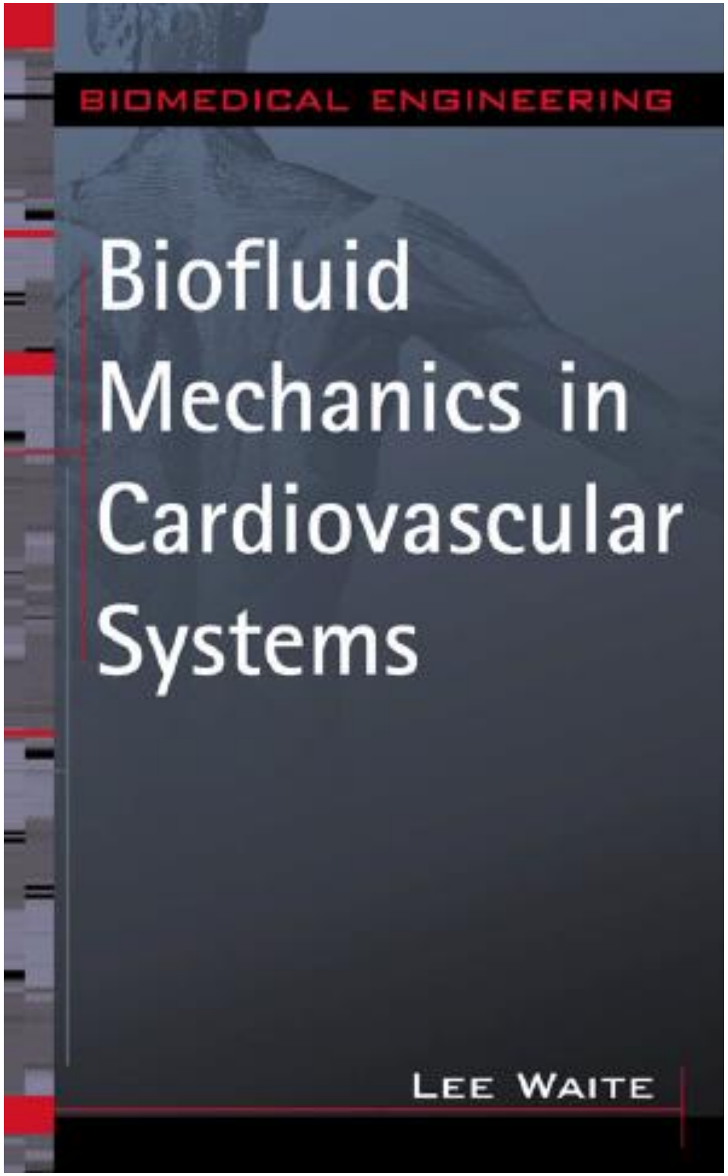
BIOCIRCULATION
Basics of fluid mechanics

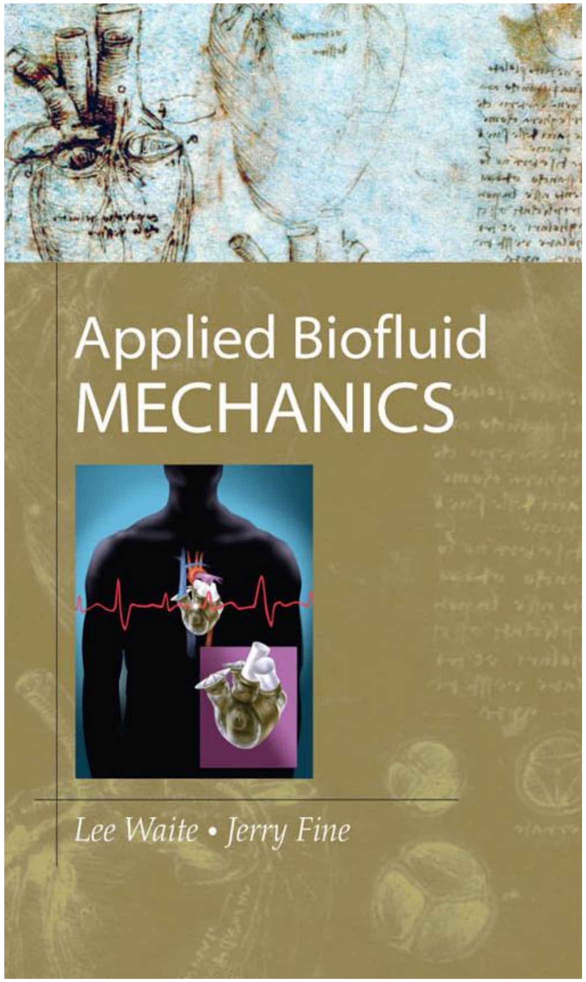
Dimosthenis Mavrilas
Professor
Lab. of Biomechanics & Biomedical Engineering
University of Patras



Biofluid Mechanics in Cardiovascular Systems

Lee Waite
Rose-Hulman Institute of Technology
Terre Haute, Indiana

McGraw-Hill
New York Chicago San Francisco Lisbon London Madrid
Mexico City Milan New Delhi San Juan Seoul
Singapore Sydney Toronto



Applied Biofluid Mechanics

Lee Waite, Ph.D., P.E.
Jerry Fine, Ph.D.

Mc Graw Hill
New York Chicago San Francisco Lisbon London Madrid Mexico City Milan New Delhi San Juan Seoul Singapore Sydney Toronto

Study Chapter 1

Biocirculation: Some historical steps

- 2700-2600 BC;China – **Huang Ti**: *Internal Classics* with issues of human circulation
- 400 BC;Greece – **Hippocrates**: Father of science-based Medicine
- 384-322;Greece – **Aristotle**: The heart was the focus of blood vessels. Breathing; "Air is thin in high mountains" (also supported by many others centuries later on worldwide…)
- Same period;Greece – **Praxagoras**: Arteries (carrying air!!!) & Veins (carrying blood)
- 1628;England - **William Harvey**: "An anatomical study of the motion of the heart and of the blood of animals" the first publication in western world on blood circulation
- 1797-1869;France – **Jan Luis Marie Poiseuille**: Flow/pressure law in long tubes
- 1865-1944;Germany – **Otto Frank**: "Windkessel theory" & "Fundamental form of the arterial pulse"

Basic fluid characteristics:

- **Fluid**
  - ➢ Continuous deformation even with a small shear stress
  - ➢ Not recovering the initial configuration (not individual “scheme” upon vanishing the stress
  - ➢ Important material characteristics:
  - ✓ Density, **ρ**: mass/volume
  - ✓ Specific weight, **S**: weight/volume
  - ✓ Specific gravity, **s**: $weight_{fluid}/weight_{water}$
  - ✓ Viscocity, **μ**-**ν**: determines the flowing properties of different fluids

- **Solid**
  - ➢ Finite deformation (strain)under an analogous finite shear stress
  - ➢ Recovering the initial scheme upon vanishing the stress
  - ➢ Important material characteristics:
  - ✓ Density
  - ✓ Specific weight
  - ✓ Specific gravity
  - ✓ Materials’ strength, Modulus of elasticity E: Determines the deformation properties of different materials

Basic fluid characteristics: Displacement & Velocity

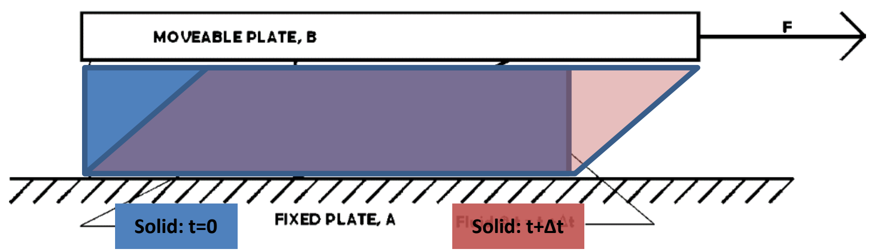
- Movable plate B suspended on a fluid above a fixed plate A
- F_{solid}: *d, τ, γ*. Motion at t+Δt zero
- F_{fluid}: constant movement; (non slip boundary condition in fluids):bottom molecules with zero velocity, upper molecules with V_B, all other “linearly” between 0-V_B
- Shearing strain, *γ*: the angle between lines at t and t+Δt
- Rate of shearing strain, $\dot{\gamma}$:

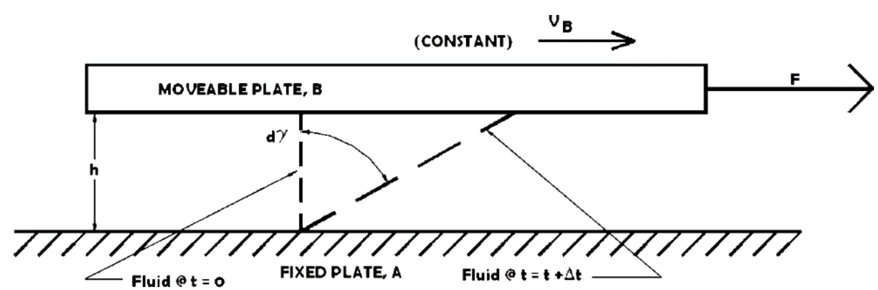
$$d\gamma/_{dt} = \tan(d\gamma)/_{dt} = \frac{V_B}{h}, (1/sec)$$

- Velocity gradient: V_B/h

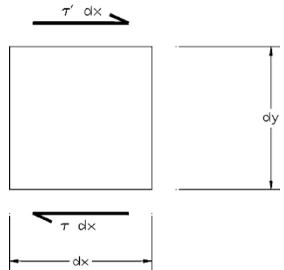
- Solid deformation under shear stress

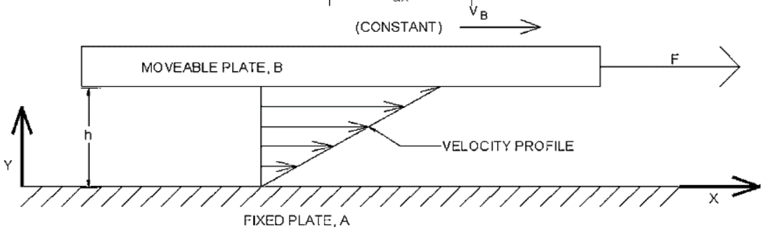
d, τ, γ



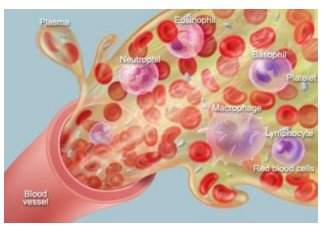


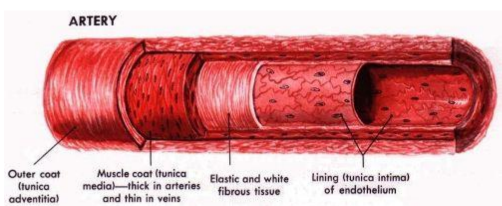
Basic fluid characteristics: Shear stress & viscosity





- Blood is a living fluid, composed of blood cells (red, white, platelets), soluble proteins, electrolyte ions & water
- The lumen of blood vessels is covered by intima, an endothelial cell monolayer, acting as interface between blood and vessel wall tissue





- Structural integrity of blood and endothelial cells is important for a healthy organism

- At any point between the plates, an element of the fluid is imposed in opposite shear forces
- The net shear force may accelerate or rotate the element (change the velocity vector)
- For an element moved in stable velocity, total τ acting is zero
- ${}^{d\tau}/_{dy} = 0 \; and \; \tau_A = \tau_{\mathrm{B}} = t_{wall}$
- Larger F results in higher velocity, higher shearing strain rate, higher shearing stress
- Viscosity correlates shearing stress with shearing strain rate

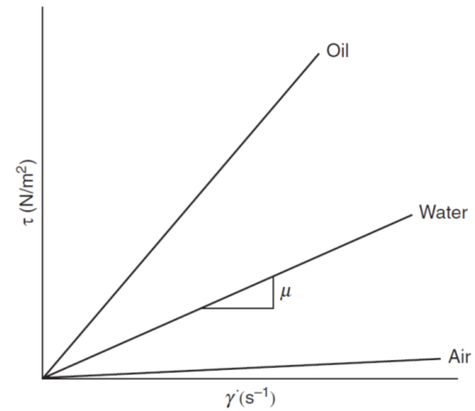
Basic fluid characteristics: Viscosity

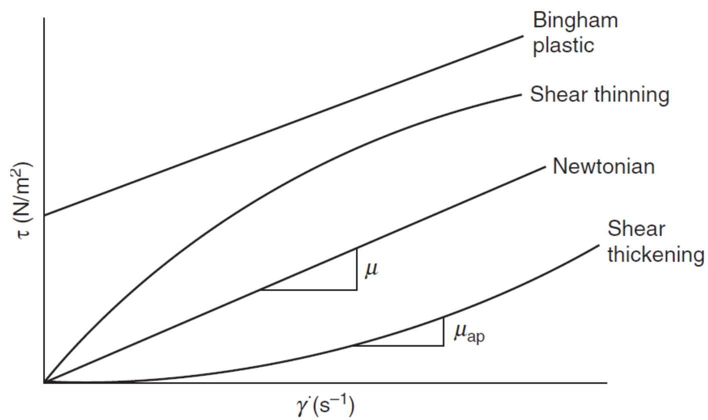
- Shear stress $\boldsymbol{\tau}$ is linearly related (supposing Newtonian fluids) with shearing stress rate $\dot{\boldsymbol{\gamma}}$ with a coefficient, $\boldsymbol{\mu}$, named as absolute or dynamic viscosity

$$\tau = \mu \times \dot{\gamma}$$

- If $\boldsymbol{\tau}$ not linear with $\dot{\boldsymbol{\gamma}}$, then apparent viscosity, $\boldsymbol{\mu_{ap}}$ is used instead of $\boldsymbol{\mu}$, defined as the tangential slope at any point of the curve
- Shear thinning vs. shear thickening fluids
- Blood in non Newtonian under 100 s^{-1} (Bingham plastic, near solid close to zero) but turns in near Newtonian for higher $\dot{\boldsymbol{\gamma}}$. Yield stress approximately in the range 0.005 - 0.01 N/m^2
- Kinematic viscosity, v:

$$\nu = \frac{\mu}{\rho} \text{ (SI units); } \mu: \frac{\text{Ns}}{m^2}, \nu: \frac{m^2}{s}$$





Basic fluid characteristics:
Measuring viscosity by rotating viscometer

- Two concentric cylinders
- Governing equations:

$$\tau = \frac{F}{A}, \dot{\gamma} = \frac{V}{h}$$

As $\mu = \frac{\tau}{\dot{\gamma}}, \nu = \frac{\mu}{\rho}, \mathrm{V} = \omega \frac{D}{2}$

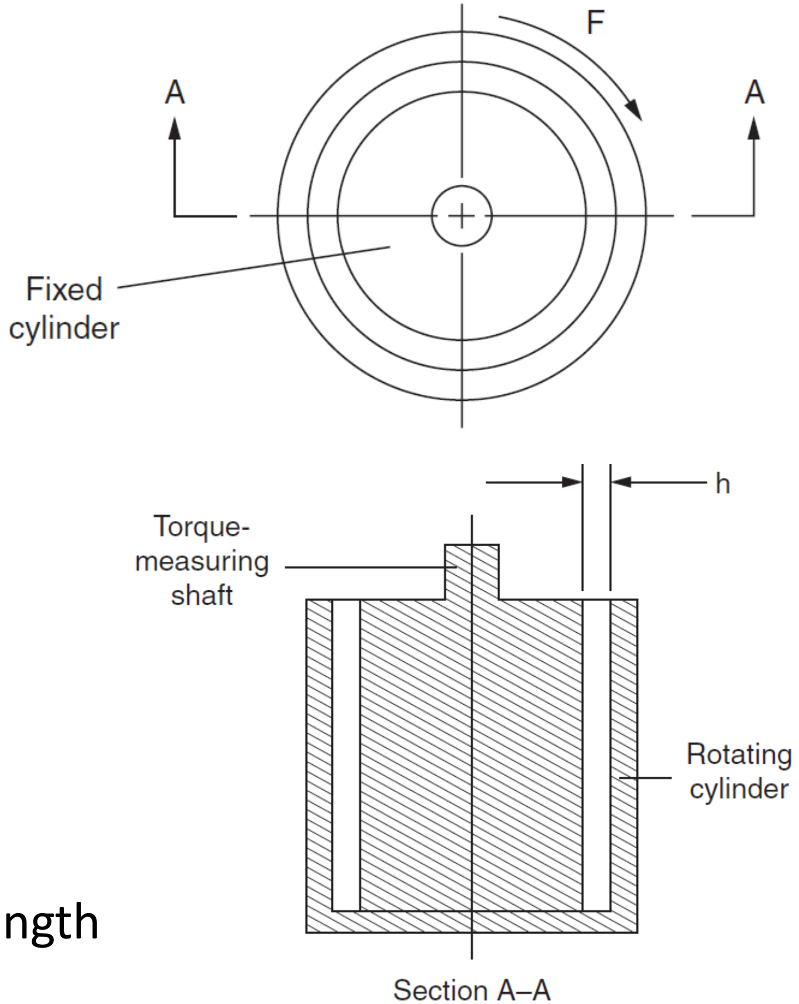
It can be shown that

$$T = F\frac{D}{2}$$

Hence, finally

$$\mu = \frac{4\mathrm{Th}}{\pi D^3 L\omega}$$

Measuring data: T=Torque at axis, h=gap,
D=internal diameter of rotating cylinder, L = length



Introduction to pipe flow
Basic approaches

- **The pipe**
  - Cross-section (circular, other?)
  - Wall material (rigid, elastic (thick-thin, modulus of elasticity)
  - Internal surface (smooth, rough)
  - Geometry (straight, curved, bifurcations)
  - Region of interest (entrance, outlet, far from edges – cross-sectional variations)
  - Stationary – moving state
  - Open – closed circuit

- **The fluid**
  - Newtonian - other?
  - Compressive - incompressive
  - Single phase - multiphase
  - High – low viscous

- **The flow**
  - Steady – pulsatile
  - Laminar – turbulent – vortices
  - Continuous – separation
  - Free – immersed objects - obstructions

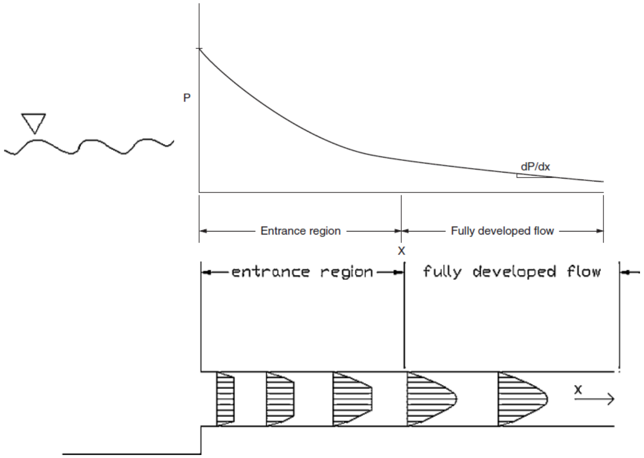
Introduction to pipe flow
Reynolds number

- **Eulorian approach**
- Fully development of a flow after entering a tube from a big reservoir: Velocity profile over a cross-section turns gradually from a straight (non steady state) to a parabolic (steady state) configuration (***p→F on the fluid fixed***)
- In steady state:

$$grad\ p = -F, {}^{dp}/_{dx} = constant$$

- In non steady state

$$ {}^{dp}/_{dx} = non\ constant$$



- Reynolds' number (dimensionless)

$$Re = \frac{\rho V D}{\mu}$$

V = average velocity across pipe cross section

Represents inertia/viscous forces

Laminar flow: Re<2000

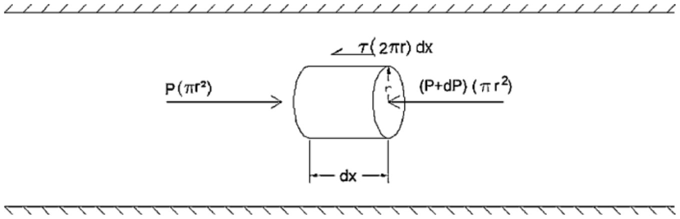
Turbulent flow: Re>4000

2000<Re<4000: Transition state

Entrance length: $X_E/D = 0.06Re$

Introduction to pipe flow
Poiseuille's law

- 1840, Jean-Marie Poiseuille
- **Conditions: Flow**: steady, laminar, incompressible; **fluid**: Newtonian; **tube**: rigid, cylindrical, straight, constant cross section, far from entrance



- S.f.: dQ/dt = 0, dp/dx = 0
- Sum of forces on the element is zero, hence

$$p(\pi r^2) - (p + dp)(\pi r^2) - \tau 2\pi r dx = 0$$

Or $-dp(\pi r^2) = 2\pi r\tau dx, and$

$$\tau = -{}^{r}/_{2}\frac{dp}{dx}\ (1), \tau_{wall} = -{}^{R}/_{2}\frac{dp}{dx}$$

But $\tau = -\mu\frac{dV}{dr}$ (2)

From (1) & (2), after separation of variables

$$dV = \frac{1}{2\mu}\frac{dp}{dx}rdr\ (3)$$

- Solution of (3)

$$V = \frac{1}{2\mu}\frac{dp}{dx}\frac{r^2}{2} + C_1\ (4)$$

- **For steady flow, dp/dx constant**, hence velocity profile is a function of r square (parabolic)

Introduction to pipe flow: Poiseuille's flow

- Boundary condition at wall: r=R, V=0

$$V = \frac{1}{2\mu}\frac{dp}{dx}\frac{r^2}{2} + C_1$$

- Then

$$C_1 = -\frac{1}{2\mu}\frac{dp}{dx}\frac{R^2}{2}$$

- Finally

$$V = \frac{1}{4\mu}\frac{dp}{dx}\left[r^2 - R^2\right]\ (5),\ {}^{dp}/_{dx}\ negative$$

Introduction to pipe flow: Flow rate

- Flow rate over a cross section of the tube. In a ring element:

$dQ = 2\pi V r dr, \qquad by\ integration$

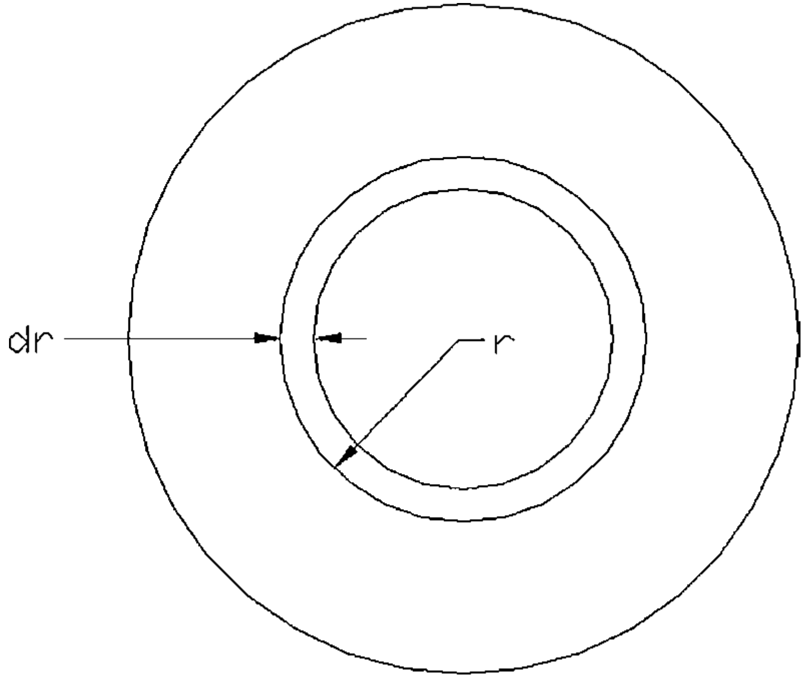
$$Q = 2\pi \int_{0}^{R} V r dr$$

$$Q = \frac{\pi}{2\mu}\frac{dp}{dx}\int_{0}^{R}(r^3 - rR^2)dr$$

$$Q = \frac{\pi}{2\mu}\frac{dp}{dx}\int_{0}^{R}\left(\frac{r^4}{4} - \frac{r^2R^2}{2}\right)[0 - \mathrm{R}]$$

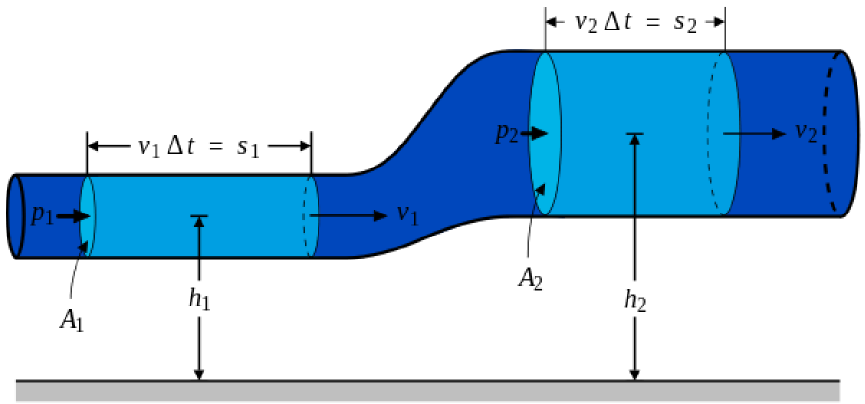
Finally

$$Q = \frac{-\pi R^4}{8\mu}\frac{dp}{dx}, Poisseuille's\ law$$



$$V_{avg} = \frac{Q}{A} = \frac{-R^2}{8\mu}\frac{dp}{dx} = \frac{V_{max}}{2}$$

Bernoulli Equation:



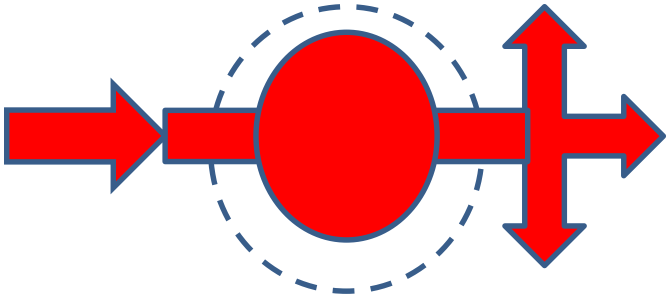
- Let's simplify
- Suppose **inviscid** (no friction), **incompressive** (ρ=constant) flow across a streamline (a line tangential to velocity of a fluid point travelling across the tube)
- P→V relationship

$$P_1 + {}^{1}/_{2}\,\rho {V_1}^2 + \rho g z_1 = P_2 + {}^{1}/_{2}\,\rho {V_2}^2 + \rho g z_2$$

- Limitation: as it is frictionless not appropriate for long pipes of constant cross section. Useful as approximation in short length stenosed pipes

Conservation of mass:

- Mass conservation law: what inserted into a stable volume has to be equally ejected
- In a non stable volume, income-outcomes computations involve volume changes +/- (volume = mass/density, volumetric flow, time)
- In a control volume element:



$$\left(\frac{m}{t}\right)_{in}\Delta \mathrm{t} = \left(\frac{m}{t}\right)_{out} + \Delta \mathrm{m}$$

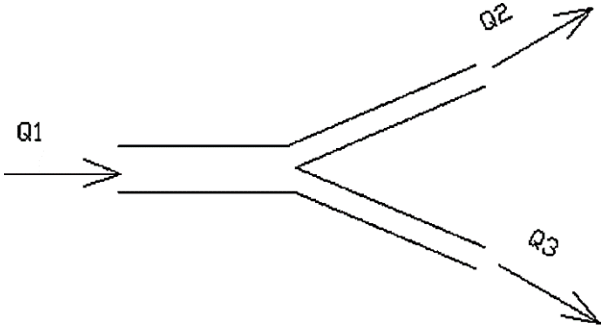
For rigid wall volume element

$$\rho_1 \mathrm{A}_1 V_1 = \rho_2 \mathrm{A}_2 V_2 = constant$$

For incompressive flow at rigid tubes

$$A_1 V_1 = A_2 V_2 = Q = constant$$

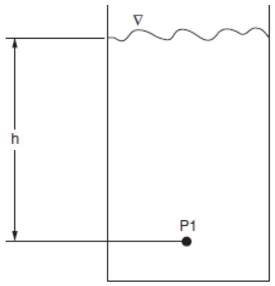
- Flow at bifurcations

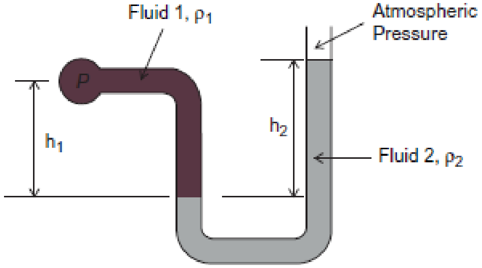


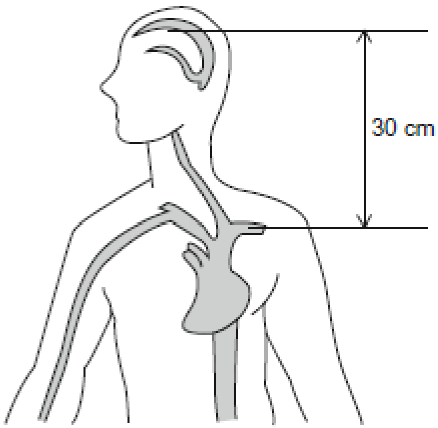
$$Q_1 = Q_2 + Q_3,$$
$$A_1 V_1 = A_2 V_2 + A_3 V_3$$

Fluid statics

- Fluid in stationary state (V=0, τ=0). Only **normal** forces act on the fluid mass **m:** $P_1 = \rho g h$
- Measuring physiological pressures with a pressure gauge (I.e. a tube manometer) we measure gauge pressure: **pressure difference**



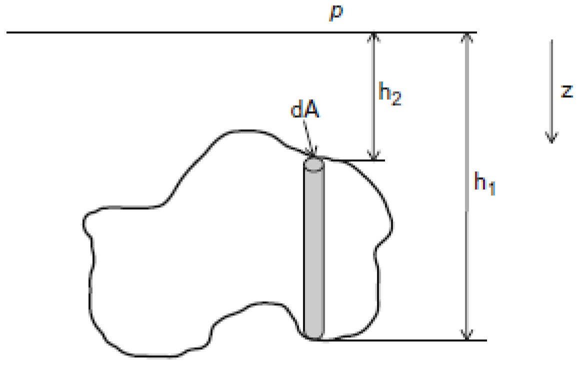




$$P_{gauge} = P_{absolute} - P_{atmosperic}$$

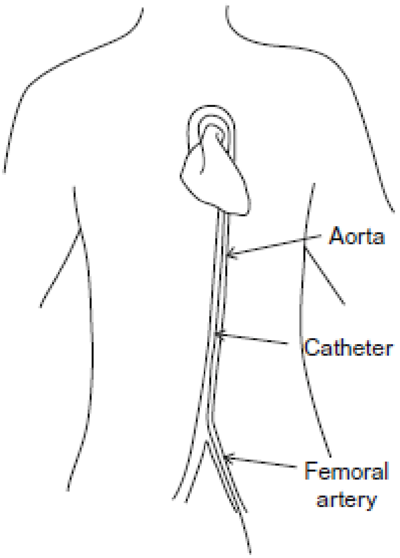
$$P_{craniac} = P_{valve} - \rho g h$$

Buoyancy



- Consider a body of volume V floating or immersed within a fluid. From Archimedes law of buoyancy, net force acting from the fluid is equal to the weight of fluid with the same volume

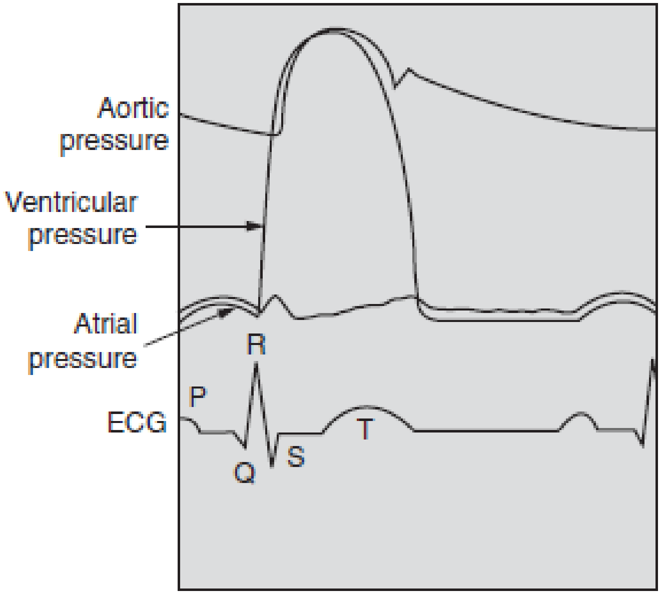
$F_Z = \rho g V$



- Example: The buoyancy of a catheter inserted from the femoral artery to coronary artery

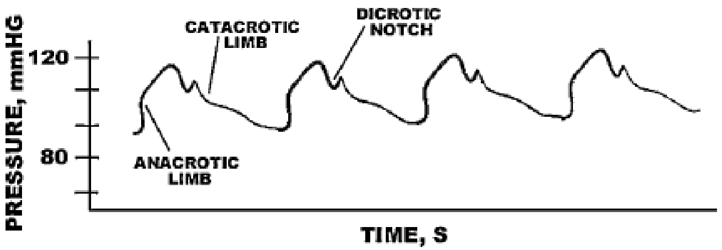
The Womersley number: Frequency of pulsed flow

- Blood flow is not steady flow – pulsatile characteristics
- In every cardiac cycle, during the systolic phase a pressure is applied, pushing the blood to the arteries
- Blood flow is therefore a periodic function of time
- Pressure waves produced into ventricular cavities and propagated downstream along the arterial tree, gradually dissipated due to bifurcations and altered compliance of the arterial walls



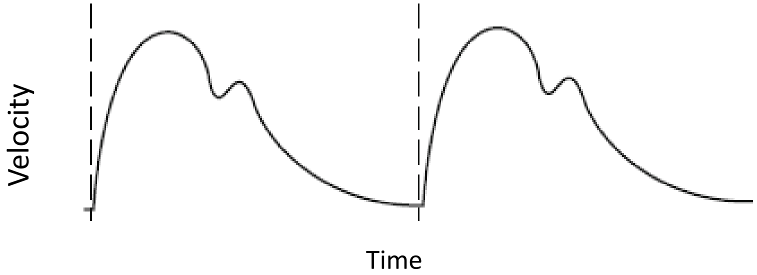
The Womersley number: Frequency of pulsed flow

- Pulsed flow is not harmonic function of time. After frequency analysis, a spectrum of harmonic functions can be derived, that superimposed to provide the actual flow waveform



- Womersley number is a measure of transient (unsteady) to viscus forces, like Re representing inertial to viscus forces in steady flow

$$a = r\sqrt{\frac{\omega}{v}} = \frac{D}{2}\sqrt{\frac{\rho\omega}{\mu}}$$



- α in human circulation ranges from 10^{-3} (capillaries) to near 20 (ascending aorta)

Basic fluid characteristics: Example in shear stress

Introduction to pipe flow
Reynolds number example