Κεφάλαιο 7

Ορθοκανονικές συναρτήσεις

Μία ομάδα συναρτήσεων που χρησιμοποιούνται συχνά στις μεθόδους ταξινόμησης προτύπων ονομάζονται ορθοκανονικές συναρτήσεις και χαρατηρίζονται απο ένα σύνολο ιδιοτήτων.

Συγκεκριμένα

Ορισμός 32 *Οι συναρτήσεις $f(x)$ και $g(x)$ είναι ορθογώνιες με συνάρτηση βαρύτητας την $w(x)$ στο διάστημα [α,β] όταν ισχύει:*

$$\int_{\alpha}^{\beta} w(x)f(x)g(x)dx = 0 \tag{7.1}$$

Ορισμός 33 *Ενα σύστημα Q εξισώσεων $\varphi_i(x), i = 1, Q$ θα ονομάζεται ορθογώνιο στο διάστημα [α,β] όταν:*

$$\int_{\alpha}^{\beta} w(x)\varphi_i(x)\varphi_j(x)dx = \begin{cases} A_i, & i = j \\ 0, & i \neq j \end{cases} \tag{7.2}$$

ο συντελεστής A_i υπολογίζεται από την σχέση:

$$A_i = \int_{\alpha}^{\beta} w(x){\varphi_i}^2(x)dx \tag{7.3}$$

Ορισμός 34 *Το σύστημα των Q εξισώσεων $\varphi_i(x), i = 1, Q$ θα ονομάζεται Ορθοκανονικό στο διάστημα [α,β], όταν:*

$$\int_{\alpha}^{\beta} w(x)\varphi_i(x)\varphi_j(x)dx = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \tag{7.4}$$

Την σχέση ανάμεσα στις ορθογώνιες και τις ορθοκανονικές συναρτήσεις μας την δίνει το ακόλουθο θεώρημα.

Θεώρημα 17 *Αν το σύστημα συναρτήσεων $\varphi_i(x), i = 1, Q$ είναι ορθογώνιο τότε το σύστημα συναρτήσεων $\varphi_i{}^*(x), i = 1, Q$ είναι ορθοκανονικό:*

$$\varphi_i{}^*(x) = \sqrt{\frac{w(x)}{A_i}}\varphi_i(x) \tag{7.5}$$

Με ένα πολύ απλό τρόπο μπορούμε να μεταβούμε από τις μονοδιάστατες ορθογώνιες συναρτήσεις, στις αντίστοιχες διανυσματικές:

Θεώρημα 18 *Αν το σύστημα συναρτήσεων $\varphi_i(x), i = 1, Q$ είναι ορθογώνιο τότε το σύστημα συναρτήσεων $f_i(\mathbf{x}), i = 1, M$ και $\mathbf{x} = (x_1, x_2, ..., x_P)^T \in \mathbb{R}^P$ είναι επίσης ορθογώνιο:*

$$f_i(\mathbf{x}) = \prod_{i=1}^{P} \varphi_{r(i)}(x_i) \tag{7.6}$$

Η συνάρτηση $r(i)$ είναι μία συνάρτηση τυχαίων φυσικών αριθμών στο διάστημα [1,Q].

Παράδειγμα 76 *Αν το σύστημα συναρτήσεων $\varphi_i(x), i = 1, Q$ είναι ορθογώνιο τότε βρείτε ένα σύστημα U ορθογώνιων συναρτήσεων στον χώρο δύο διαστάσεων ($\mathbb{R}^2$).*

Με την βοήθεια του προηγούμενου θεωρήματος ορίζω ένα σύνολο συναρτήσεων δύο μεταβλητών επιλέγοντας τυχαία ζεύγη από τις ορθογώνιες συναρτήσεις μιας μεταβλητής:

$$\begin{aligned}
f_1(\mathbf{x}) &= \varphi_1(x_1)\varphi_1(x_2)\\
f_2(\mathbf{x}) &= \varphi_3(x_1)\varphi_2(x_2)\\
f_3(\mathbf{x}) &= \varphi_Q(x_1)\varphi_3(x_2)\\
f_3(\mathbf{x}) &= \varphi_1(x_1)\varphi_{Q-1}(x_2)\\
&\dots\\
f_U(\mathbf{x}) &= \varphi_3(x_1)\varphi_1(x_2)
\end{aligned}$$

7.1 Χρήσιμες ορθοκανονικές συναρτήσεις

Η αναζήτηση ορθογώνιων συναρτήσεων αποτελεί ένα πρόβλημα της μαθηματικής ανάλυσης. Σε πρακτικά προβλήματα χρησιμοποιούμε συνήθως γνωστά σύνολα ορθογώνιων συναρτήσεων. Τα πιο γνωστά σύνολα ορθογώνιων συναρτήσεων είναι πολυωνυμικής μορφής διότι έχουν το πλεονέκτημα της μικρότερης υπολογιστικής πολυπλοκότητας.

Στην συνέχεια δίνονται τα πλέον γνωστές ομάδες ορθογώνιων πολυωνυμικών συναρτήσεων.

7.1.1 Πολυώνυμα Hermite

Τα πολυώνυμα Hermite είναι ορθογώνια με συνάρτηση βαρύτητας την $w(x) = e^{-x^2}$ στο διάστημα $-\infty < x < +\infty$ και υπολογίζονται από τον ακόλουθη αναδρομική σχέση:

$$H_{i+1}(x) - 2xH_i(x) + 2iH_{i-1}(x) = 0, \qquad i = 1, 2... \tag{7.7}$$

με $H_0(x) = 1$ και $H_1(x) = 2x$.

Τα πρώτα πέντε πολυώνυμα Hermite είναι τα ακόλουθα:

$$\begin{aligned} H_0(x) &= 1 \\ H_1(x) &= 2x \\ H_2(x) &= 4x^2 - 2 \\ H_3(x) &= 8x^3 - 12x \\ H_4(x) &= 16x^4 - 48x^2 + 12 \end{aligned} \tag{7.8}$$

7.1.2 Πολυώνυμα Laguerre

Τα πολυώνυμα Laguerre είναι ορθογώνια με συνάρτηση βαρύτητας την $w(x) = e^{-x}$ στο διάστημα $0 \leq x < +\infty$ και υπολογίζονται από τον ακόλουθη αναδρομική σχέση:

$$L_{i+1}(x) - (2i + 1 - x)L_i(x) + i^2 H_{i-1}(x) = 0, \qquad i = 1, 2... \tag{7.9}$$

με $L_0(x) = 1$ και $L_1(x) = 1 - x$.

Τα πρώτα πέντε πολυώνυμα Laguerre είναι τα ακόλουθα:

$$\begin{aligned} L_0(x) &= 1 \\ L_1(x) &= 1 - x \\ L_2(x) &= x^2 - 4x + 2 \\ L_3(x) &= -x^3 + 9x^2 - 18x + 6 \\ L_4(x) &= x^4 - 16x^3 + 72x^2 - 96x + 24 \end{aligned} \tag{7.10}$$

7.1.3 Πολυώνυμα Legendre

Τα πολυώνυμα Legendre είναι ορθογώνια με συνάρτηση βαρύτητας την $w(x) = 1$ στο διάστημα $x \in [-1, 1]$. Υπολογίζονται από τον ακόλουθη αναδρομική σχέση:

$$(i + 1)G_{i+1}(x) - (2i + 1)xG_i(x) + iG_{i-1}(x) = 0, \qquad i = 1, 2... \tag{7.11}$$

με $G_0(x) = 1$ και $G_1(x) = x$.

Τα πρώτα πέντε πολυώνυμα Legendre είναι τα ακόλουθα:

$$\begin{aligned} G_0(x) &= 1 \\ G_1(x) &= x \\ G_2(x) &= \tfrac{3}{2}x^2 - \tfrac{1}{2} \\ G_3(x) &= \tfrac{5}{2}x^3 - \tfrac{3}{2}x \\ G_4(x) &= \tfrac{35}{8}x^4 - \tfrac{15}{4}x^2 + \tfrac{3}{8} \end{aligned} \tag{7.12}$$

7.2 Αλυτα Προβλήματα

Ασκηση 33 *Βρείτε τα πρώτα δέκα πολυώνυμα Hermite.*

Ασκηση 34 *Αποδείξτε το θεώρημα 18.*

Ασκηση 35 *Φτιάξτε ένα σύνολο οχτώ ορθοκανονικών συναρτήσεων για τον χώρο δύο διαστάσεων με βάσει των πολυωνύμων Hermite.*

Ασκηση 36 *Βρείτε το σύνολο των ορθοκανονικών συναρτήσεων με βάσει τα ορθογώνια πολυώνυμα Laguerre.*

Ασκηση 37 *Βρείτε το σύνολο των ορθοκανονικών συναρτήσεων με βάσει τα ορθογώνια πολυώνυμα Hermite.*

Ασκηση 38 *Βρείτε το σύνολο των ορθοκανονικών συναρτήσεων με βάσει τα ορθογώνια πολυώνυμα Legendre.*

Ασκηση 39 *Βρείτε τα πρώτα δέκα πολυώνυμα Legendre.*

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