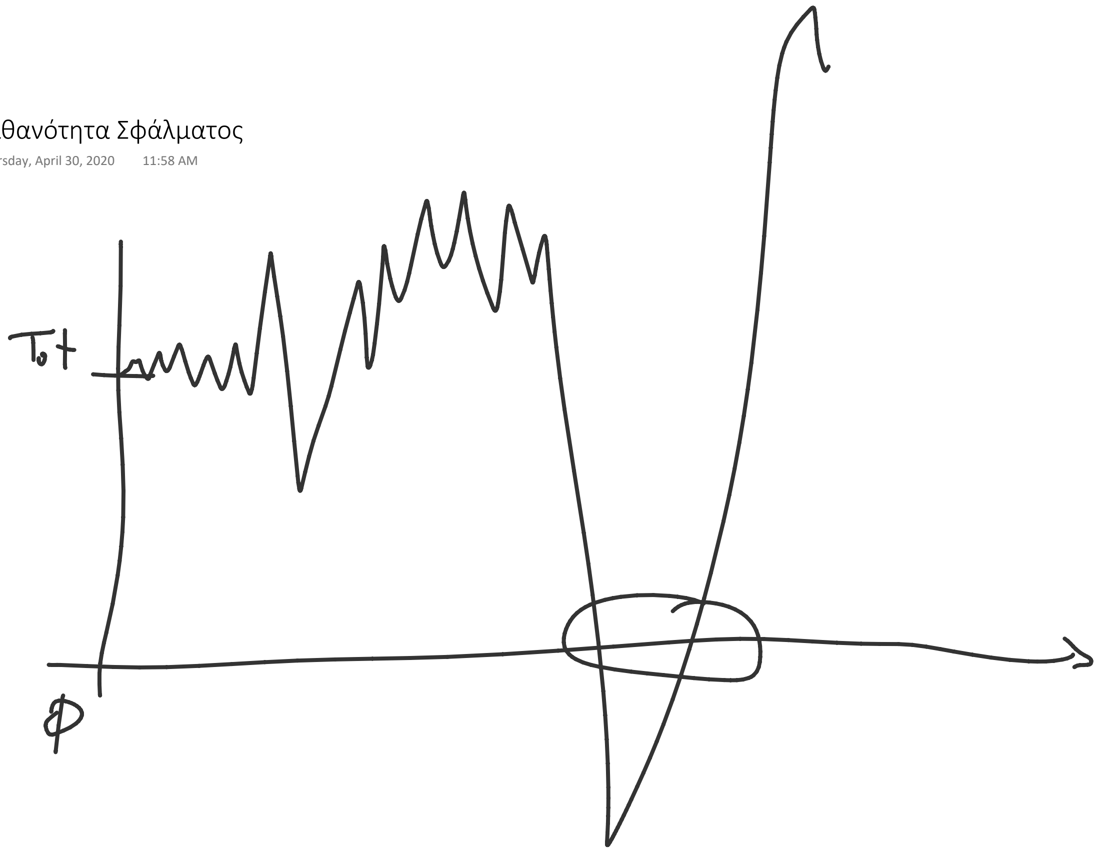
Πιθανότητα Σφάλματος

Thursday, April 30, 2020 11:58 AM



$$\boxed{P(\Phi_i \mid \underline{x})} \qquad \underline{x} \to \boxed{} \to \Phi_i, \quad i=1,M$$

ΕΤ. ΤΑΞΙΝ.

$$\boxed{\Phi_{best} = \arg\max_{\Phi_i} P(\Phi_i \mid \underline{x})}$$

$$\max_{\Phi_i} P(\Phi_i \mid \underline{x}) = \max_{\Phi_i} \frac{p(\underline{x} \mid \Phi_i) \cdot P(\Phi_i)}{\cancel{P(\underline{x})}} =$$

ω_i ω_i $\cancel{p(\underline{x})}$

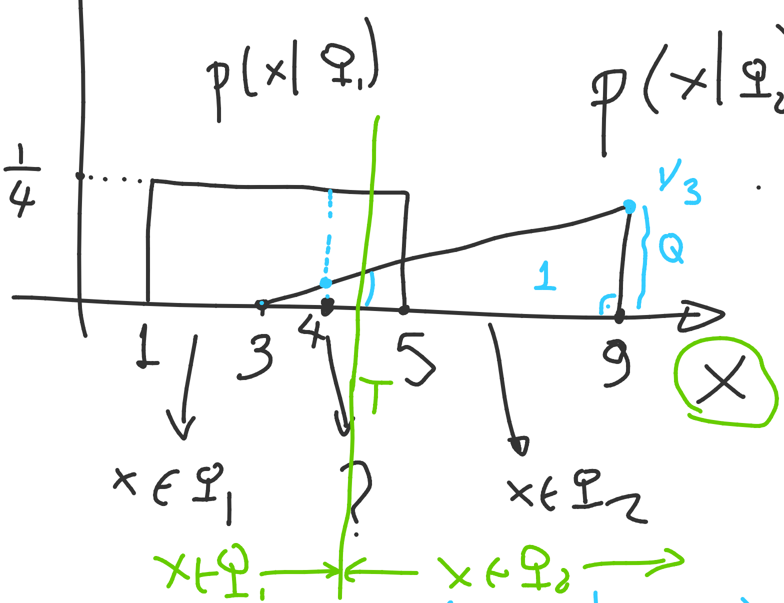
$$\boxed{= \max_{\omega_i} \underbrace{p(\underline{x}|\omega_i)}\cdot p(\omega_i)}$$

$p(\omega_1) = \frac{1}{3}$ $p(\omega_2) = \frac{2}{3}$

$1/4 \cdot \frac{1}{3} = \frac{1}{12}$

$/\!/$

$p(x=4|\omega_1)\cdot p(\omega_1)$



$p(x=4|\omega_2)\, p(\omega_2)$

$= \left(\frac{1}{18}\cdot 4 - \frac{1}{6}\right)\cdot 2/3 = (\quad)$

$p(x|\omega_2) = \begin{cases} \frac{1}{18}x - \frac{1}{6} & 3<x<9 \\ 0, & \dots \end{cases}$

$Q = p(x=9|\omega_2)$

$\frac{1}{2}(9-3)\cdot Q = 1 \Rightarrow \boxed{Q = \frac{1}{3}}$

$(3, \emptyset), \ (9, 1/3)$

$y = ax + b$

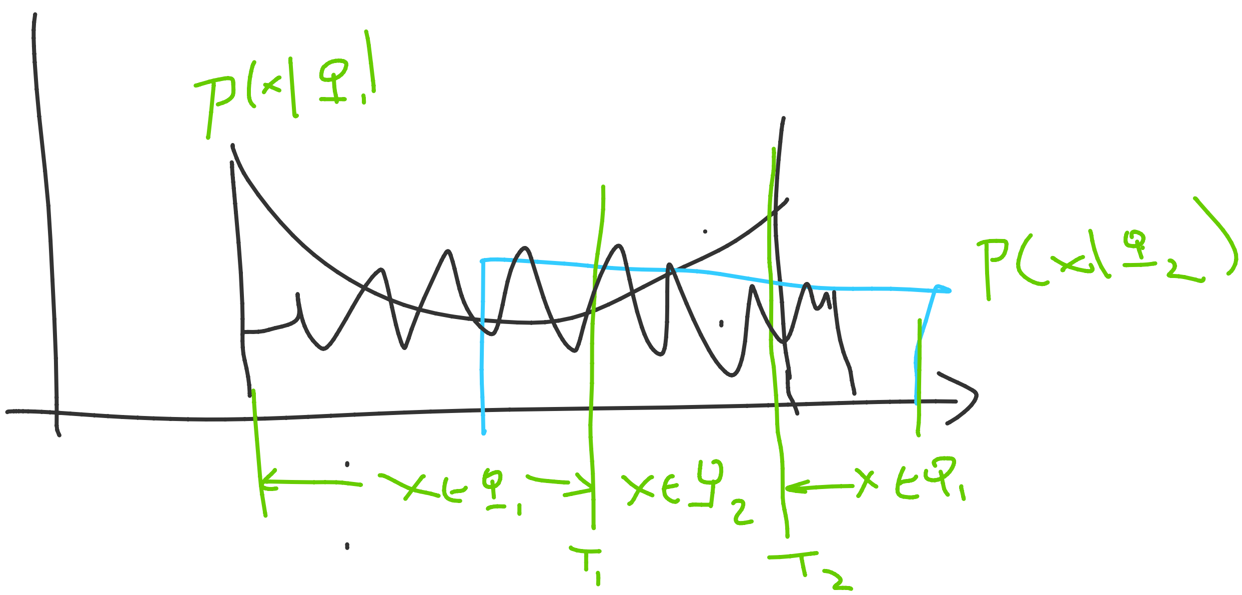
$\phi = 3\alpha + b$ $\qquad \frac{r}{3} = 9\alpha + \beta$

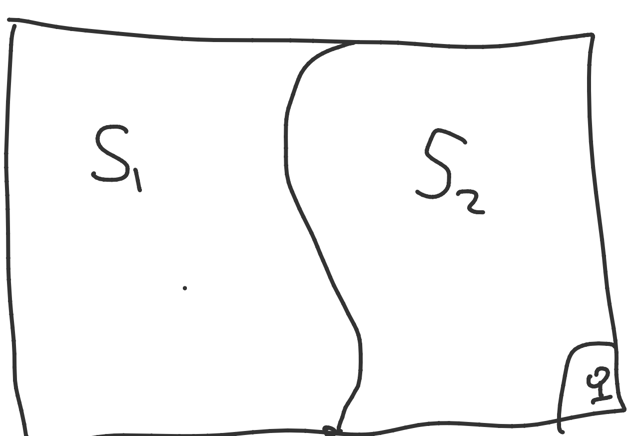
$b = -3\alpha$

$\Downarrow$

$\frac{1}{3} = 9\alpha - 3\alpha = 6\alpha \Rightarrow \boxed{\alpha = \frac{1}{18}}$

$b = -3\frac{1}{3\cdot 6} = \boxed{-\frac{1}{6} = b}$





$\underline{X}$

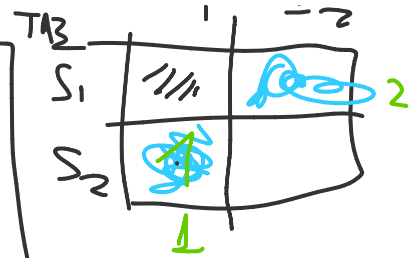
$S_1 \cup S_2 = \Phi$

ПРАГ

Φ_1 $\quad \Phi_2$

ТАЗ

S_1



$$\text{Αν } \underline{x} \in S_1 \xrightarrow{\text{ΟΡΙΖΩ}} \underline{x} \in \Omega_1$$

$$\text{Αν } \underline{x} \in S_2 \Longrightarrow \underline{x} \in \Omega_2$$

$$P(\underline{x} \in S_1, \underline{x} \in \Omega_1)$$

$$P(\text{ΣΦΑΛΜΑΤΟΣ}) = P(\underbrace{\underline{x} \in S_1, \underline{x} \in \Omega_2}_{S_1 \quad \Omega_2}) + P(\underbrace{\underline{x} \in S_2, \underline{x} \in \Omega_1}_{S_2 \quad \Omega_1}) =$$

ΠΟΙΟ ΕΙΝΑΙ ΤΟ S_1 ΕΤΣΙ ΩΣΤΕ ΤΟ P(ΣΦΑΛΜΑ) ΝΑ ΕΙΝΑΙ ΕΛΑΧΙΣΤΟ

$$= P(S_1, \Omega_2) + P(S_2, \Omega_1) =$$

$$= P(S_1 \mid \Omega_2) \cdot P(\Omega_2) + P(S_2 \mid \Omega_1) \cdot P(\Omega_1)$$

$$= \int_{S_1} f(x \mid \Omega_2) \cdot P(\Omega_2)\, dx + \int_{S_2} f(x \mid \Omega_1) \cdot P(\Omega_1)\, dx =$$

$$P(\Omega_2) \int_{S_1} f(x \mid \Omega_2)\, dx + P(\Omega_1) \int_{S_2} f(x \mid \Omega_1)\, dx =$$

S_1 $\qquad\qquad\qquad\qquad S_2$

$$= P(\varphi_2)\left[1-\int_{S_2} f(x|\varphi_2)\,dx\right] + \quad -\|- \quad =$$

$$\int_{\Omega} f(x|\varphi_1)\,dx = 1 = \boxed{\int_{S_1} f(x|\varphi_1)\,dx} + \int_{S_2} f(x|\varphi_1)\,dx$$

$$\boxed{P(\varphi_2) + \int_{S_2} \left(p(\varphi_1)\, f(x|\varphi_1) - p(\varphi_2)\, f(x|\varphi_2)\right) dx =}$$

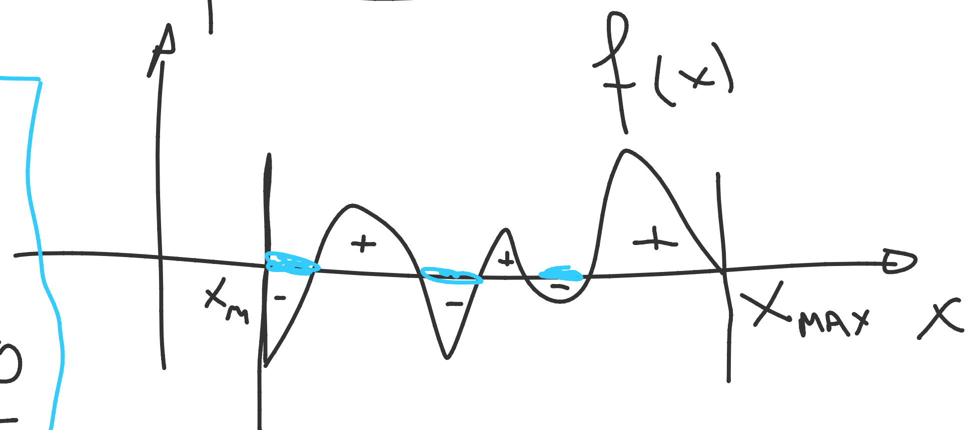
$S_2 : f(x|\varphi_1)\cdot p(\varphi_1) < f(x|\varphi_2)\cdot p(\varphi_2)$

$p \qquad\qquad\qquad p$

$$\min_{S_2} \int f(x)\,dx \Leftrightarrow$$

S_2

$\underline{S_2 : f(x) < 0}$

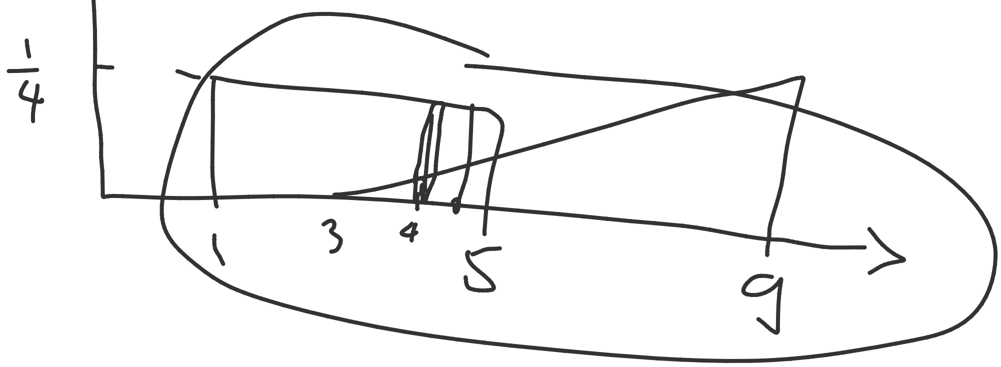


$$\frac{f(\underline{x}|\varphi_1)}{f(\underline{x}|\varphi_2)}$$

$$f(x|\varphi) = \begin{cases} \frac{1}{4}, & 1 < x < 5 \end{cases}$$

$$f(x|\varphi_1) = \begin{cases} \frac{1}{4}, & 1<x<5 \\ \varnothing, & \text{αλλού} \end{cases}$$

$$f(x|\varphi_2) = \begin{cases} \frac{1}{18}x - \frac{1}{6}, & 3<x<9 \\ \varnothing, & -''- \end{cases}$$



$$P([x,\Delta x] \,|\, \varphi_1)\cdot P(\varphi_1) > P([x,\Delta x] \,|\, \varphi_2)\cdot P(\varphi_2)$$

$$\underline{f(x=4|\varphi_1)}\cdot \cancel{\Delta} x \cdot P(\varphi_1) > f(x=4|\varphi_2)\cdot \cancel{\Delta x}\cdot P(\varphi_2)$$

$x=4$

$x=4.5$

A B , κοκ , κοκ

$$P(\varphi_1 \,|\, \underbrace{4, 4.5}) > P(\varphi_2 \,|\, \underbrace{4, 4.5}) \Rightarrow x \in \varphi_1$$

$$\downarrow$$

$$\frac{P(4,4.5|\varphi_1)\cdot \overset{1/3}{P(\varphi_1)}}{\cancel{P(4,4.5)}} > \frac{P(4,4.5|\varphi_2)\cdot \overset{2/3}{P(\varphi_2)}}{\cancel{P(4,4.5)}}$$

$\cancel{p(4, 4.5)}$ $\cancel{p(4, 4.5)}$

$\Downarrow$

$$p(4|\omega_1)\,p(4.5|\omega_1)\cdot p(\omega_1) > p(4|\omega_2)\cdot p(4.5|\omega_2)\cdot p(\omega_2)$$

$$\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{3} > \left(\frac{1}{18}\cdot 4-\frac{1}{6}\right)\left(\frac{1}{18}\cdot 4.5-\frac{1}{6}\right)\cdot\frac{2}{3}$$