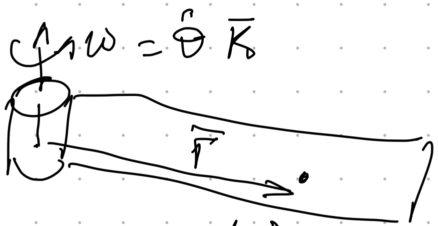
VELOCITY KINEMATICS – MANIPULATOR JACOBIAN



$\bar{v} = \bar{\omega} \times \bar{r}$

skew symmetric matrix $\quad S^T + S = 0 \quad s_{ij} + s_{ji} = 0$

$$\bar{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \qquad S(\bar{a}) \triangleq \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \qquad S(\bar{k}) = S\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

1 $S(\alpha\bar{a} + \beta\bar{b}) = \alpha S(\bar{a}) + \beta S(\bar{b})$

2 $S(\bar{a}) \cdot \bar{p} = \bar{a} \times \bar{p}$

3 If R orthogonal $\quad R(\bar{a} \times \bar{b}) = R\bar{a} \times R\bar{b}$

4 $R S(\bar{a}) R^T = S(R\bar{a})$

$$R S(\bar{a}) R^T \bar{b} = R(\bar{a} \times R^T \bar{b}) = (R\bar{a}) \times (R R^T b)$$
$$= (R\bar{a}) \times \bar{b} = S(R\bar{a})\bar{b}$$

$$R(\theta)\, R^T(\theta) = I$$

$$\frac{dR}{d\theta} R^T(\theta) + R(\theta) \frac{dR^T}{d\theta} = \phi$$

$$S \quad + \quad S^T = \phi$$

$$S \triangleq \frac{dR}{d\theta} R^T(\theta)$$

$$S^T = R(\theta) \frac{dR^T}{d\theta}$$

$$\boxed{\frac{dR}{d\theta} = S\, R(\theta)}$$

\# $$R = R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix}$$

$$\frac{dR}{d\theta} R^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -s\theta & -c\theta \\ 0 & c\theta & -s\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & s\theta \\ 0 & -s\theta & c\theta \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}}_{S(\bar{i})}$$

$$R = R_{z,\theta} \qquad \boxed{\frac{dR_{z,\theta}}{d\theta} = S(\bar{k}) R_{z,\theta}}$$

$$\dot{R}(t) = S(\bar{\omega}(t))\, R(t)$$

$\bar{\omega}(t)$ ↳ angular velocity

$$\dot{R} = \frac{dR}{dt} = \frac{dR}{d\theta}\frac{d\theta}{dt} = \dot{\theta}\, S(\bar{k})\, R(t) = S(\bar{\omega}(t))\, R(t)$$

$$A_0^1 = \begin{bmatrix} R_0^1 & \\ 0\ 0\ 0 & 1 \end{bmatrix} \qquad A_1^2 = \begin{bmatrix} R_1^2 & \\ 0\ 0\ 0 & 1 \end{bmatrix}$$

$$A_0^2 = \begin{bmatrix} R_0^1 R_1^2 & \\ 0\ 0\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_0^2 & \\ 0\ 0\ 0 & 1 \end{bmatrix}$$

$$R_0^2(t) = R_0^1(t)\, R_1^2(t)$$

$$\dot{R}_0^2(t) = \dot{R}_0^1(t)\, R_1^2(t) + R_0^1(t)\, \dot{R}_1^2(t)$$

current

$$S(\bar{\omega}_{0,0}^2) R_0^2 = S(\bar{\omega}_{0,0}^1) R_0^1 R_1^2 + R_0^1(t)\, S(\bar{\omega}_{1,1}^2) R_1^2$$

$$= S(\bar{\omega}_{0,0}^1) R_0^2 + R_0^1(t) S(\bar{\omega}_{1,1}^2) (R_0^1)^T R_0^1 R_1^2$$

$$= S(\bar{\omega}^1_{0,0}) R^2_0 + R^1_0 \, S(\bar{\omega}^2_{1,1})(R^1_0)^T R^2_0$$

$$S(\bar{\omega}^2_{0,0}) R^2_0 = S(\bar{\omega}^1_{0,0}) R^2_0 + S(R^1_0 \, \bar{\omega}^2_{1,1}) \quad R^2_0$$

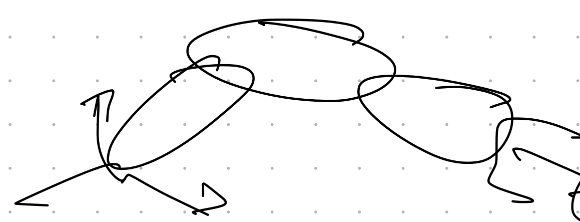
$$\bar{\omega}^2_{0,0} = \bar{\omega}^1_{0,0} + R^1_0 \, \bar{\omega}^2_{1,1}$$

$$R^n_0 = R^1_0 R^2_1 \cdots R^n_{n-1}$$

$$\bar{\omega}^n_{0,\phi} = \bar{\omega}^1_{0,\phi} + \bar{\omega}^2_{1,\phi} + \cdots + \bar{\omega}^n_{n-1,\phi}$$

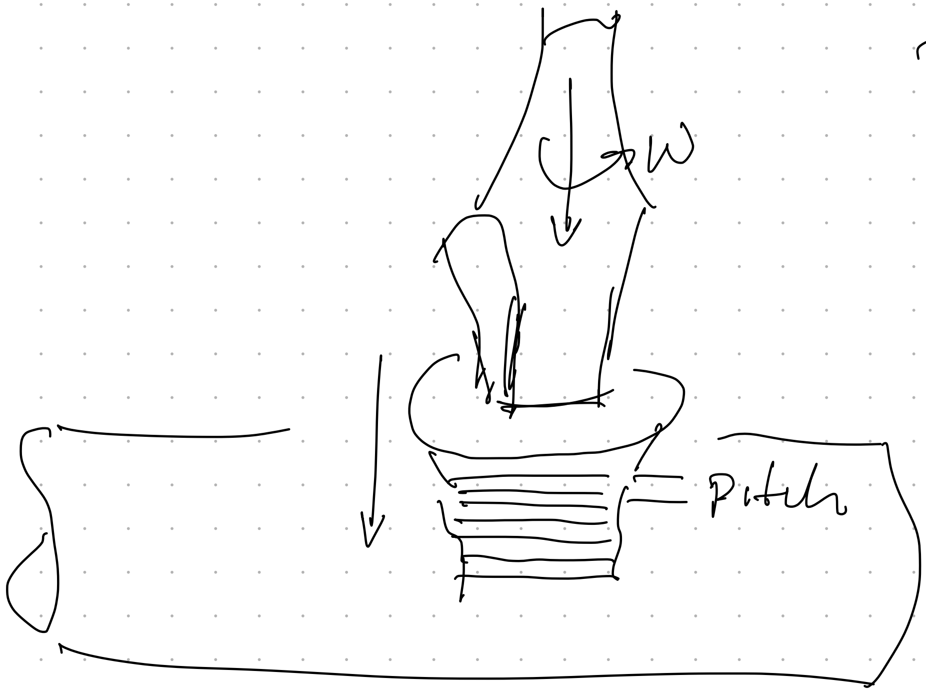
$$\boxed{\bar{\omega}^n_{0,\phi} = \bar{\omega}^1_{0,\phi} + R^1_0 \, \bar{\omega}^2_{1,1} + \cdots + R^{n-1}_0 \, \bar{\omega}^n_{n-1,n}}$$

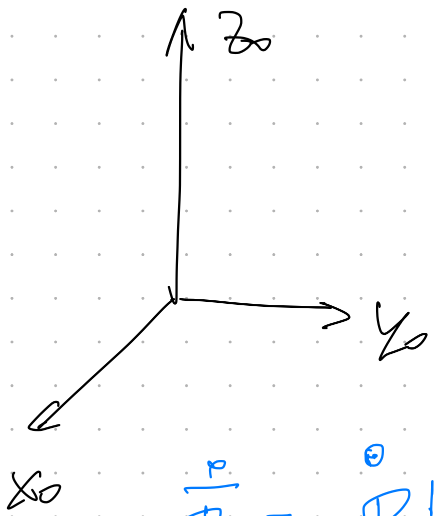
LINEAR VELOCITY

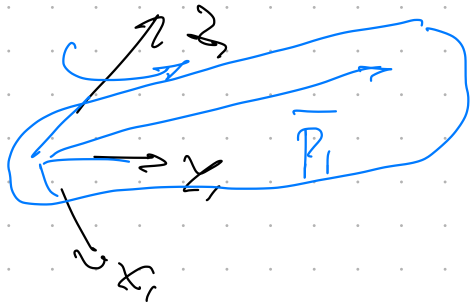


$\bar{\omega}^n_{0,\phi}$

PLATFORM ILIZAROV







$O_1 x_1 y_1 z_1$ rotates relative to $O_0 x_0 y_0 z_0$

$$\bar{p}_0 = R_0^1(t)\,\bar{p}_1$$

$$\dot{\bar{p}}_0 = \dot{R}_0^1(t)\,\bar{p}_1 + R_0^1(t)\,\dot{\bar{p}}_1$$

$$= S(\bar{\omega}_0)\,R_0^1\,\bar{p}_1 = S(\bar{\omega}_0)\,\bar{p}_0 = \bar{\omega}_0 \times \bar{p}_0$$

$$A_0^1 = \left[\begin{array}{c|c} R_0^1(t) & \bar{O}_0^1(t) \\ \hline 0 \quad 0 \quad 0 & 1 \end{array}\right]$$

$$\bar{P}_0 = R_0^1(t)\bar{P}_1 + \bar{O}_0^1(t)$$

$$\dot{\bar{P}}_0 = \dot{R}_0^1(t)\bar{P}_1 + \dot{\bar{O}}_0^1(t)$$

$$= \bar{\omega} \times \bar{P}_1 + \bar{v}$$

$$\bar{q} = \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix}$$ joint variables

$$A_0^n(\bar{q}) = \left[\begin{array}{c|c} R_0^n(\bar{q}) & \bar{O}_0^n(\bar{q}) \\ \hline 0 \quad 0 \quad 0 & 1 \end{array}\right]$$

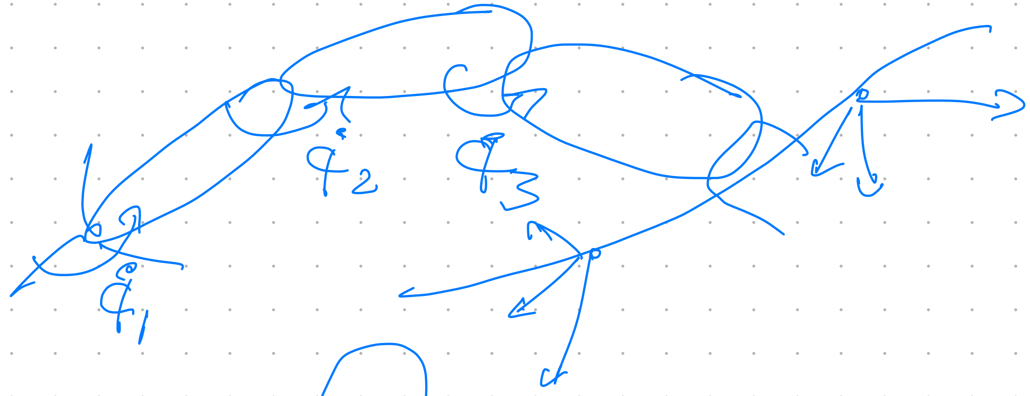
$$S(\bar{\omega}_0^n) = \dot{R}_0^n \left(R_0^n\right)^T$$

linear velocity $\rightarrow$ $\bar{v}_0^n \triangleq \dot{\bar{O}}_0^n = \underset{3\times n}{J_v}\,\dot{\bar{q}}_{n\times 1}$

angular $\rightarrow$ $\bar{\omega}_0^n = \underset{3\times n}{J_\omega}\,\dot{\bar{q}}_{n\times 1}$

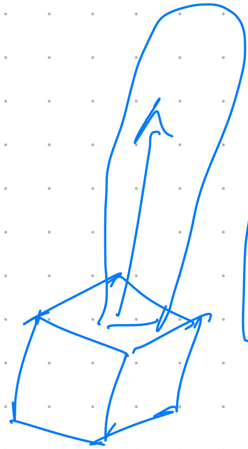
$$\begin{bmatrix} \bar{V}_0^n \\ \bar{w}_0^n \end{bmatrix}_{6\times 1} = J_{6\times n} \begin{bmatrix} \dot{q} \end{bmatrix}_{n\times 1}$$

Jacobian matrix



$$\bar{w}_{i-1}^{i} = \dot{q}_i \, \bar{z}_{i-1}^{i} = \dot{q}_i \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{\bar{w}_{i-1}^{i} = \dot{q}_i \bar{K}}$$



$$\boxed{\bar{w}_{i-1}^{i} = \bar{0}_{3\times 1}}$$

$$\bar{w}_0^n = \rho_1 \dot{q}_1 \bar{K} + \rho_2 \dot{q}_2 R_0^1 \bar{K} + \dots + \rho_n \dot{q}_n R_0^{n-1} \bar{K}$$

$$\rho_i = \begin{cases} 1 & \text{if } i\text{th is revolute} \\ 0 & \text{is prismatic} \end{cases}$$

$$\bar{w}_0^{n} = \sum_{i=1}^{n} \rho_i \, \dot{q}_i \, \bar{z}_0^{i-1}$$

$$\bar{z}_0^{i-1} = R_0^{i-1} \, \bar{k} \quad \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \bar{k} \right)$$

$$\bar{w}_0^{n} = I_w \, \dot{q}$$

$$\underset{3\times 1}{=} \left[\rho_1 \bar{z}_0^{0} \;\middle|\; \rho_2 \bar{z}_0^{1} \;\middle|\; \cdots \;\middle|\; \rho_n \bar{z}_0^{n-1} \right]_{3\times n} \underbrace{\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}}_{n\times 1}$$

$$\bar{z}_0^{0} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\bar{z}_0^{i-1}$$

$$A_0^{i} = \left[\begin{array}{c|c} R_0^{i-1} \; \boxed{} & \bar{O}_0^{i-1} \\ \hline 0 \quad 0 \quad 0 & 1 \end{array} \right]$$

KUKA iiWA

7DoF

Gorke