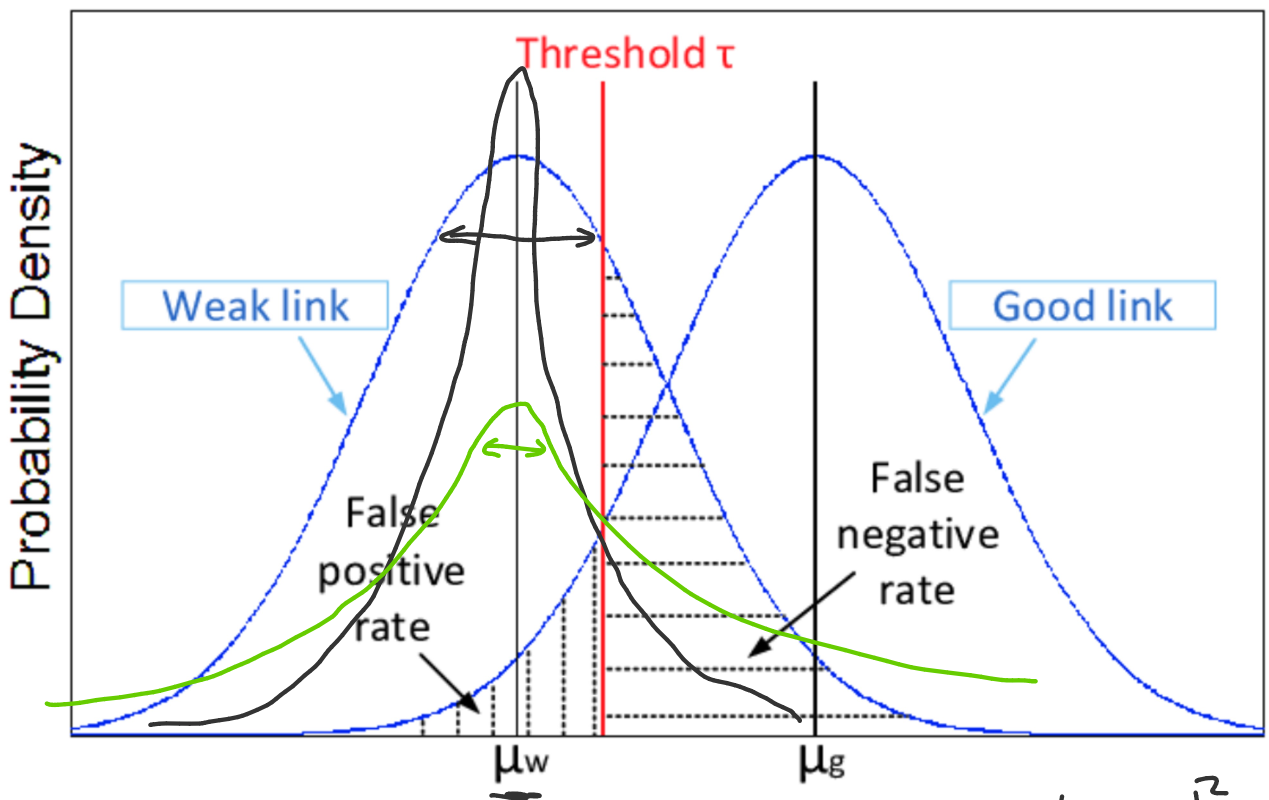
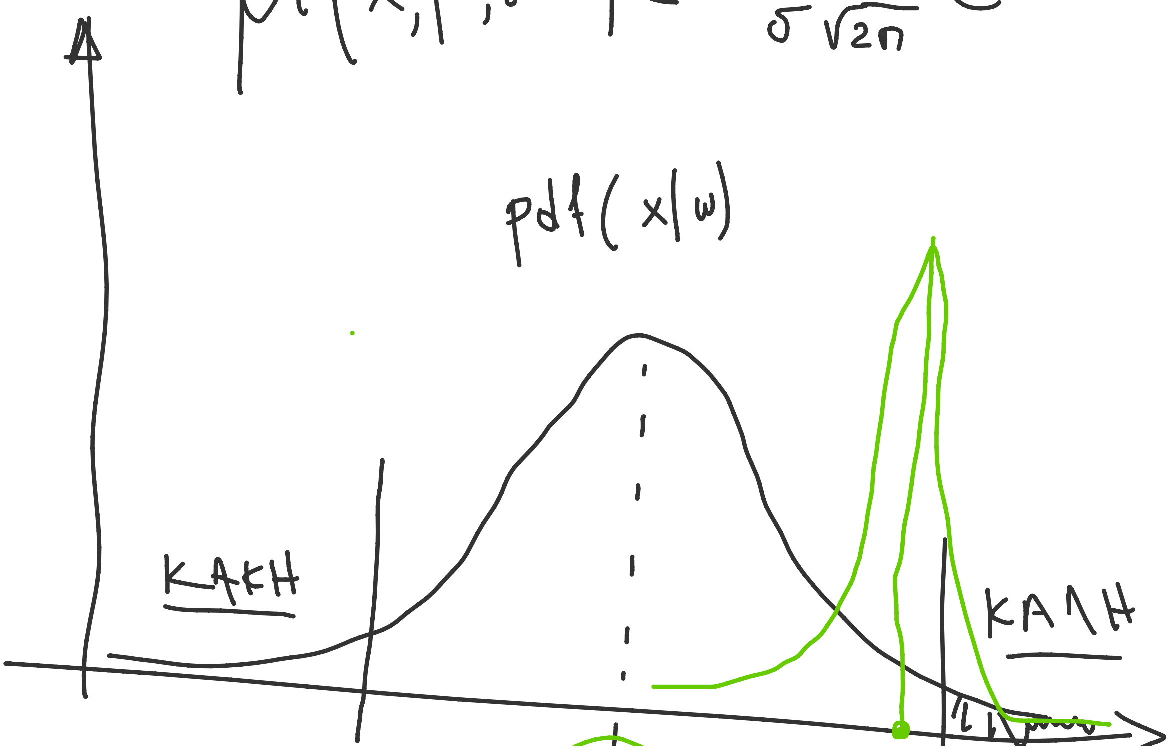
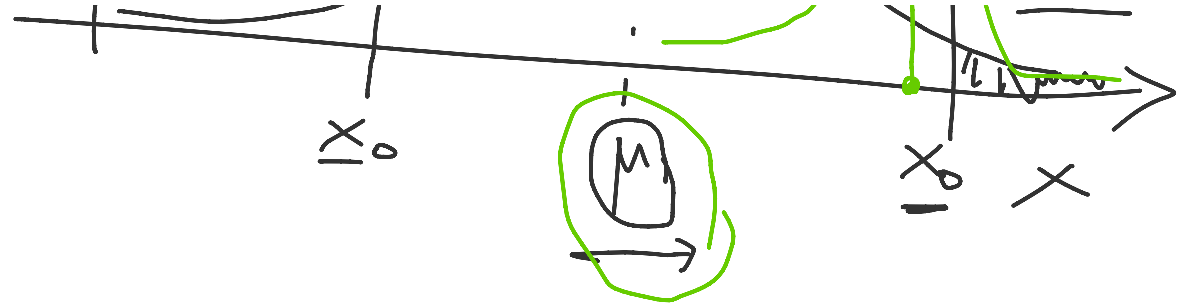
# Αλλαγή Μεταβλητής

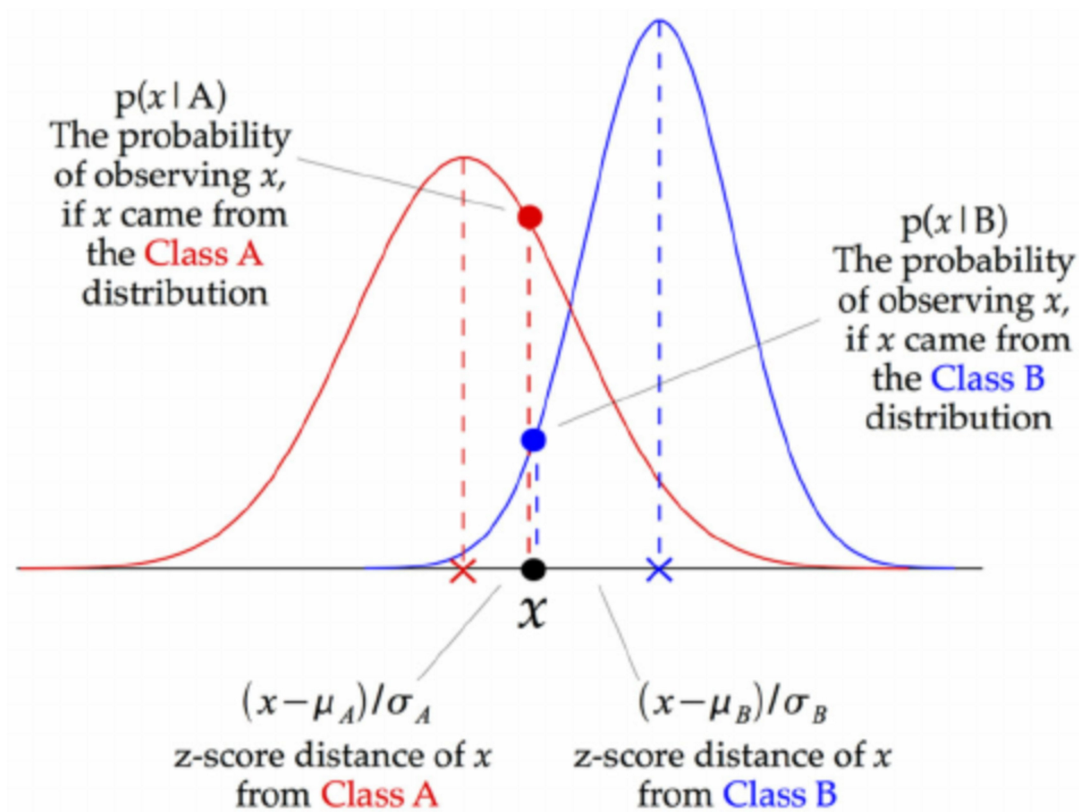
Wednesday, May 6, 2020 12:07 PM




$$M(x,\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

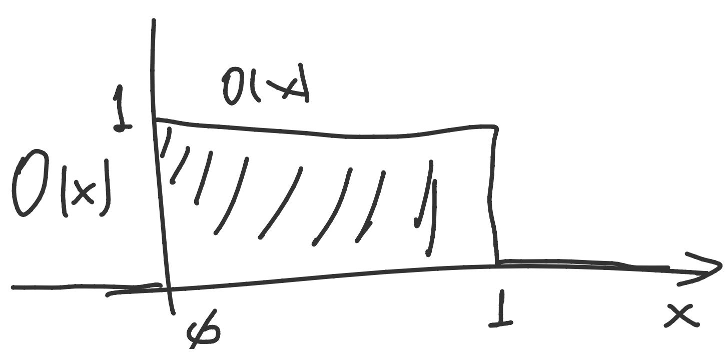





$$X \sim f_X(x) \longrightarrow y = \underbrace{g(x)} \sim f_y(y)$$

$X \sim$ ομοιομ$(0,1) = O(x)$




$$\boxed{D(x) = \begin{cases} 1, & \phi \le x \le 1 \\ \phi, & \text{αλλού} \end{cases}}$$

$\Rightarrow f_Y(y)$

$$x \longrightarrow \boxed{\alpha x + \beta} \xrightarrow{\ y\ }$$

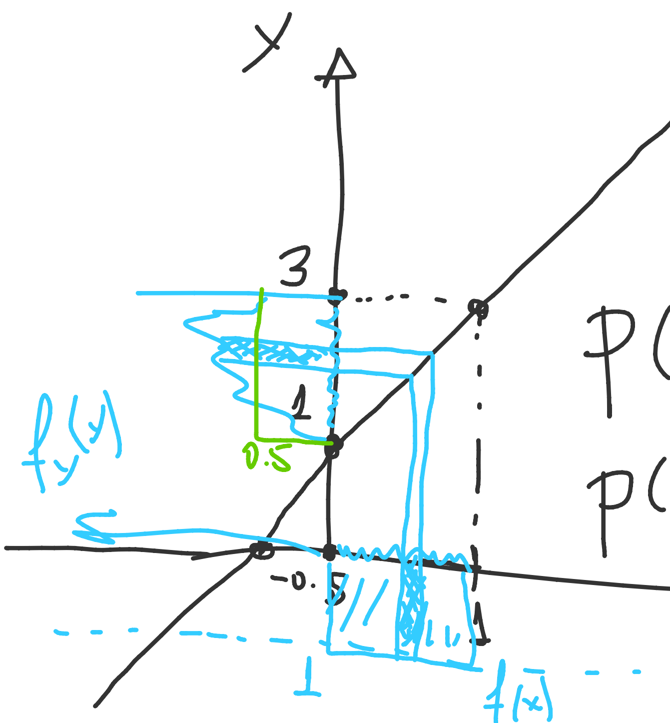
$$\boxed{y = \alpha x + \beta}$$

$$\underline{y = 2x + 1}$$

$$\boxed{x = \frac{y}{\alpha} - \frac{b}{\alpha}}$$

$$\boxed{x \in [x_0, x_0 + \Delta x]}$$

$$y \in [y_0, y_0 + \Delta y$$




$$P(x \in [x_0, x_0 + \Delta x]) =$$

$$p(y \in [y_0, y_0 + \Delta y])$$

$x = \phi \rightarrow y = 1$

$x = 1 \rightarrow y = 3$

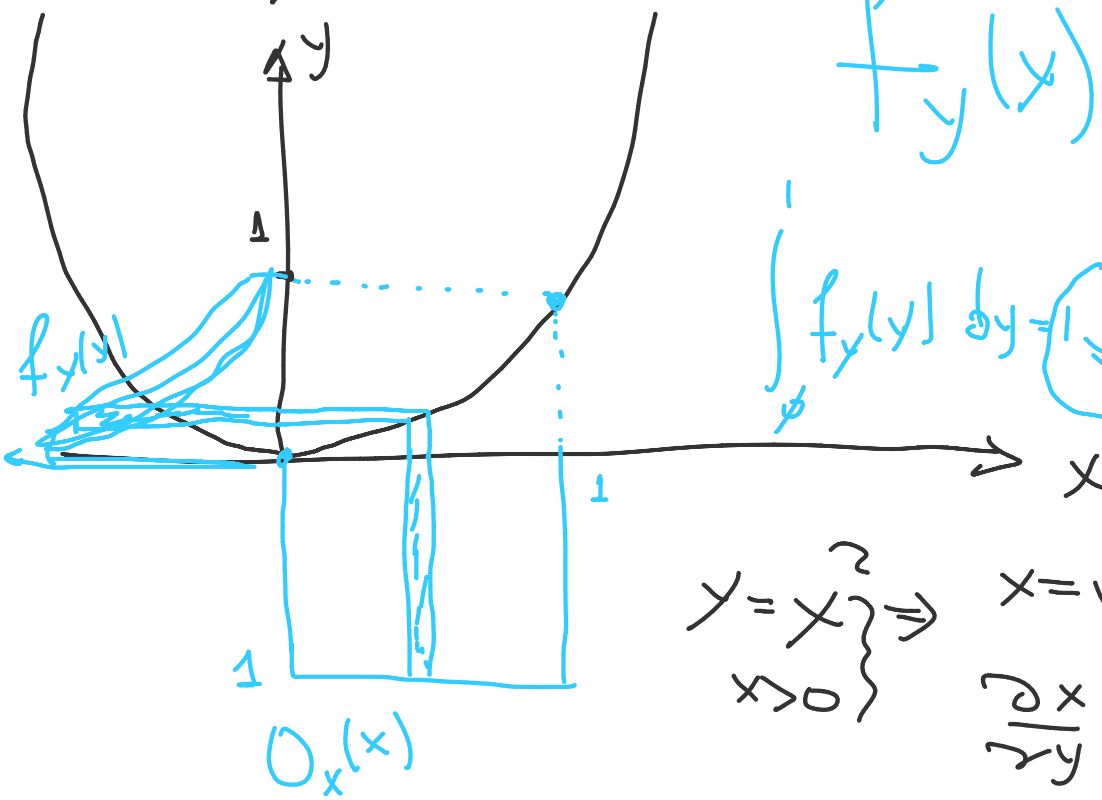
$$\boxed{f_X(x)|dx| = f_Y(y)|dy|} \Rightarrow f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|$$

$$f_y(y) = O_x(x)\left|\frac{1}{\alpha}\right| \Rightarrow \boxed{f_y(y) = \begin{cases} \frac{1}{2}, & \underline{1 \leq y \leq 3} \\ \emptyset, & \text{αλλού} \end{cases}}$$

$x = \emptyset \longrightarrow y = \emptyset$

$x = 1 \longrightarrow y = 1$

$$y = x^2$$




$$f_y(y) = \begin{cases} \frac{1}{2\sqrt{y}}, & 0 \leq y \leq 1 \\ \emptyset, & \text{αλλού} \end{cases}$$

$$\int_\emptyset^1 f_y(y)\, dy = 1 = \int_\emptyset^1 \frac{1}{2\sqrt{y}}\, dy$$

$$\left.\begin{array}{l} y = x^2 \\ x > 0 \end{array}\right\} \Rightarrow x = \sqrt{y}$$

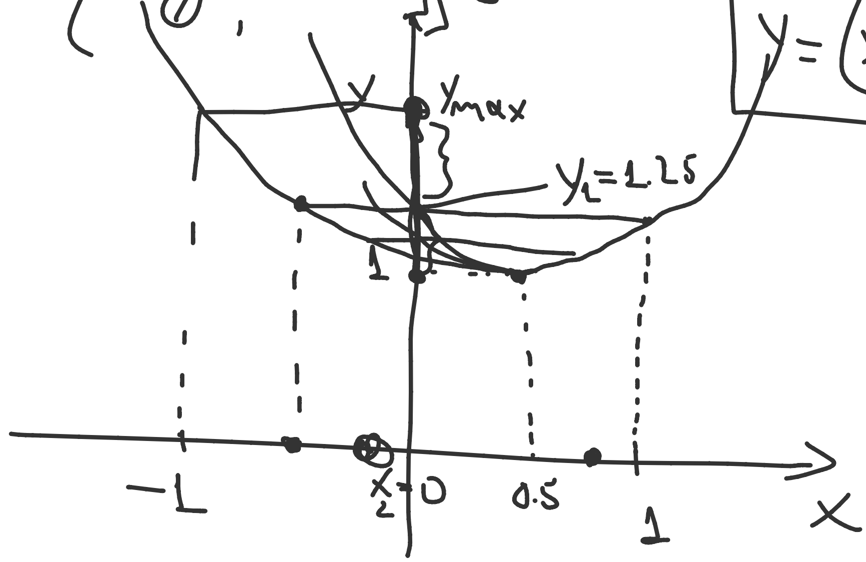
$$\frac{\partial x}{\partial y} = \frac{\partial}{\partial y}\left(y^{1/2}\right) = +\frac{1}{2}y^{-1/2} = \frac{1}{2\sqrt{y}}$$

$$f_y(y) = \overset{1}{f_x(x)}\left|\frac{\partial x}{\partial y}\right| = \frac{1}{2\sqrt{y}}$$

$$0 \leq y \leq 1$$

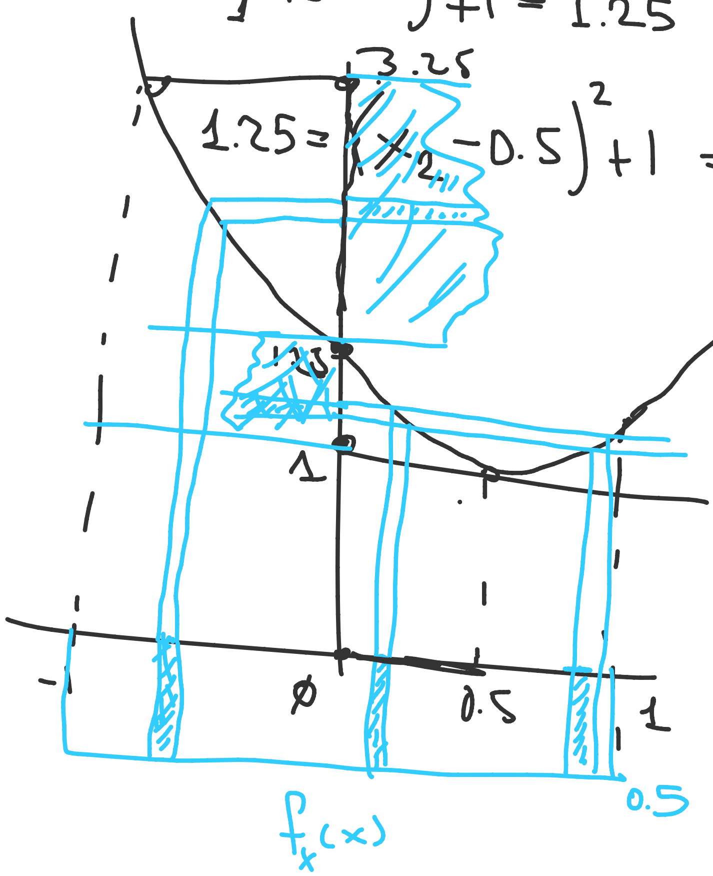
$$f(x) = \begin{cases} \frac{1}{2} & -1 \leq x \leq 1 \end{cases}$$

$$f_X(x)=\begin{cases}\frac{\;}{2} & -1\le x\le 1\\ \emptyset, & \text{אחרת}\end{cases}$$




$$\boxed{y=(x-0.5)^2+1}$$

$$y_1=(1-0.5)^2+1=1.25$$




$$1.25=(x_2-0.5)^2+1 \Rightarrow (x_2-0.5)^2=0.25 \Rightarrow$$

$$\Rightarrow x_2=\emptyset$$

$$y_{max}=(-1.5)^2+1=3.25$$

$$f_Y(y)=\begin{cases}\frac{1}{4\sqrt{y-1}} & , 1.25\le y\le 3.25\\ \boxed{\quad} & , \underline{1\le y\le 1.25}\\ \emptyset & , \text{אחרת}\end{cases}$$

0.5

$$f_Y(y)\,|dy|=f_X(x)\,|dx|+f_X(x)\,|dx|$$

$|dx|$

$$f_y(y)|\partial y| = f_x(x)|\partial x| + f_x(x)|\partial x| \qquad x \in (0, 0.5) \quad x \in (0.5, 1)$$

$$f_y(y) = f_x(x)\left|\frac{\partial x}{\partial y}\right| =$$

$$\boxed{y = (x-0.5)^2 + 1} \Rightarrow y - 1 = (x-0.5)^2 \Rightarrow$$

$(-1, 0)$

$$|x - 0.5| = \sqrt{y-1} \Rightarrow 0.5 - x = \sqrt{y-1} \Rightarrow$$

$$x = 0.5 - \sqrt{y-1} \Rightarrow \frac{\partial x}{\partial y} = -\frac{1}{2\sqrt{y-1}} \Rightarrow \left|\frac{\partial x}{\partial y}\right| = \frac{1}{2\sqrt{y-1}}$$

$$f_y(y) = \frac{1}{4}\,\frac{1}{\sqrt{y-1}}$$

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$$f_X(x) \xrightarrow{y = g(x)} f_y(y)$$

$$X \sim f_X(x) \longrightarrow H(f_x)$$

$$\left.\begin{array}{l} X \sim f_X(x) \\ y \sim f_y(y) \end{array}\right\} \quad Z = x + y \Longrightarrow f_z(z)$$

$$y \sim f_y(y) \longrightarrow H(f_y)$$

$$f_z(z) = f_x(x) \otimes f_y(y)$$

$$= \int_{z \in \Phi_z} f_x(x)\, f_y(x-z)\, dx$$

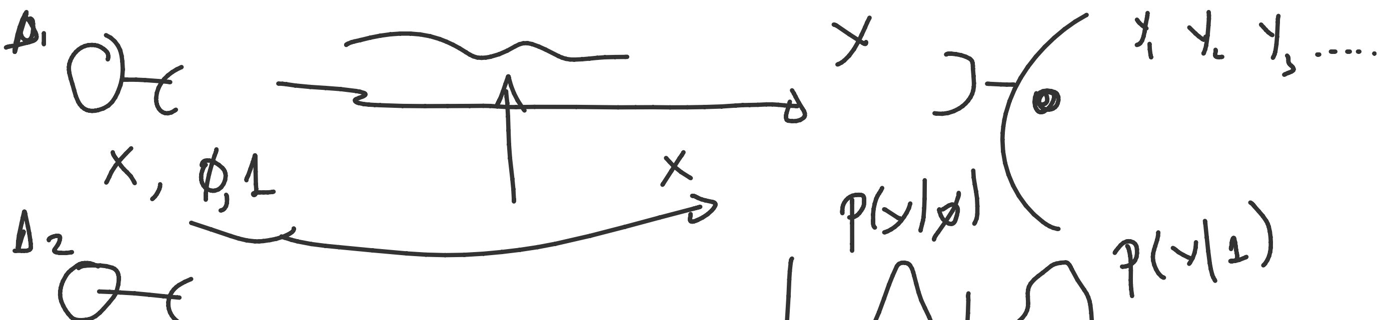
$$X + \underbrace{Y + Z}$$

$$y = \frac{z}{N}$$

$$\frac{\overbrace{(x_1 + x_2 + x_3 + \cdots\cdots + x_N)}^{=z}}{N} = y$$

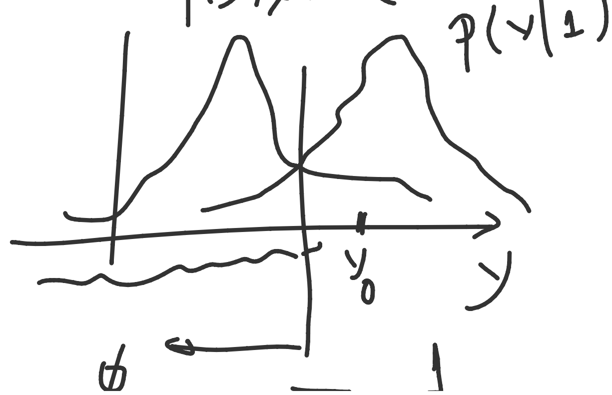
$$f_y(y) = ?$$

$$y = A + x, \quad x \sim N$$




$P(\phi|y_0)$

$P(1|y_0)$



$p(z|y_0)$

$\phi \longleftarrow\!\!\mid \quad \}$

$y < T$