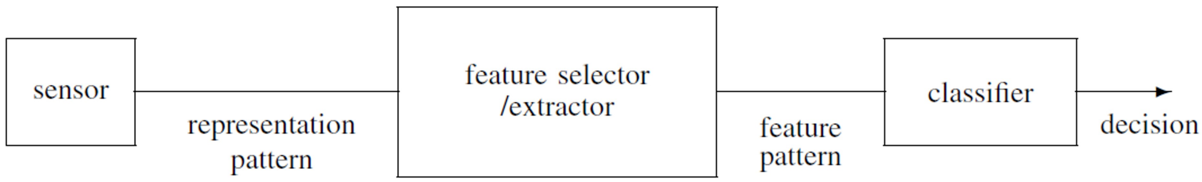
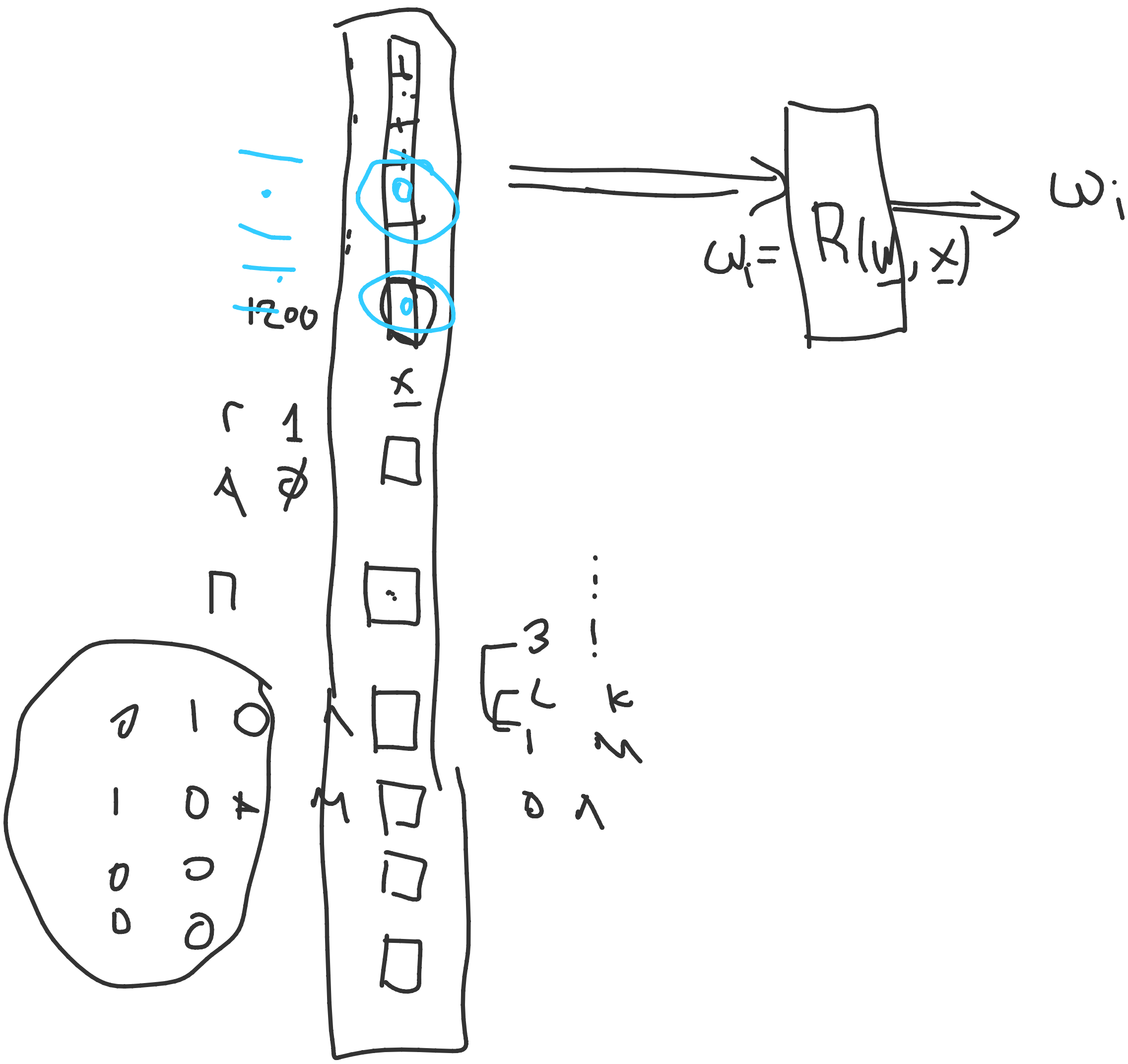
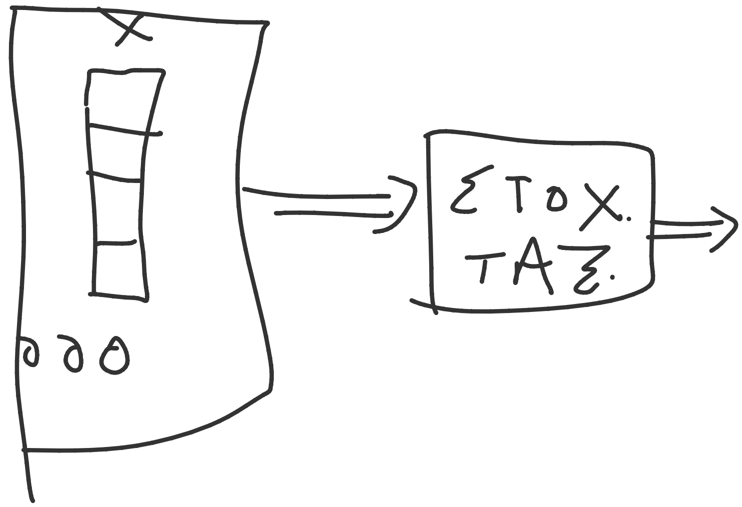
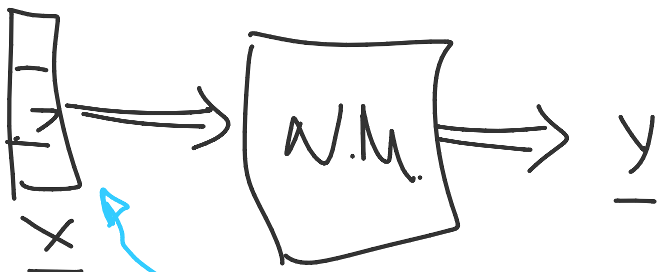
Βασική πιθανοθεωρία

Thursday, April 23, 2020 10:51 AM









$$p(X) = A + w$$

Θόρυβος

$$K, \Gamma \quad P(K) = 0.5 \qquad P(\Gamma) = 0.5$$

$1, \ldots, 6 \rightarrow p(1) = \frac{1}{6} = p(2) = \ldots\ldots p(6)$

Img. $\longrightarrow \boxed{6}$ $\quad$ 0×1 6 $\longrightarrow$ ΣAN $\begin{bmatrix} 1 \\ 2 \\ \vdots \\ 6 \end{bmatrix}$

$$6000 \to N(1) = 996$$
$$\to N(2) = 1010$$
$$\vdots$$
$$N(6) = \ldots$$
$$\overline{}$$
$$6000$$

0.306567 $\quad$ 0.30555..... $\quad$ 0.305397

$$X \in \{\underbrace{x_1, \ldots, x_N}\} \quad N \in \mathbb{N}.$$

$$P(x_N)$$

$X \in \mathbb{R} \rightarrow$ ΣΠΠ pdf $p(x)$

$$\boxed{P(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} p(x)\,dx}$$

$\Downarrow$

$P(\cdot) \in [0,1]$

$$\sum_{\forall x_i} P(x_i) = 1 \qquad \int_{x \in \Omega} p(x)\,dx = 1$$

$\forall x_i : P(x_i) \geq 0 \qquad p(x) \geq 0 \quad \forall x \in \Omega$

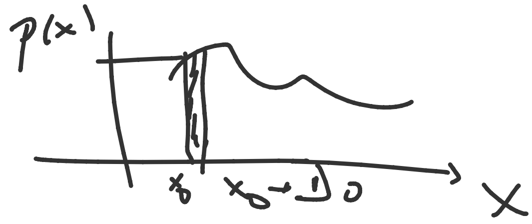
X ΣΥΝΕΧΗΣ ΣΤΟ $\mathbb{R}$ $\qquad p(x)$ ΣΥΝ ΠΑΡ

$| X = x_0 \quad x_0 \in \mathbb{R}$

$$\boxed{P(X = x_0) = 0}$$

$$P(x|y)$$

ɟxt' ΣYMBel



Ɛ.ה.ה

$$P(x_0 < x \le x_0 + \Delta x_0) = \int_{x_0}^{x_0+\Delta x_0} p(x)\,dx \triangleq p(x_0)\underbrace{\Delta x_0}_{\to 0}$$

$$P(x|y) = \frac{P(x,y)}{p(y)} \Rightarrow P(x,y) = P(x|y)\cdot p(y)$$

$$\Rightarrow P(y,x) = P(y|x)\cdot p(x)$$

$$\Downarrow$$

$$P(x|y) = \frac{P(x,y)}{p(y)} = \frac{P(y,x)}{p(y)}$$

$$\boxed{P(x|y) = \frac{P(y|x)\cdot p(x)}{p(y)} \quad \text{BAYES}}$$

ΣΤΟΧΑΣΤΙΚΑ ΜΟΝΤ. | ΣΤΑΤΙΣΤΙΚΗ

$x, \ p(x)$

$$\langle x \rangle = \mu = \int x\,p(x)\,dx \quad ?$$

$$\sigma^2 = \int (x-\mu)^2 p(x)\,dx$$

$$\{2, 3, 4, 1, 1, 2, 5\} = x_i$$

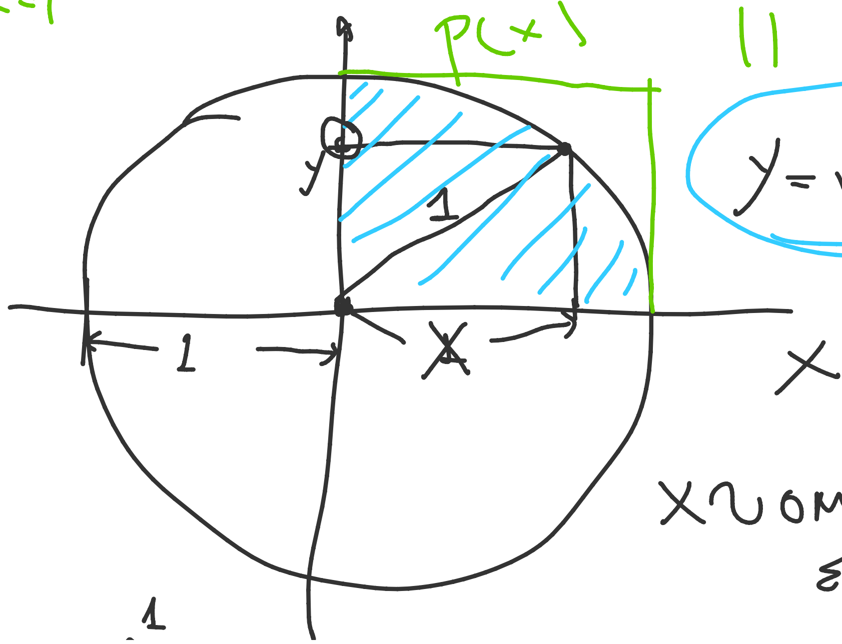
$$\mu_\sigma = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{18}{7} = 2.57$$

$$g(x) = y$$

$$\langle y \rangle = \langle g(x) \rangle = \int g(x)\,p(x)\,dx \simeq \frac{1}{N}\sum_{i=1}^{N} g(x_i)$$

$$\langle x \rangle = \cancel{\frac{1}{N}} \sum_{i=1}^{N} x_i\,p(x_i)$$

$$\cancel{\frac{1}{6}} \sum_{k=1}^{6} k\,\frac{1}{6} = \frac{1}{6}(1+2+3+4+5+6) = \frac{21}{6} = 3.5$$



$$y = \sqrt{1-x^2}$$

$X \sim$ ΟΜΟΙΟΜΟΡΦΗ

Σ.Π.Π $[0,1]$

1 2

ε.π.π [0,1]

$$E_\eta = \pi \cdot r^2 = \frac{\pi}{2} \cdot d^2$$

ΜΠΛΕ ΕΠΙΦ

$$\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}$$

$$\int_0^1 \underbrace{\sqrt{1-x^2}}_{g(x)} \cdot \underbrace{1}_{p(x)} \cdot dx \simeq \boxed{\frac{1}{N}\sum_{i=1}^{N}\sqrt{1-x_i^2} \simeq \frac{\pi}{4}}$$

$$\Rightarrow \pi \simeq \frac{4}{N}\sum_{i=1}^{N}\sqrt{1-x_i^2}$$

$$x_1, x_2, x_3, \dots\dots, x_N,$$