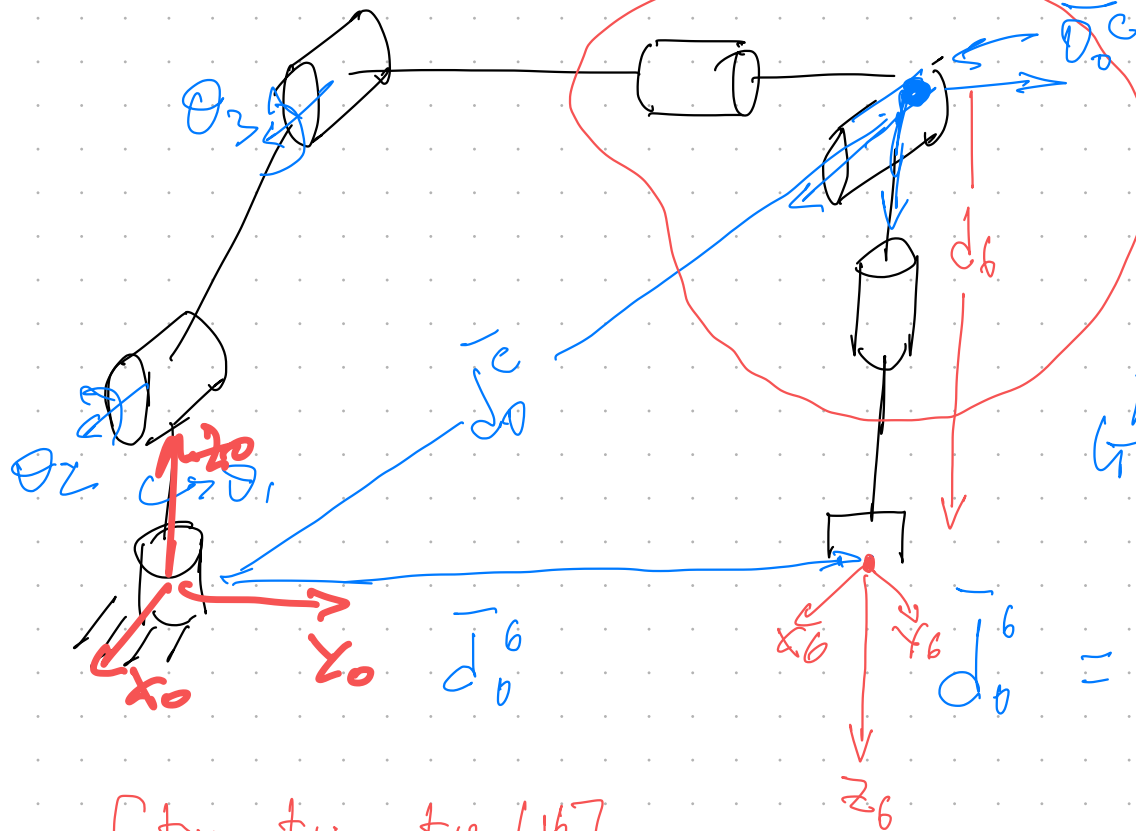


# KINEMATIC DECOUPLING

spherical wrist

by Mark Spong  
Vidyasagar

$$\bar{J}_0^c = f(\theta_1, \theta_2, \theta_3)$$



$$A_0^6 = \begin{bmatrix} R_0^6 & | & \bar{J}_0^6 \\ \hline 0 & 0 & 0 & | & 1 \end{bmatrix}$$

Given  $d_0^6(\theta_1, \dots, \theta_6) \rightarrow$  Compute  $\bar{J}_0^c$

$$\bar{J}_0^6 = \bar{J}_0^c + d_6 R_0^6 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$T_0^6 = \begin{bmatrix} t_{11} & t_{12} & t_{13} & | & d_{0,x} \\ & t_{23} & & | & d_{0,y} \\ & & t_{33} & | & d_{0,z} \\ \hline & & & & 0 & | & 1 \end{bmatrix}$$

$$\bar{J}_0^c = \begin{bmatrix} d_{0,x} \\ d_{0,y} \\ d_{0,z} \end{bmatrix} - d_6 \begin{bmatrix} t_{13} \\ t_{23} \\ t_{33} \end{bmatrix}$$

Computed

$$\bar{J}_0^c$$



Inverse Kinematics  
 $\theta_1, \theta_2, \theta_3$

Unit vector  
along  $Z_6$

Given  $\theta_1, \theta_2, \theta_3 \rightarrow A_0^3 = \left[ \begin{array}{ccc|c} R_0^3(\theta_1, \theta_2, \theta_3) & \vec{d}_0 \\ \hline 0 & 0 & 0 & 1 \end{array} \right]$

$\downarrow$   
 $R_0^3 \leftarrow$  compute

$$A_0^6 = A_0^3 A_3^6$$

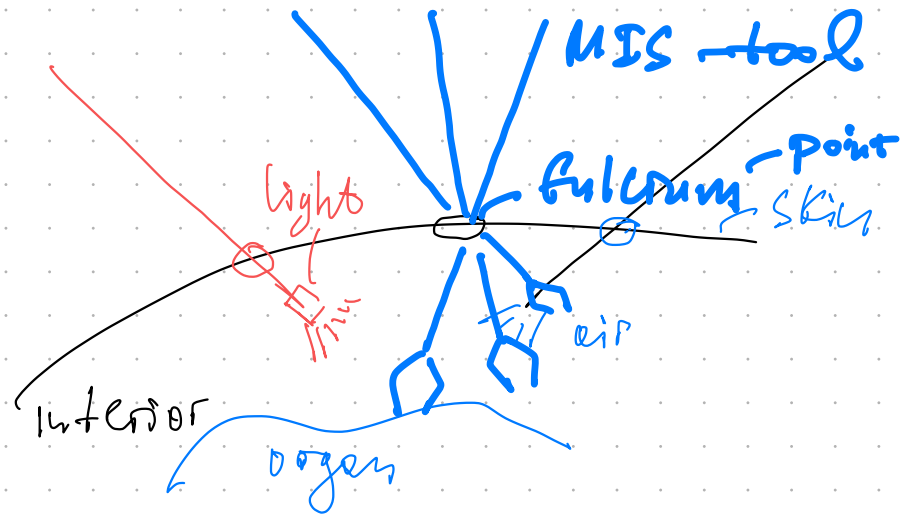
$$\left[ \begin{array}{ccc|c} t_{11} & t_{12} & t_{13} & \vec{d}_0 \\ \vdots & & & \\ t_{33} & & & \\ \hline & & & 1 \end{array} \right] = \left[ \begin{array}{ccc|c} R_0^3 & & & \vec{d}_0 \\ \hline & & & 1 \end{array} \right] \left[ \begin{array}{ccc|c} R_3^6 & & & \vec{d}_3 \\ \hline & & & 1 \end{array} \right]$$

↑ given                      ↑ just computed                      ↑ unknown

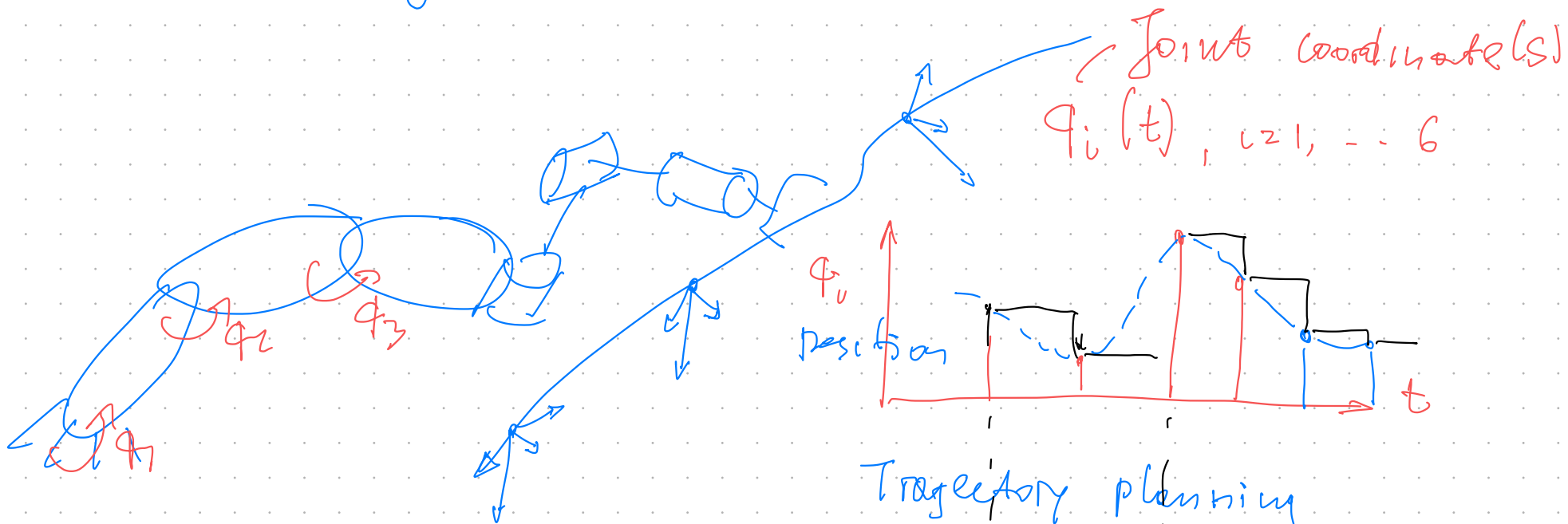
$$R_3^6 = [R_0^3]^{-1} \left[ \begin{array}{ccc} t_{11} & t_{12} & t_{13} \\ & & t_{23} \\ & & t_{33} \end{array} \right]$$

$\theta_4, \theta_5, \theta_6$

$$R_3^6 = (R_0^3)^T \left[ \begin{array}{ccc} t_{11} & t_{12} & t_{13} \\ & & 1 \\ & & t_{33} \end{array} \right]$$



# MINIMAL INVASIVE SURGERY



acceleration

jerk

