

Distributed Energy Resources Optimization for Demand Response using MILP

Hemanth Singabhattu
Junior Research Fellow
Power Systems Division
CPRI, Bangalore
hemanth-jrf@cpri.in

Amit Jain
Joint Director
Power Systems Division
CPRI, Bangalore
amitjain@cpri.in

Tulika Bhattacharjee
Engineering Officer
R&D Management Division
CPRI, Bangalore
tulikab@cpri.in

Abstract—Demand response, over the years, has emerged as a key feature of smart grid. This paper investigates the problem of optimal demand response of residential customer equipped with smart loads, distributed storage and distributed generation which together form distributed energy resources (DER). A novel way of linking distributed storage i.e., battery operation to real time prices via a price threshold is proposed and incorporated in Mixed Integer Linear Programming (MILP) formulation to optimally schedule smart loads and battery. Simulation results validate that the price threshold constraint is effective in optimizing the battery charging/discharging cycles and MILP formulation optimally scheduled the loads for bill reduction within scheduling requirements. Finally, to show how distributed generation coupled with smart loads and distributed storage can further bring down energy costs, a comparison is drawn for various scenarios of customer DER set up.

Index Terms—Day-Ahead Real Time Prices, Demand Response, Distributed Generation, Distributed Storage, Mixed Integer Linear Programming, Smart Load.

I. INTRODUCTION

Electricity storage in huge quantities for use at a later time is still a challenge. So, the traditional way to balance production and consumption is to make generation follow demand at all the time. As a result, the expensive plants set up to meet the high peak demand, which occurs only few hours a year, are underutilized. But, their maintenance and operation costs reflect in tariff and tend to increase the customer bill. On the other hand, electricity prices at times in a day are lower than marginal cost of generation, but consumers lack incentives to use electricity in those times. Demand Response (DR) helps in addressing these issues by deferring investments on capital intensive capacity addition or at least aids in buying time until capacity already under construction comes up. It also helps lower consumers electricity bill.

According to FERC [1], Demand Response is defined as "Changes in electric use by demand-side resources from their normal consumption patterns in response to changes in the price of electricity or to incentives designed to induce lower electricity use at times of potential peak load, high cost periods, or when systems reliability is jeopardized". Demand response is of two types: Incentive based demand response and Price based demand response. The former is suitable for

industrial and large commercial customers, whereas the latter is popular with residential customers.

Since residential customers load form about 40% of the total demand and given the time flexibility offered by smart appliances in completing their tasks, energy consumption scheduling algorithms for demand response from residential customers have garnered much attention in recent times. A group of residential customers load was scheduled using game theory in [2] to reduce the peak to average ratio and individual customers bill. To meet the same objectives and reduce the waiting time of appliances, the authors in [3] employed price prediction for customers on RTP to decide on demand response for the subsequent hours. A demand response model considering customers utility (satisfaction) along with price prediction was explored in [4]. Algorithms like Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) approaches to develop demand response algorithms for residential customers were proposed in [5] and [6]. To overcome the problems of peak rebound, measures like imposition of maximum load hourly constraints and random scheduling among customers were proposed in [7]. The residential load scheduling problem was formulated as an Integer Linear Programming (ILP) in [8], as Mixed Integer Linear Programming (MILP) in [9] and as Mixed Integer Non-Linear Programming (MINLP) in [10], [11], [12]. The benefits offered by optimization of demand in conjunction with energy storage devices like battery were highlighted in [13]. A parallel load optimization for customers aggregated load in presence of renewable generation or customers premises was proposed in [14].

Most of these works concentrated only on optimizing the load to reduce consumers bill safeguarding their comforts. Only a few of these considered the presence of renewable energy generation and battery storage, which is the ubiquitous in the present context, but only as centralized ones either for a group of customers in an area or as part of microgrids. This paper attempts to solve the demand optimization problem of residential dwelling considering it as a microgrid in itself equipped not only with smart appliances but also Distributed Storage (DS) and Distributed Generation (DG) installed in their premises.

The main contributions of this paper are as follows:

- Formulation of shifting and reduction of smart load consumption, battery charging/discharging for demand response as an MILP problem.
- Propose price threshold as a criteria to discharge battery to take off load from grid during high price hours.
- Demonstration of role of customer owned distributed generation in further reducing energy bill.

The rest of the paper is organized in the following manner. The model of a smart home equipped with Distributed Energy Resources is introduced in section II, formulation of MILP problem to reduce electricity bill and shift consumption is done in section III. The simulation results on a sample smart home model are discussed in section IV and concluding remarks are presented in section V.

II. MODELLING OF SMART HOME

Smart home equipped with smart loads, battery storage and micro wind turbine and rooftop solar PV as distributed generation is considered in this paper. A smart meter capable of net metering is installed which communicates with the utility using Advanced Metering Infrastructure (AMI). The smart meter relays the 15 minute interval consumption data to the utility and receives the day-ahead real time prices and grid related emergency events notifications from the utility. The smart meter communicates with an In-Home Display (IHD) installed inside the home and relays the information received from utility which the customer can view through Graphical User Interface (GUI) of the IHD. Battery controller and smart home appliances, categorized as uninterruptible, interruptible and thermostatic loads, communicate with the IHD over a Home Area Network (HAN). The proposed energy consumption optimization program runs inside the IHD enabling it to function as an Energy Management System (EMS). Battery controller and smart loads wirelessly communicate with this IHD-cum-EMS over any of the HAN like Open ADR, ZigBee SEP, Wi-Fi, IEEE 2030.5 etc., to transmit the energy consumption information and receive the scheduled control signals.

A. Smart loads

The loads installed in a typical home can be categorized as unschedulable, schedulable and thermostatic loads based on the scheduling flexibility they offer. An unschedulable load is one which the user wants to run at his/ her discretion at any time of the day. These loads typically run all day and are not so power intensive devices e.g., refrigerator, oven, lighting, television etc. So these devices are not scheduled to operate at specific instances of the day. Schedulable appliances, smart appliances which are DR ready, are required to perform a specific job within a user-defined time frame and hence operate only a few hours a day. These schedulable appliances are further classified as interruptible, A_i , and uninterruptible devices, A_{ui} . Interruptible devices viz., electric vehicle (EV), pool pump etc., demand certain fixed number of slots for operation, whereas uninterruptible devices viz., washing machine,

dishwasher need continuous fixed number of time slots to complete the job.

The energy consumed by the schedulable loads usually do not vary continuously with time, but draw a fixed amount of power once they start and is assumed to remain constant for the entire period of operation. So the EMS should decide on the binary state of when to turn on and off the appliances based on the prices received day ahead but not on the energy allocation to them. This binary state, denoted by binary variable, y_a^t , can be linked to its energy consumption, x_a^t , by,

$$x_a^t = (1 - y_a^t) E_{\min} + y_a^t E_{\max}, \quad \forall a \in \{A_i, A_{ui}\} \quad (1)$$

Under the schedulable loads, the interruptible loads can operate intermittently i.e., by breaking their operation when grid prices are high but are bound to finish the job within the user specified timeframe, T_{sh}^i . Also, they are required to complete the job in fixed number of slots, N , given by,

$$\sum_{t=T_{start}}^{T_{stop}} y_a^t = N \begin{cases} y_a^t \in \{0, 1\}, \forall t \in T_{sh}^i = \{T_{start}, \dots, T_{stop}\} \\ y_a^t = 0, \quad \forall t \notin T_{sh}^i \end{cases} \quad (2)$$

On the other hand, uninterruptible loads also once start within the user specified time frame, T_{sh}^{ui} , have to complete the job in a fixed number of slots N , as given in (3). But, they are required to run continuously without breaking the operation for those fixed number of slots. This uninterruptibility feature of these devices can be modelled by the condition given in (4) as,

$$\sum_{t=T_{start}}^{T_{stop}} y_a^t = N \begin{cases} y_a^t \in \{0, 1\}, \forall t \in T_{sh}^{ui} = \{T_{start}, \dots, T_{stop}\} \\ y_a^t = 0, \quad \forall t \notin T_{sh}^{ui} \end{cases} \quad (3)$$

$$\text{if } y_a^t = 1 \text{ then } y_a^{t+1} > y_a^t, \dots, > y_a^{t+(T_{stop}-T_{start})-1} > y_a^{t+(T_{stop}-T_{start})-2} \quad \forall t \in T_{sh}^{ui} \quad (4)$$

Finally, thermostatically controlled loads, A_{Th} , such as ACs and water heaters participate in demand response and the preferred loads for direct load control of utilities load management programs historically. The operation of thermostatic loads can be emulated by the linear dynamic model,

$$T(t) = (1 - \alpha)T(t-1) + \alpha T_a(t) - \beta x_a^t \quad \forall a \in A_T \quad (5)$$

Where, β is positive for AC loads and negative for water heater loads, $T(t)$ is the hourly room temperature, $T_a(t)$ is hourly ambient temperature, α and β represent thermal appliance characteristics and operating environment conditions. In order to avoid inconvenience for customers lifestyle, the EMS adjusts the room temperature, $T(t)$, via the smart thermostat between the customer-set comfort settings,

$$T_{\min} \leq T(t) \leq T_{\max}. \quad (6)$$

B. Distributed Storage

Distributed energy storage technologies like Lithium-ion batteries are emerging as grid supportive home storage system for self- produced electricity. Due to the intermittent nature of renewable energy sources, they can be used to store electricity

and discharge to supply load during high electricity price hours. Battery usage can be modelled to meet mainly two requirements:

- 1) Maximum charging rate, $r(t)^{max}$, and discharging rate, $r(t)^{min}$, given by,

$$r(t)^{min} \leq r(t) \leq r(t)^{max}. \quad (7)$$

- 2) Maximum storage capacity, given by

$$\sum_{t \in T} r(t) \leq b^{max}. \quad (8)$$

In order to effectively use the battery storage to minimize the customers energy bill, the battery controller should precharge the battery during the periods of low grid prices. When the prices reach a $P_{threshold}$, which is set by the customer, the battery should discharge to supply the entire the loads scheduled to be in operation during that hour. This requirement can be modelled inside the EMS by,

$$r(t) \geq 0, \quad \forall p(t) < P_{threshold}, \quad (9)$$

$$r(t) \leq 0, \quad \forall p(t) \geq P_{threshold}, \quad (10)$$

$$\sum_{a \in A} x_a^t = r(t), \quad \forall p(t) \geq P_{threshold}. \quad (11)$$

Conditions (7)-(11) not only ties the distributed storage operation to grid through price signals but also optimizes the charge/discharge cycles to secure the life of battery.

C. Distributed Generation

A future smart home customer can no longer be a consumer but may evolve into prosumer through the installation of distributed renewable generation like grid connected rooftop solar PV and micro wind turbines. Utilities plans to increase renewable energy generation to reduce carbon footprint and decarbonize power sector and policy initiatives like net metering and gross metering etc. are encouraging this transformation.

1) *Roof top solar PV*: The amount of solar output, $g_{pv}(t)$, from a solar PV is dependent on many factors but the irradiance and temperature at a given condition directly affect it. So considering the model used in [9],

$$g_{pv}(t) = J \begin{cases} \frac{e_c}{k_c} (\gamma(t))^2, & 0 \leq \gamma(t) \leq k_c \\ e_c \cdot \gamma(t), & \gamma(t) > k_c \end{cases} \quad (12)$$

Where, J is the number of solar cells in the panel, e_c is the efficiency of the cell, $\gamma(t)$ is the solar irradiance in W/m^2 and k_c is critical irradiance point.

2) *Wind Energy*: The output of wind turbine, $g_w(t)$, is highly dependent on wind velocity, a 20% change in wind velocity changes the wind turbine output by 73%, which is given by [9],

$$g_w(t) = \frac{1}{2} \rho A v(t)^3 C_p \quad (13)$$

Where, ρ is the density of air (1.25 kg/m^3), A is the swept area of wind turbine in m^2 , $v(t)$ is the hourly wind velocity in m/s , C_p is the Betz constant (max of 0.59).

III. PROBLEM FORMULATION

The energy consumption scheduling problem in order to minimize customers bill can be modelled as an optimization problem with the objective function given as,

$$\min_{y_a^t, x_a^t, r(t)} \sum_{t \in T} \sum_{a \in A} p(t) \cdot \{x_a^t + y_a^t + r(t) - g(t)\} \quad (14)$$

subject to the constraints (1) - (11).

Where x_a^t denotes the energy consumption of thermo-static loads, y_a^t is the binary variable indicating the status of schedulable appliances, $r(t)$ denotes the battery charging and discharging and $g(t)$ is the sum of $g_{pv}(t)$ and $g_w(t)$. The renewable energy generation sources considered here are stochastic in nature whose generation cannot be dispatched or controlled by the EMS. But, the amount of generation can be predicted with the models considered in (12) and (13). Here, it is assumed that they are prioritized to be consumed as and when energy is generated from them. So, the EMS tries to optimize the load and battery operation with the modified objective function given as,

$$\min_{y_a^t, x_a^t, r(t)} \sum_{t \in T} \sum_{a \in A} p(t) \cdot \{x_a^t + y_a^t + r(t)\}. \quad (15)$$

But, in attempting to meet this objective, the EMS tries to schedule all the loads to low price periods which may give rise to peak rebounds or shifted peaks. To overcome this peak rebound problem, a maximum hourly load constraint is imposed on the schedulable load by the EMS given as,

$$\sum_{a \in A} x_a^t = E_{max}^t, \quad \forall a \in \{A_i, A_{ui}\}. \quad (16)$$

With the objective function formulated as in (15) and including constraints (1)-(11) and (16) with the decision variables involving both binary and continuous variables, the above optimization problem is a mixed integer linear programming (MILP) problem. This optimization problem can be solved using branch and bound method in any of the commercially available optimization solvers such as in MATLAB, GAMS or CPLEX packages to generate optimum schedules, temperatures for smart loads and optimize charge/discharge cycles of battery operation.

IV. SIMULATION RESULTS AND DISCUSSIONS

In order to demonstrate the effectiveness of the proposed energy consumption scheduling program, smart home equipped with smart washing machine, dishwasher (uninterruptible loads), electric vehicle (interruptible load), air conditioner and water heater (thermostatic loads), distributed storage like Li-ion battery and distributed generation like roof-top solar PV and micro wind turbine is considered.

The day-ahead Real Time Prices are taken from [15] and ambient temperatures are taken from [16]. The ratings, preferred schedules of interruptible and uninterruptible loads, comfort settings of thermostatic loads, and battery parameters of distributed storage considered are given in Table I.

Assuming that the consumer remains faithful without overriding the preferred schedules and comfort settings, the smart

TABLE I
RATINGS AND PREFERRED SCHEDULES OF SMART APPLIANCES AND
DISTRIBUTED STORAGE CONSIDERED

Uninterruptible loads			
Appliance	Rating	Operational Slots	Preferred Schedule
Washing Machine	1 kW	3	8am-12pm
Dish Washer	1.2 kW	4	8am-6pm
Interruptible loads			
Electric Vehicle	3 kW	3	6pm-12am
Thermostatic loads			
Appliance	Comfort settings	Preferred Schedule	
Air Conditioner	70 ⁰ F-79 ⁰ F	12am-11pm	
Water Heater	130 ⁰ F-160 ⁰ F	12am-11pm	
Battery Parameters			
b^{max}	$r(t)^{max}$	$r(t)^{min}$	
10	6	-4	

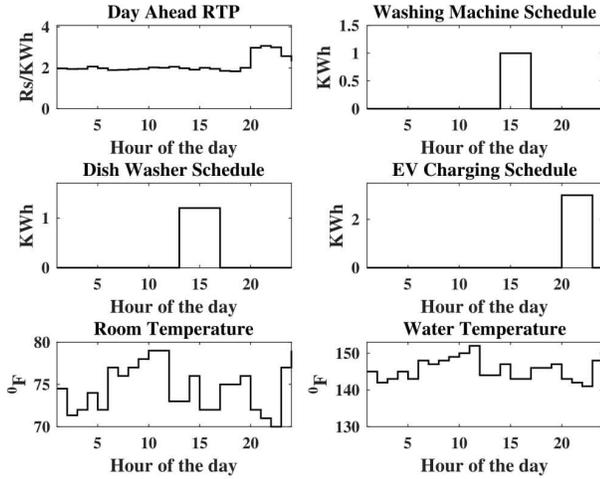


Fig. 1. Schedules for various smart appliances

loads optimally scheduled by the EMS using the MILP formulation is shown in Fig. 1.

For interruptible loads, the EMS, on receipt of day-ahead RTP, searches for possible combinations of N slots in the users preferred timeframe whose sum of prices amount to the lowest. Each interruptible load is scheduled to the respective N slots thus found. A similar search for N consecutive slots in the user preferred timeframe whose sum would be the lowest is made for the uninterruptible loads. All the uninterruptible loads are thus scheduled in the N consecutive slots found for each of them. For thermostatic loads, the EMS dynamically adjusts the hourly room temperatures within the user comfort settings for the respective prices.

From Fig. 1, it can be seen that the schedulable and thermostatic loads are scheduled to operate well within the users preferred timeframes and comfort settings as mentioned in Table I. The maximum hourly loading constraint imposed by the EMS on schedulable loads avoided them operating all

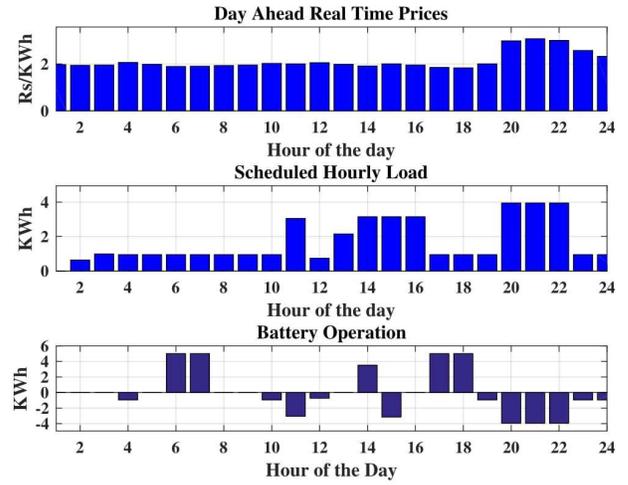


Fig. 2. Battery charging/discharging cycles w.r.t. to real time prices

at the same period as can be seen in Fig. 1. This optimally distributed the load along the scheduled time horizon and effectively mitigated the occurrence of peaks getting shifted to low price periods or the peak rebound condition. This contributes to the peak shaving and load flattening objectives of utility when aggregated loads of all other residences in an area are considered.

The battery storage operation tied to grid through real time prices is shown in Fig. 2. The price threshold, $P_{threshold}$, a criteria for discharging the battery is set as $2 \text{ } \$/kwh$ in the simulation. This stimulated the battery controller, controlled by EMS, charge the battery during price periods below this threshold. Whenever the hourly prices reached $P_{threshold}$, the battery is discharged and took over all the load scheduled to be in operation at that hour as can be seen from Fig 2. For a consumer, from billing perspective, this means that the energy consumed by loads during those hours is billed to the comparatively low price periods of battery charging. Moreover, it can be seen be that not everyhour there is charging/discharging but there are periods in between where battery operation is idle. This optimized the charging/ discharging cycles and enables the battery operate near to its warranted lifetime.

The wind speed data taken from [16] for the micro wind turbine model, explained in section II, is shown in Fig. 3 and other parameters of the model are taken as $\rho = 1.25 \text{ } kg/m^2$, $r = 1 \text{ } m$ and $C_p = 0.30$. Similarly, the irradiance data taken from [17] of a random day for inclusion in rooftop solar PV solar model, explained in section II, is shown in Fig. 3 and other parameters of the model are taken as $J = 20$, $e_c = 0.15$ and $K_c = 1000$. Using this data, the energy in KWh , predicted to be generated from the models, is also shown in Fig3.

This energy produced from the renewables considered here can be stored for later use or as assumed here, consumed as and when generated by the loads scheduled to those hours. The net energy at any hour when in excess, is assumed to be exported to the grid via net-metering which further reduces

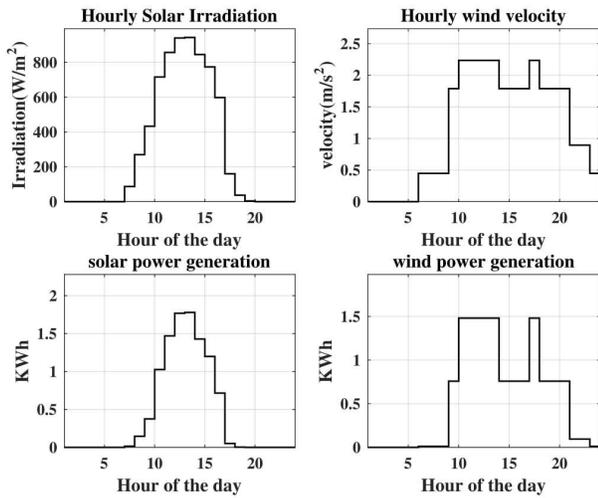


Fig. 3. Predicted renewable generation

TABLE II
CUSTOMER DAILY TENTATIVE BILL UNDER VARIOUS SCENARIOS IN RS.

Only appliance scheduling	Appliance scheduling + DS	Appliance scheduling + DS + DG
92.20	76.24	30.00

the customer bill.

Table II presents the tentative consumer daily bill obtained by incorporating the proposed MILP demand response strategy for various scenarios of customer DER set-up.

In the first scenario, in the absence of any DS and DG the bill is due to only the smart appliances operation shifted to low price periods. In the second scenario, the reduction is because battery charging at low price hours avoided consumption from grid during hours that reached RTP threshold set in EMS. Finally in third scenario, the large reduction in bill is due to the consumption from DG and DS.

V. CONCLUSION

In this paper, the problem of optimal demand response from residential dwelling equipped with resources such as smart loads, distributed storage and distributed generation is studied under the day-ahead real time pricing environment. MILP is used to formulate the model to minimize the consumer's energy bill by adjusting and shifting the load demands to low price hours, within the customers comfort settings and scheduling requirements. This contributes in fulfilling the load reduction and load shifting objectives of demand response. The proposed approach to link the use of distributed storage to the real time prices via a price threshold is found to be effective in optimizing the charging/discharging cycles of the battery, thereby ensuring further reduction in the customer bill. Finally, a comparison of energy bill drawn for various scenarios of customer DER set up presented shows that the maximum reduction in energy cost for the customer is achieved when demand response from smart loads and distributed storage is

coupled with distributed generation. This long term monetary benefit encourages the customer to install DG despite the high capital cost involved. The effects of aggregated DR for a group of residential customers with DER setup on distribution networks and their coordinated operation to benefit the system as a whole are further being investigated by authors and will be presented in future publications.

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