int u^6 \cdot du - \int 2u^4 du + \int u^2 du \right] =$$

$$\boxed{\begin{aligned} x^a \cdot x^b &= x^{a+b} \\ (x^a)^b &= x^{ab} \end{aligned}}$$

$$= 2 \left[\frac{u^7}{7} - 2 \cdot \frac{u^5}{5} + \frac{u^3}{3} \right] + C =$$

$$= \frac{2}{7} u^7 - \frac{4}{5} u^5 + \frac{2}{3} u^3 + C =$$

$$= \frac{2}{7} \left(\sqrt{x+1}\right)^7 - \frac{4}{5} \left(\sqrt{x+1}\right)^5 + \frac{2}{3} \left(\sqrt{x+1}\right)^3 + C =$$

$$= \boxed{\frac{2}{7} (x+1)^{7/2} - \frac{4}{5} (x+1)^{5/2} + \frac{2}{3} (x+1)^{3/2} + C}$$

$$\int 3x\cdot\sqrt{1-2x^2}\,dx =$$

$$= 3\cdot\int x\cdot\sqrt{1-2x^2}\,dx =$$

$$= 3\cdot\int -\frac{1}{4}\sqrt{u}\,du = -\frac{3}{4}\int\sqrt{u}\,du =$$

$$= -\frac{3}{4}\int u^{1/2}\,du = -\frac{\cancel{3}}{\cancel{4}_{2}}\cdot\frac{2}{\cancel{3}}\,u^{3/2} + c$$

$$= \boxed{-\frac{1}{2}\left(1-2x^2\right)^{3/2} + c}$$

$$-\frac{1}{2}\left(1-2x^2\right)\cdot\sqrt{1-2x^2} + c$$

$$= -\frac{1}{2}\cdot\left(1-2x^2\right)^{3/2} + c' \checkmark$$

Θέτω

$$1-2x^2 = u \Rightarrow$$

$$\Rightarrow \frac{du}{dx} = -4x \Rightarrow$$

$$\Rightarrow du = -4x\,dx$$

$$\Rightarrow \boxed{-\frac{du}{4} = x\,dx}$$

$$\left(\quad\right)' = u^{1/2}$$

$$n-1 = 1/2$$

$$n = 3/2$$

$$\left(u^{3/2}\right)' = \frac{3}{2}u^{1/2}$$

$$\int\frac{x^2}{1-2x^3}\,dx = \cancel{\int x^2\,dx\cdot\int\frac{1}{1-2x^3}\,dx}$$

$$\boxed{x^2\cdot\frac{1}{1-2x^3}} = \int\frac{\frac{du}{-6}}{\frac{u}{1}} = \int -\frac{1}{6}\cdot\frac{du}{u}$$

$$\frac{\ln|1-2x^3|}{-6} + c$$

$$= -\frac{1}{6}\int\frac{du}{u} = -\frac{1}{6}\int\frac{u'}{u}\,du =$$

$$= -\frac{1}{6}\ln|u| + c =$$

$$= \boxed{-\frac{1}{6}\ln\left|1-2x^3\right| + c'}$$

Θέτω

$$\boxed{1-2x^3 = u} \Rightarrow \frac{du}{dx} = -6x^2$$

$$du = -6x^2\,dx \Rightarrow \boxed{\frac{du}{-6} = x^2\,dx}$$

$$\int \frac{x+3}{(x^2+6x)^{1/3}}\,dx = \int \frac{\frac{3}{2}\cdot u^2\,du}{u} =$$

$$= \frac{3}{2}\int \frac{u^{\cancel{2}}}{\cancel{u}}\,du = \frac{3}{2}\int u\,du =$$

$$= \frac{3}{2}\cdot\frac{u^2}{2} + c' =$$

$$= \frac{3}{4}u^2 + c' =$$

$$= \frac{3}{4}\left(\left(x^2+6x\right)^{1/3}\right)^2 + c' =$$

$$= \boxed{\frac{3}{4}\cdot\left(x^2+6x\right)^{2/3} + c'}$$ ✓

$$\frac{3}{4}u^2 + c'$$ ✓

Θέτω

$$(x^2+6x)^{1/3} = u.$$

$$x^2+6x = u^3$$

$$(2x+6)\,dx = 3\cdot u^2\,du.$$

$$\cancel{2}(x+3)\,dx = \boxed{\frac{3\cdot u^2}{2}\,du.}$$

ή ισοδύναμα

$$x^2+6x = u$$

...