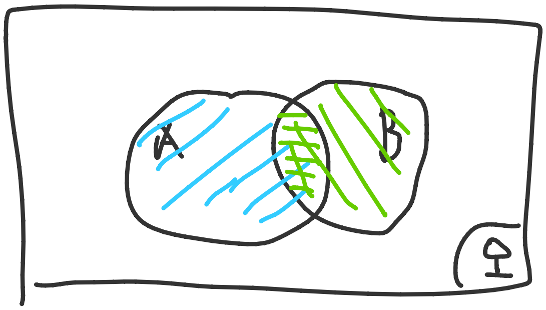
Ιδιότητες Πιθανοτήτων

Wednesday, April 29, 2020 9:58 AM



$$P(A \vee B) = P(A) + P(B) - P(A,B)$$

$$P(A,B) = P(A) + P(B) - P(A \vee B)$$

$$P(A,B \mid C) = P(A \mid C) + P(B \mid C) - P(A \vee B \mid C)$$

BAYES:

$$P(A \mid B) = \frac{P(A,B)}{P(B)} = \frac{P(B,A)}{P(B)} = \boxed{\frac{P(B \mid A) \cdot P(A)}{P(B)}}$$

$$\boxed{P(B \mid \Omega) = \frac{P(B)}{P(\Omega)} = \frac{P(B)}{1}}$$

ΠΑΡΑΔΕΙΓΜΑ

	Υγιής	Πάσχων
A_ρ	9900	3
$\Theta_{\varepsilon\tau}$ Test	5	92

10.000

$$P(\Pi) = \frac{95}{10.000} = 0.0095$$

$$P(Y) = \frac{9905}{10.000} = 1 - 0.0095$$

$$P(A) = \frac{9900+3}{10.000} = \frac{9903}{10000}$$

$$P(\Theta) = \frac{97}{10.000}$$

1). ΠΟΙΑ Η ΠΙΘΑΝΟΤΗΤΑ ΝΑ ΠΑΣΧΕΙ ΚΑΠΟΙΟΣ

(ΧΩΡΙΣ ΕΞΕΤΑΣΕΙΣ !!!)

(ΧΩΡΙΣ ΕΞΕΤΑΣΕΙΣ !!!)

$$p(\Pi|\Theta) = \frac{P(\Pi,\Theta)}{P(\Theta)} = \frac{\frac{92}{10000}}{\frac{97}{10000}} = \frac{92}{97} = 0.948$$

$$p(\Pi) = 0.0099$$

$$P(\Pi) = 10^{-6}$$

$$P(\Theta|\Pi) = 0.99$$

$$P(\Theta|Y) = 0.01$$

$$\boxed{p(\Pi|\Theta)} = \frac{p(\Theta|\Pi)\cdot p(\Pi)}{P(\Theta)} =$$

$$9.8\cdot 10^{-5}$$

$$= \frac{p(\Theta|\Pi)\cdot p(\Pi)}{p(\Theta,Y)+p(\Theta,\Pi)} = \frac{p(\Theta|\Pi)\cdot p(\Pi)}{p(\Theta|Y)\,p(Y)+p(\Theta|\Pi)\cdot p(\Pi)} =$$

$$= \frac{0.99\cdot 10^{-6}}{10^{-2}(1-10^{-6})+0.99\cdot 10^{-6}} = \frac{0.99\cdot 10^{-4}}{1-10^{-6}+0.99\cdot 10^{-4}} =$$

$$= \frac{99\cdot 10^{-6}}{1-10^{-6}+99\cdot 10^{-6}} = \frac{99\cdot \cancel{10^{-6}}}{10^{6}\cdot\cancel{10^{-6}}+98\cdot\cancel{10^{-6}}} =$$

$$= \frac{99}{10^{6}+98} = \frac{99}{1000098} = 9.899\cdot 10^{-5}$$

$$P(\pi \mid \theta_1, \theta_2) > P(\pi \mid \theta)$$

$$P(A \mid B, C) = \frac{P(A, B \mid C)}{P(B \mid C)} = \text{------ ...}$$

$$\boxed{P(A \mid B) = \frac{P(A, B)}{P(B)}}$$

$$I(x) = f(p(x))$$

$$I(x) \overset{?}{=} \frac{1}{p(x)} \geq \phi$$

$$P(A, B) = p(A) \cdot p(B) \quad \text{U!}$$

$$\boxed{I(A, B) = I(A) + I(B)}$$

$$I(A) \qquad I(B)$$

$$I(A) = \frac{1}{} \rightarrow \quad I(A, B) = \frac{1}{} \overset{\text{U!}}{=} \frac{1}{} \cdot \frac{1}{} \cdot =$$

$$I(A) = \frac{1}{p(A)} \Rightarrow I(A,B) = \frac{1}{p(A,B)} = \frac{1}{p(A)} \cdot \frac{1}{p(B)} =$$

$$I(A,B) = I(A) \cdot I(B)$$

$$\boxed{I(A) \equiv -\log_2 p(A)}$$

$$I(r) = -\log_2 \frac{1}{2} = \log_2 2 = 1$$

$$I(4) = -\log_2 \frac{1}{6} = \log_2 6 = \dots$$

$$I(4,3) = I(4) + I(3) = 2\log_2 6$$

$$\underline{I(x)} = \underline{-\log_2 p(x)}$$

$r \quad K$

x_1, x_2

$$\boxed{\langle I(x) \rangle = \sum_{x_i} I(x_i)\, p(x_i)}$$

$$H(p)$$

$$\langle I(x) \rangle = \int I \dots \quad \int p(x) \log p(x)\, dx$$

$$\langle I(x)\rangle = \int_{x\in\Phi} I(x)\,p(x)\,dx = \boxed{-\int_{x\in\Phi} p(x)\log_2 p(x)\,dx \quad \text{ΕΝΤΡΟΠΙΑΣ}}$$

$$K,\Gamma \qquad p_K = \frac{1}{4} \quad p_\Gamma = \frac{3}{4}$$

$$I(K) = -\log_2\frac{1}{4} = -(-\log_2 4) = \log_2 2^2 = 2\underbrace{\log_2 2}_{1} = 2$$

$$I(\Gamma) = -\log_2\frac{3}{4} = \log_2\frac{4}{3} = \log_2 4 - \log_2 3 =$$

$$= 2 - \log_2 3 = 0.415\ldots$$

$$H = -p_K\log_2 p_K - p_\Gamma \log_2 p_\Gamma = -\frac{1}{4}\log_2\frac{1}{4} - \frac{3}{4}\log_2\frac{3}{4} =$$

$$\frac{1}{4}\cdot 2 + \frac{3}{4}\cdot 0.415 = .81125 \rightarrow \text{bits} = H(\text{ΜΙΤ ΤΙΜΙΟ})$$

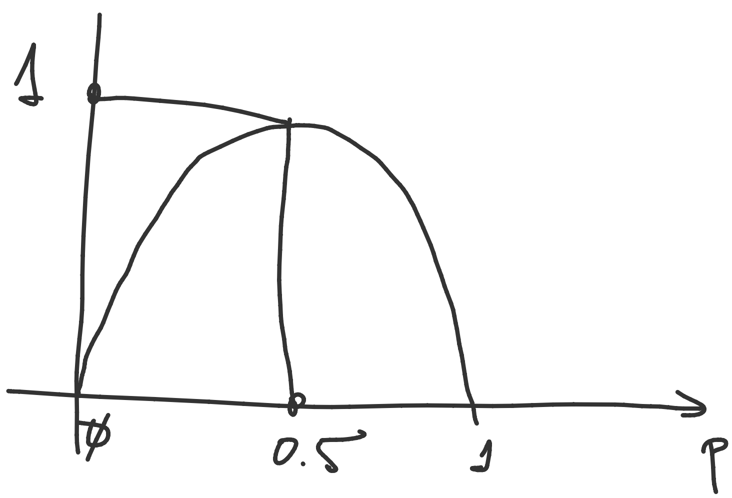
$$H = -2\,\frac{1}{2}\log_2\frac{1}{2} = 1 \text{ bit}$$

$$H = -6\cdot\frac{1}{6}\log_2\frac{1}{6} = \log_2 6 = \ldots$$

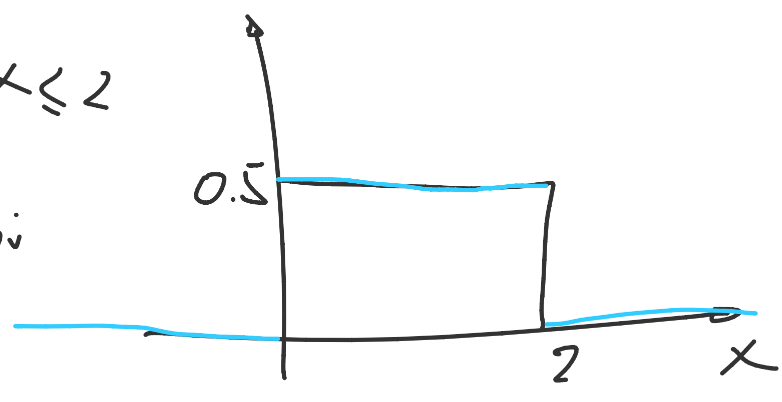
$$H = -6 \cdot \frac{1}{6} \log_2 \frac{1}{6} = \log_2 6 = \dots$$

$$p(k) = p$$
$$p(r) = 1 - p$$
$$0 \le p \le 1$$

$$H(p) = -p \log_2 p - (1-p) \log_2 (1-p)$$



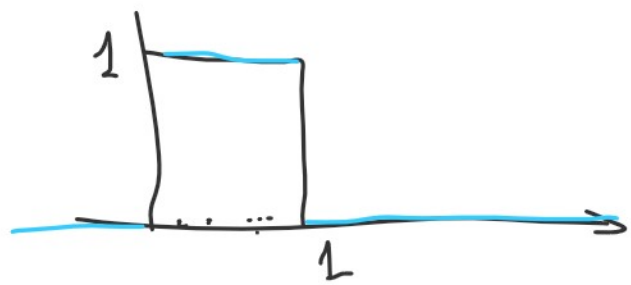
$$p_1(x) = \begin{cases} \frac{1}{2}, & 0 \le x \le 2 \\ 0, & \text{αλλού} \end{cases}$$



$$H(p_1) = -\int_{x\in[0,2]} p_1(x)\log_2(p_1(x))\,dx =$$

$$= -\int_0^2 \frac{1}{2}\log_2\frac{1}{2}\,dx = +\frac{1}{2}\int_\phi^2 1\,dx = 1$$

$$p_2(x) = \begin{cases} 1, & 0 \le x \le 1 \\ \phi, & \text{αλλού} \end{cases}$$

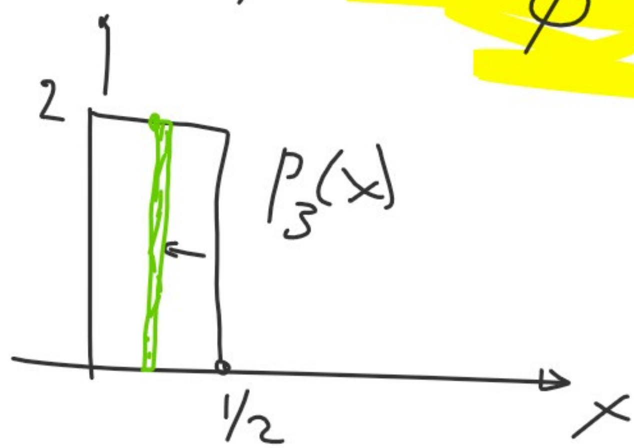


ΨΥΧΟΛΟΓΙΚΟ
↓ ΠΡΟΒΛΗΜΑ

$$H(p_2) = -\int_0^1 1\log_2\frac{1}{1}\,dx = \phi$$

ΕΝΤΡΟΠΙΑ
ϕ

$$p_3 = \begin{cases} 2, & 0 < x < 1/2 \\ \phi, & \text{αλλού} \end{cases}$$



$$H(p_3) = -\int_0^{1/2} 2\log_2 2\,dx < \phi$$

$= -1$