LINEAR VELOCITY

$$\dot{o}_0^n = \sum_{i=1}^{n} \frac{\partial o_0^n}{\partial q_i} \dot{q}_i \qquad J_{v,i} = \frac{\partial o_0^n}{\partial q_i}$$

– prismatic joint (i^{th} joint is prismatic)

$$A_0^n = \left[\begin{array}{c|c} R_0^n & o_0^n \\ \hline 0\ 0\ 0 & 1 \end{array}\right] = A_0^{i-1} A_{i-1}^{i} A_i^n$$

$$= \left[\begin{array}{c|c} R_0^{i-1} & o_0^{i-1} \\ \hline & 1 \end{array}\right] \left[\begin{array}{c|c} R_{i-1}^{i} & o_{i-1}^{i} \\ \hline & 1 \end{array}\right] \left[\begin{array}{c|c} R_i^n & o_i^n \\ \hline & 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} R_0^n & R_0^i o_i^n + R_0^{i-1} o_{i-1}^{i} + o_0^{i-1} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$$o_0^n = R_0^i o_i^n + R_0^{i-1} o_{i-1}^{i} + o_0^{i-1}$$

$$\frac{\partial o_0^n}{\partial q_i} = \frac{\partial}{\partial q_i} R_0^{i-1} o_{i-1}^{i} \qquad q_i \triangleq d_i$$

$$= R_0^{i-1} \frac{\partial}{\partial d_i} \begin{bmatrix} a_i c_i \\ a_i s_i \\ d_i \end{bmatrix} = \dot{d}_i R_0^{i-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \dot{d}_i z_0^{i-1}$$

$$\boxed{J_{v_i} = z_0^{i-1}}$$

- revolute joint $\quad q_i \triangleq \theta_i$

$$\frac{\partial}{\partial \theta_i} O_0^n = \frac{\partial}{\partial \theta_i}\left[R_0^i \bar{O}_i^n + R_0^{i-1} O_{i-1}^i \right]$$

$$= \frac{\partial}{\partial \theta_i} R_0^i O_i^n + R_0^{i-1} \frac{\partial}{\partial \theta_i} O_{i-1}^i$$

$$= \dot{\theta}_i S(Z_0^{i-1}) R_0^i O_i^n + \dot{\theta}_i S(Z_0^{i-1}) R_0^{i-1} O_{i-1}^i$$

$$= \dot{\theta}_i \; Z_0^{i-1} \times (O_0^n - O_{i-1}^n)$$

$$J_{v_i} = Z_0^{i-1} \times (O_0^n - O_{i-1}^n)$$

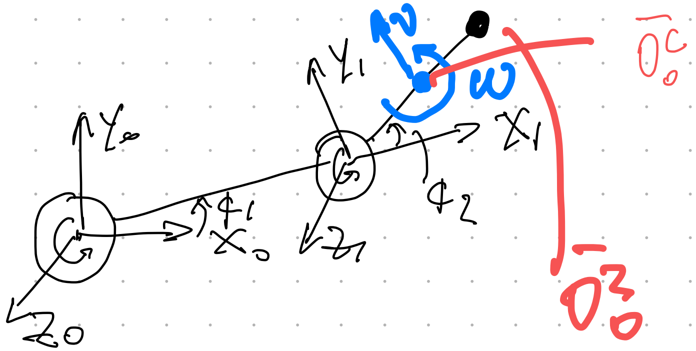
$$A_0^n = \left[\begin{array}{c|c} & O_0^n \\ \hline & \end{array}\right] \qquad A_{i-1}^n = \left[\begin{array}{c|c} & O_{i-1}^n \\ \hline & \end{array}\right]$$

$$A_0^{i-1} \left[\begin{array}{c|c} \; Z_0^{i-1} & \\ \hline & \end{array}\right]$$

$$J = \left[\cdots \left| \frac{J_{v_i}}{J_{w_i}} \right| \cdots \right]$$

i^{th} joint revolute vs $\left[\frac{Z_0^{i-1} \times (O_0^n - O_{i-1}^n)}{Z_0^{i-1}}\right]$

prismatic $\quad \left[\frac{Z_{i-1}}{0}\right]$ 3×1 3×1



$$\overset{\circ}{q}_1 = \overset{\circ}{\theta}_1$$

$$\overset{\circ}{q}_2 = \overset{\circ}{\theta}_2$$

$$J\begin{bmatrix}\theta_1\\ \theta_2\\ \ \end{bmatrix} = \left[\begin{array}{c|c} \bar{z}_0^0 \times (\bar{O}_0^c - O_0^0) & \bar{z}_0^1 \times (\bar{O}^c - O_0^1) \\ \hline \bar{z}_0^0 & \bar{z}_0^1 \end{array}\right]$$

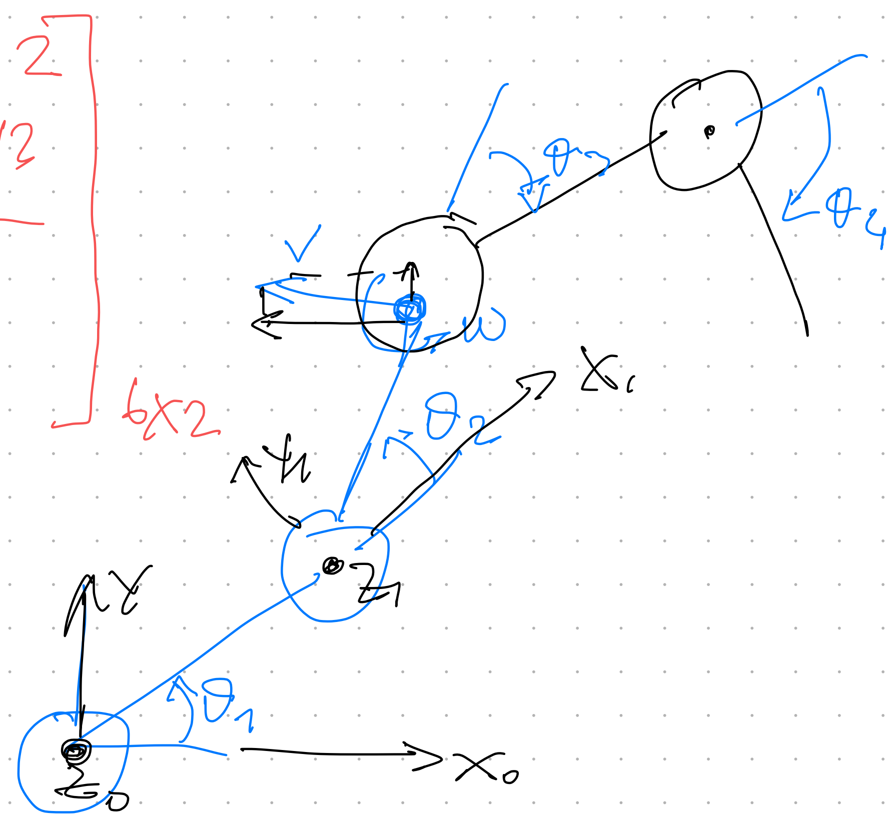
$$J = \left[\begin{array}{c|c} \bar{z}_0 \times (\bar{O}^2 - \bar{O}^0) & \bar{z}_0^1 \times (\bar{O}^2 - \bar{O}^1) \\ \hline \bar{z}_0 & \bar{z}_0^1 \end{array}\right]$$

$$\bar{O}^0 = \begin{bmatrix}0\\0\\0\end{bmatrix} \qquad \bar{O}^1 = \begin{bmatrix}a_1 C_1\\ a_1 S_1\\ 0\end{bmatrix} \qquad \bar{O}^2 = \begin{bmatrix}a_1 C_1 + a_2 C_{12}\\ a_1 S_1 + a_2 S_{12}\\ 0\end{bmatrix}$$

$$A_0^1 = \left[\begin{array}{c|c} \begin{matrix} & & 0\\ & & 0\\ & & 1\end{matrix} & \begin{matrix}a_1 C_1\\ a_1 S_1\\ 0\end{matrix} \\ \hline & \end{array}\right] \qquad \bar{z}_0 = \begin{bmatrix}0\\0\\1\end{bmatrix} \qquad \bar{z}_0^1 = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$A_0^2 = \left[\begin{array}{c|c} & \begin{matrix}a_1 C_1 + a_2 C_{12}\\ a_1 S_1 + a_2 S_{12}\\ 0\end{matrix} \\ \hline & 1 \end{array}\right]$$

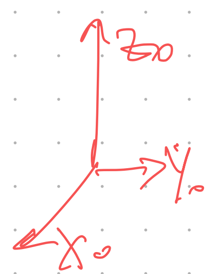
$$J = \left[\begin{array}{c|c} -a_1 S1 - a_2 S12 & -a_2 S12 \\ a_1 C1 + a_2 C12 & a_2 C12 \\ 0 & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{array}\right]_{6\times 2}$$



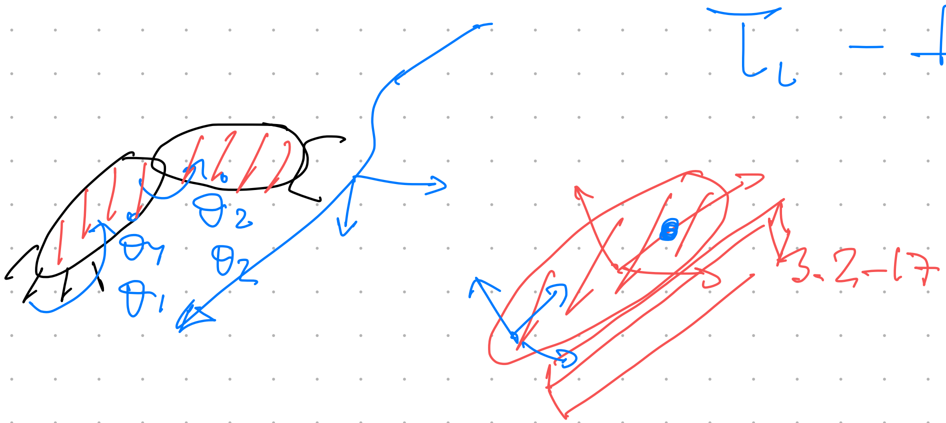
$$\begin{bmatrix} V \\ \hline W \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\begin{bmatrix} V \\ \hline W \end{bmatrix} = \left[\begin{array}{cccc} & & 0 & 0 \\ & & 0 & 0 \\ & & 0 & 0 \\ \hline & & 0 & 0 \\ & & 0 & 0 \\ & & 1 & 1 \end{array}\right]_{6\times 4} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} & \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$A_0^4 = \left[\begin{array}{c|c} R_0^4 & \\ \hline & 1 \end{array}\right]$$



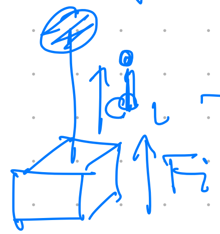
ROBOT DYNAMICS (Energy = Kinetic + Potential energy)



τ_i – torque if ith joint is revolute

$\rightarrow \theta_i$

if ith joint is prismatic



$\rightarrow d_i$

$$L = K + P$$

Lagrange-Euler

Chapter 3.2 L-E formulation

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i}$$

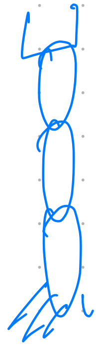
$$U_{ij} = \frac{\partial A_0^i}{\partial q_j}$$

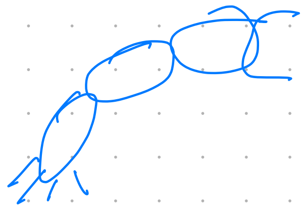
$$L = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{i} \left(Tr\left(U_{ij}\, \underbrace{J_i}_{\text{inertia matrix}}\, U_{ik}^T\right) \dot q_j \dot q_k \right) + \sum_{i=1}^{n} m_i \bar g A_0^i \bar r_i^i$$

$$T_i = \sum_{k=1}^{n} D_{ik}\ddot{q}_k + \sum_{k=1}^{n}\sum_{m=1}^{n} h_{ikm}\dot{q}_k\dot{q}_m + C_i(\bar{q})$$

inertial accelerations

Coriolis
centripetal
centri fugal



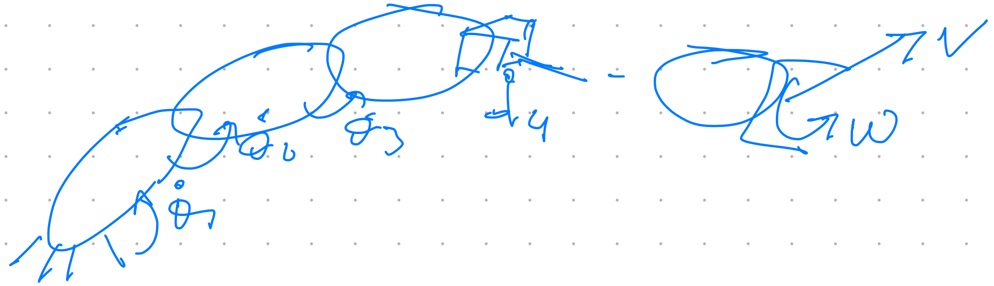


$$C_i = \sum_{j=i}^{n} \left(-m_j \bar{g}\, U_{ji}\, \bar{r}_j\right)$$

$$D_{ik} = D_{ki}$$

$$D_{ii}$$

$$T_i = D_{ii}\ddot{q}_i + C_i(\bar{q})$$



$$\begin{bmatrix} V \\ \hline \omega \end{bmatrix}_{6\times1} = J_{6\times n} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

let $n=6$

$$\dot{q} = J^{-1}_{6\times n} \begin{bmatrix} V \\ \hline \omega \end{bmatrix}$$

pseudoinverse
($n=7$, KUKA iiWA)

prosthetics