ΠΑΝΕΠΙΣΤΗΜΙΟ ΠΑΤΡΩΝ
UNIVERSITY OF PATRAS

ΑΝΟΙΚΤΑ ακαδημαϊκά μαθήματα ΠΠ

Τεχνητή Νοημοσύνη ΙΙ

Ενότητα : Μηχανική Μάθηση

Σγάρμπας Κυριάκος
Πολυτεχνική Σχολή
Τμήμα Ηλεκτρολόγων Μηχανικών και Τεχνολογίας Υπολογιστών

Σκοποί ενότητας

- Μηχανική Μάθηση

Περιεχόμενα ενότητας

- Μηχανική Μάθηση

Machine Learning

Μηχανική Μάθηση

Ορισμός

- “A field of study that gives computers the ability to learn without being explicitly programmed”

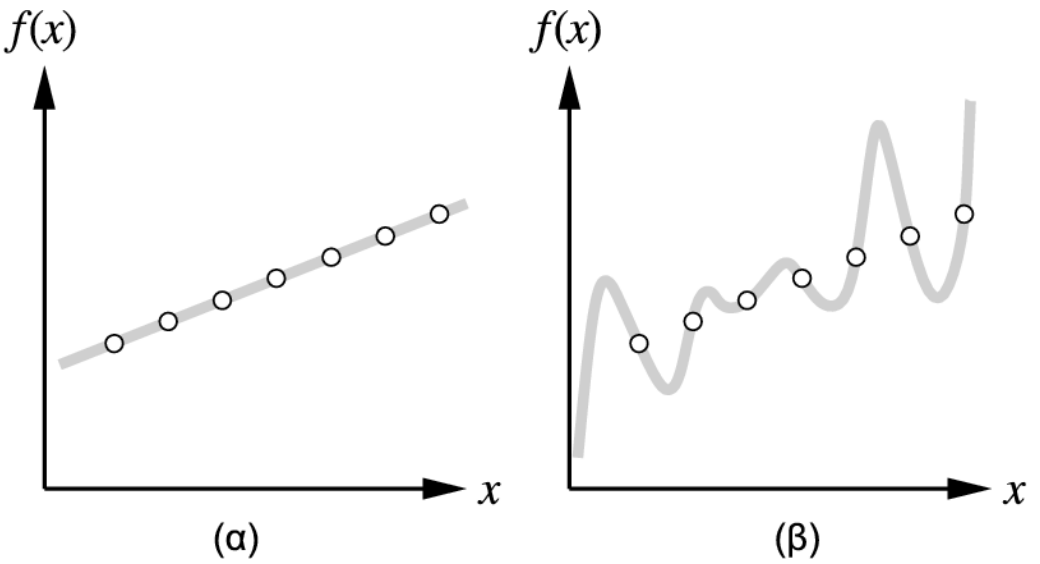
  1959, Arthur Samuel

- “A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E”

  1997, Tom M. Mitchell

Είναι Δυνατόν;
(Το Πρόβλημα της Επαγωγής)

- y = f(x)
- Δείγματα: (*x*, y)
- Υπόθεση: *h(x) ~ f(x)*
  - Χώρος υποθέσεων
- Συνεπείς (με τα δείγματα) υποθέσεις
- Ξυράφι του Ockham
- Γενίκευση



Στη Γενική Περίπτωση

Πολλές μεταβλητές

Relation: Glass

No.	RI Numeric	Na Numeric	Mg Numeric	Al Numeric	Si Numeric	K Numeric	Ca Numeric	Ba Numeric	Fe Numeric	Type Nominal
1	1.51793	12.79	3.5	1.12	73.03	0.64	8.77	0.0	0.0	build wind float
2	1.51643	12.16	3.52	1.35	72.89	0.57	8.53	0.0	0.0	vehic wind float
3	1.51793	13.21	3.48	1.41	72.64	0.59	8.43	0.0	0.0	build wind float
4	1.51299	14.4	1.74	1.54	74.55	0.0	7.59	0.0	0.0	tableware
5	1.53393	12.3	0.0	1.0	70.16	0.12	16.19	0.0	0.24	build wind non-flo
6	1.51655	12.75	2.85	1.44	73.27	0.57	8.79	0.11	0.22	build wind non-float
7	1.51779	13.64	3.65	0.65	73.0	0.06	8.93	0.0	0.0	vehic wind float
8	1.51837	13.14	2.84	1.28	72.85	0.55	9.07	0.0	0.0	build wind float
9	1.51545	14.14	0.0	2.68	73.39	0.08	9.07	0.61	0.05	headlamps
10	1.51789	13.19	3.9	1.3	72.33	0.55	8.44	0.0	0.28	build wind non-float
11	1.51625	13.36	3.58	1.49	72.72	0.45	8.21	0.0	0.0	build wind non-float
12	1.51743	12.2	3.25	1.16	73.55	0.62	8.9	0.0	0.24	build wind non-float
13	1.52223	13.21	3.77	0.79	71.99	0.13	10.02	0.0	0.0	build wind float
14	1.52121	14.03	3.76	0.58	71.79	0.11	9.65	0.0	0.0	vehic wind float
15	1.51665	13.14	3.45	1.76	72.48	0.6	8.38	0.0	0.17	vehic wind float
16	1.51707	13.48	3.48	1.71	72.52	0.62	7.99	0.0	0.0	build wind non-float
17	1.51719	14.75	0.0	2.0	73.02	0.0	8.53	1.59	0.08	headlamps
18	1.51629	12.71	3.33	1.49	73.28	0.67	8.24	0.0	0.0	build wind non-float
19	1.51994	13.27	0.0	1.76	73.03	0.47	11.32	0.0	0.0	containers
20	1.51811	12.96	2.96	1.43	72.92	0.6	8.79	0.14	0.0	build wind non-float
21	1.52152	13.05	3.65	0.87	72.22	0.19	9.85	0.0	0.17	build wind float
22	1.52475	11.45	0.0	1.88	72.19	0.81	13.24	0.0	0.34	build wind non-float
23	1.51841	12.93	3.74	1.11	72.28	0.64	8.96	0.0	0.22	build wind non-float
24	1.51754	13.39	3.66	1.19	72.79	0.57	8.27	0.0	0.11	build wind float
25	1.52058	12.85	1.61	2.17	72.18	0.76	9.7	0.24	0.51	containers
26	1.51569	13.24	3.49	1.47	73.25	0.38	8.03	0.0	0.0	build wind non-float
27	1.5159	12.82	3.52	1.9	72.86	0.69	7.97	0.0	0.0	build wind non-float
28	1.51683	14.56	0.0	1.98	73.29	0.0	8.52	1.57	0.07	headlamps
29	1.51687	13.23	3.54	1.48	72.84	0.56	8.1	0.0	0.0	build wind non-float
30	1.5161	13.33	3.53	1.34	72.67	0.56	8.33	0.0	0.0	vehic wind float

Undo OK Cancel

Πολλές κλάσεις

...και συχνά χωρίς αριθμητικές τιμές

Viewer

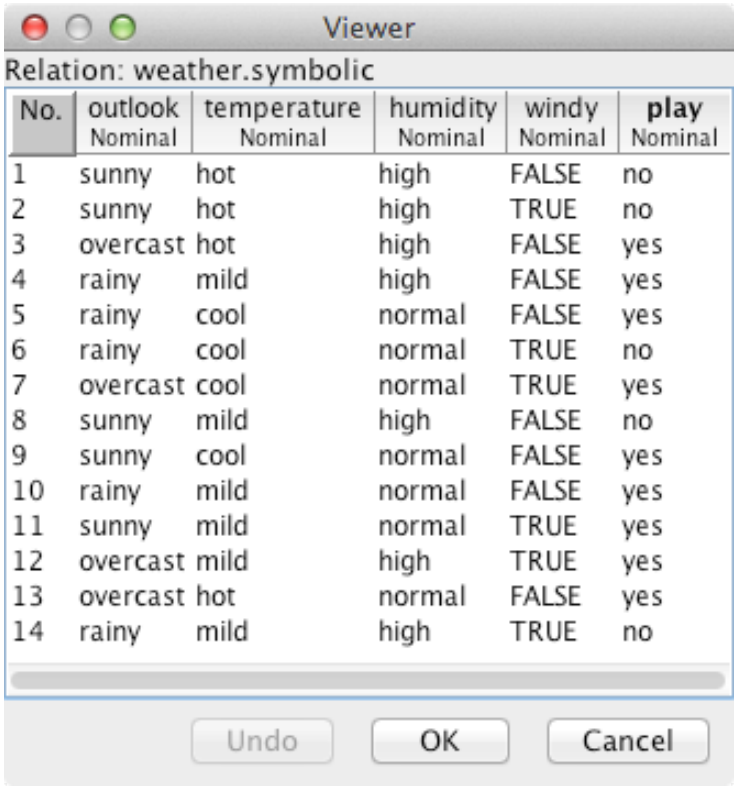
Relation: weather.symbolic

No.	outlook Nominal	temperature Nominal	humidity Nominal	windy Nominal	**play** Nominal
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
3	overcast	hot	high	FALSE	yes
4	rainy	mild	high	FALSE	yes
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
7	overcast	cool	normal	TRUE	yes
8	sunny	mild	high	FALSE	no
9	sunny	cool	normal	FALSE	yes
10	rainy	mild	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
13	overcast	hot	normal	FALSE	yes
14	rainy	mild	high	TRUE	no

Undo OK Cancel

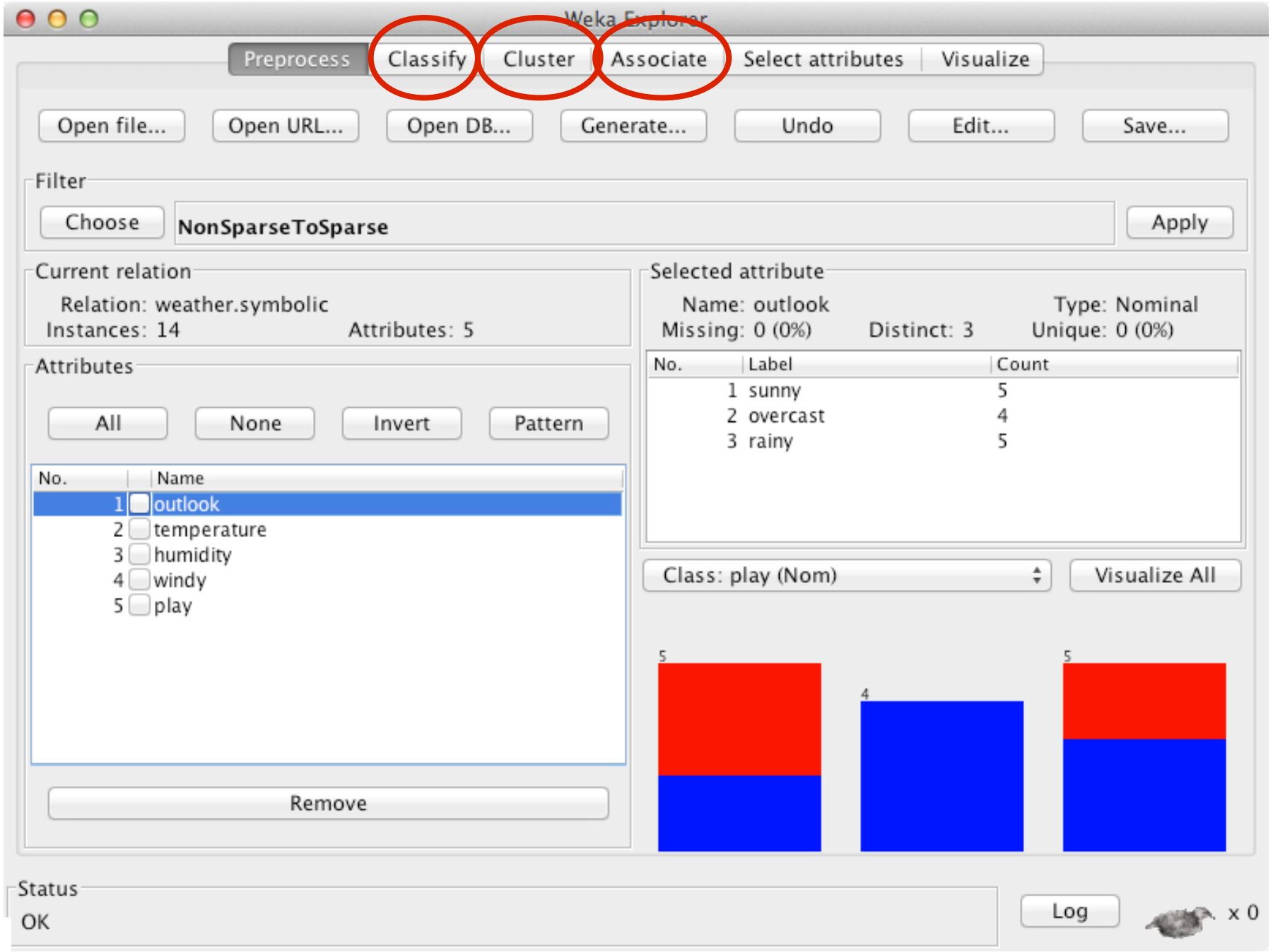
Έννοιες

- Μοντέλο (Model, Concept)
- Στιγμιότυπο (Instance, Δείγμα, Example)
- Χαρακτηριστικό (Feature, Μεταβλητή)
- Κλάση (Class)
- Επίβλεψη (Supervision)



Τί μπορούμε να μάθουμε από τέτοια δεδομένα;

- Κατηγορίες (Classification)
- Ομάδες (Clustering)
- Συσχετίσεις (Association)


Weka Explorer
Preprocess
Classify
Cluster
Associate
Select attributes
Visualize
Open file...
Open URL...
Open DB...
Generate...
Undo
Edit...
Save...
Filter
Choose
NonSparseToSparse
Apply
Current relation
Relation: weather.symbolic
Instances: 14
Attributes: 5
Attributes
All
None
Invert
Pattern
No.
Name
1 outlook
2 temperature
3 humidity
4 windy
5 play
Remove
Selected attribute
Name: outlook
Type: Nominal
Missing: 0 (0%)
Distinct: 3
Unique: 0 (0%)
No.
Label
Count
1 sunny 5
2 overcast 4
3 rainy 5
Class: play (Nom)
Visualize All
5
4
5
Status
OK
Log
x 0

Αλγόριθμοι ML

1. Supervised learning

AODE
Artificial neural network
  Backpropagation
Bayesian statistics
  Naive Bayes classifier
  Bayesian network
  Bayesian knowledge base
Case-based reasoning
Decision trees
Inductive logic programming
Gaussian process regression
Gene expression programming
Group method of data handling (GMDH)
Learning Automata
Learning Vector Quantization
Logistic Model Tree
Minimum message length
  (decision trees, decision graphs, etc.)
Lazy learning
Instance-based learning
  Nearest Neighbor Algorithm
  Analogical modeling
Probably approximately correct learning (PAC) learning
Ripple down rules, a knowledge acquisition methodology
Symbolic machine learning algorithms
Subsymbolic machine learning algorithms
Support vector machines
Random Forests
Ensembles of classifiers
  Bootstrap aggregating (bagging)
  [illegible]osting (meta-algorithm)
  [illegible]al classification
  [illegible]ession analysis
  [illegible]mation fuzzy networks (IFN)

1.1 Statistical classification

AODE
Linear classifiers
  Fisher's linear discriminant
  Logistic regression
  Naive Bayes classifier
  Perceptron
  Support vector machines
Quadratic classifiers
k-nearest neighbor
Boosting
Decision trees
  C4.5
  Random forests
Bayesian networks
Hidden Markov models

2. Unsupervised learning

Artificial neural network
Data clustering
Expectation-maximization algorithm
Self-organizing map
Radial basis function network
Vector Quantization
Generative topographic map
Information bottleneck method
IBSEAD

2.1 Association rule learning

Apriori algorithm
Eclat algorithm
FP-growth algorithm

2.2 Hierarchical clustering

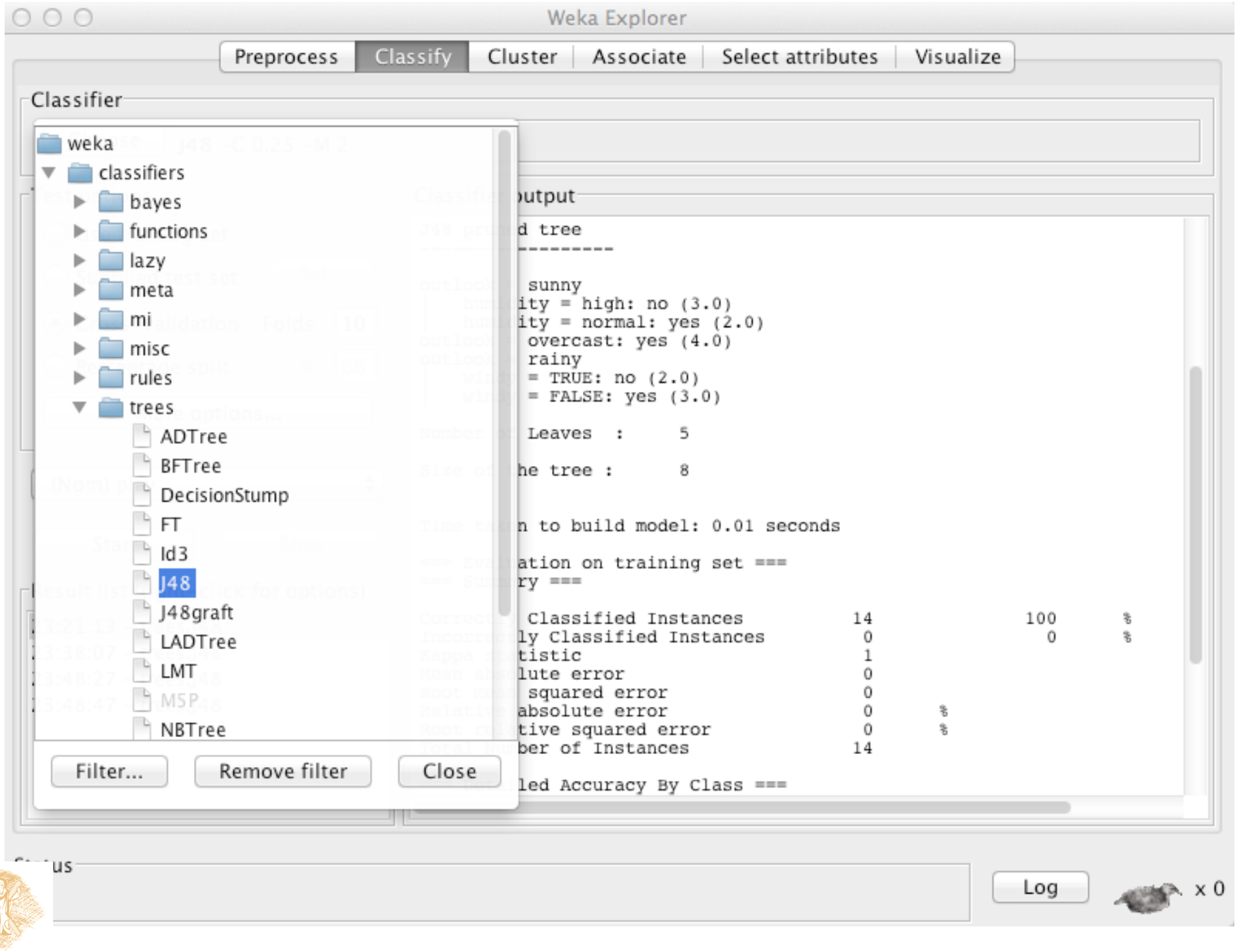
Single-linkage clustering
Conceptual clustering

2.3 Partitional clustering

K-means algorithm
Fuzzy clustering

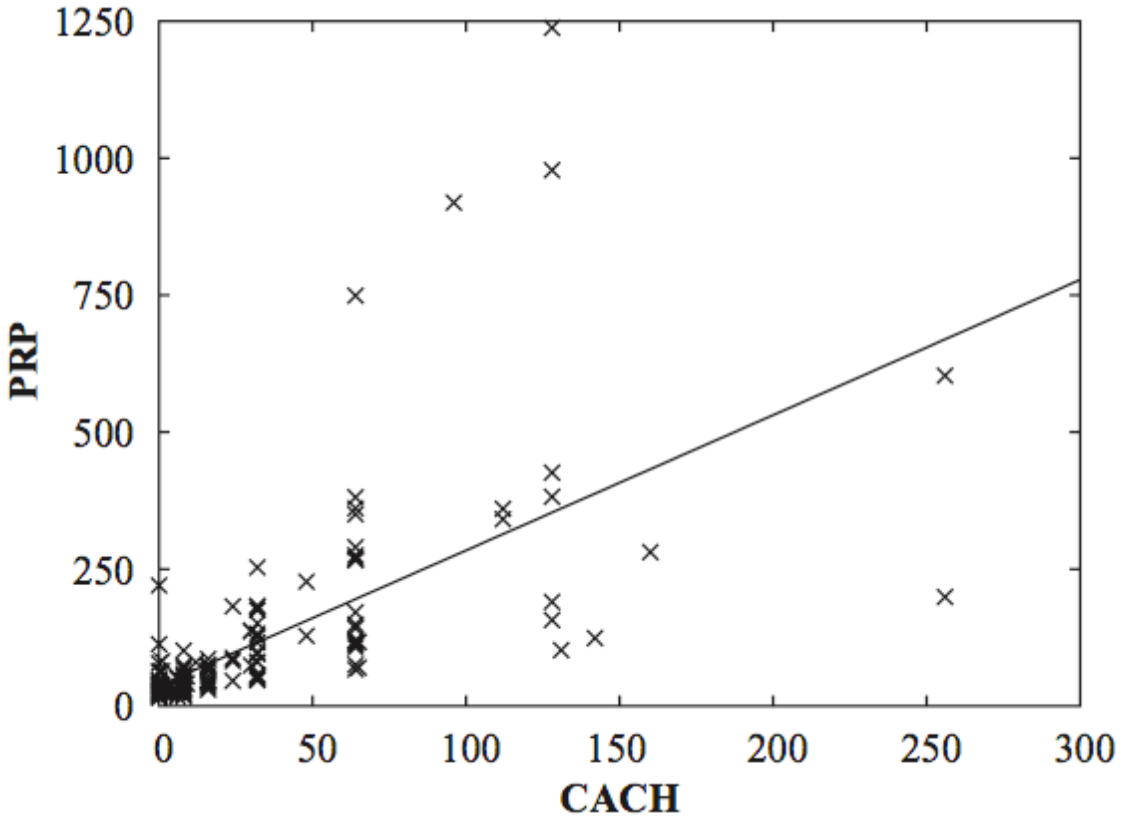
3. Reinforcement learning

Temporal difference learning
Q-learning
Learning Automata


Weka Explorer
Preprocess
Classify
Cluster
Associate
Select attributes
Visualize
Classifier
weka
classifiers
bayes
functions
lazy
meta
mi
misc
rules
trees
ADTree
BFTree
DecisionStump
FT
Id3
J48
J48graft
LADTree
LMT
M5P
NBTree
Filter...
Remove filter
Close
output
d tree
----
sunny
ity = high: no (3.0)
ity = normal: yes (2.0)
overcast: yes (4.0)
rainy
= TRUE: no (2.0)
= FALSE: yes (3.0)
Leaves : 5
he tree : 8
n to build model: 0.01 seconds
ation on training set ===
ry ===
Classified Instances 14 100 %
ly Classified Instances 0 0 %
tistic 1
lute error 0
squared error 0
absolute error 0 %
tive squared error 0 %
ber of Instances 14
led Accuracy By Class ===
Status
Log
x 0

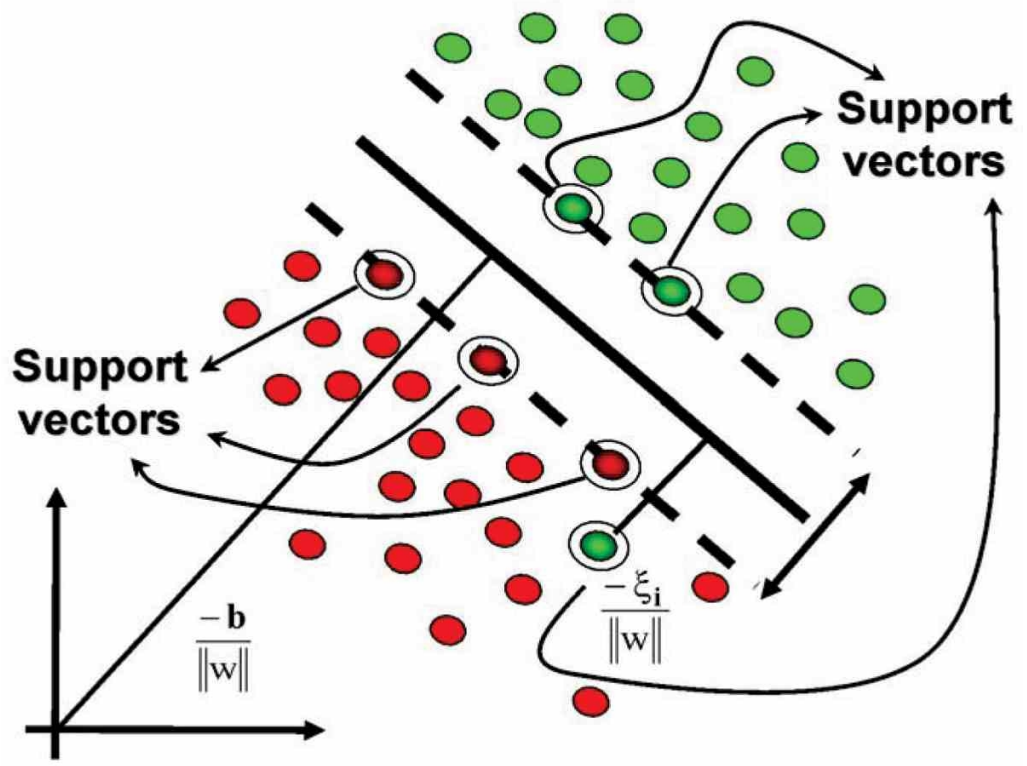
Linear Models

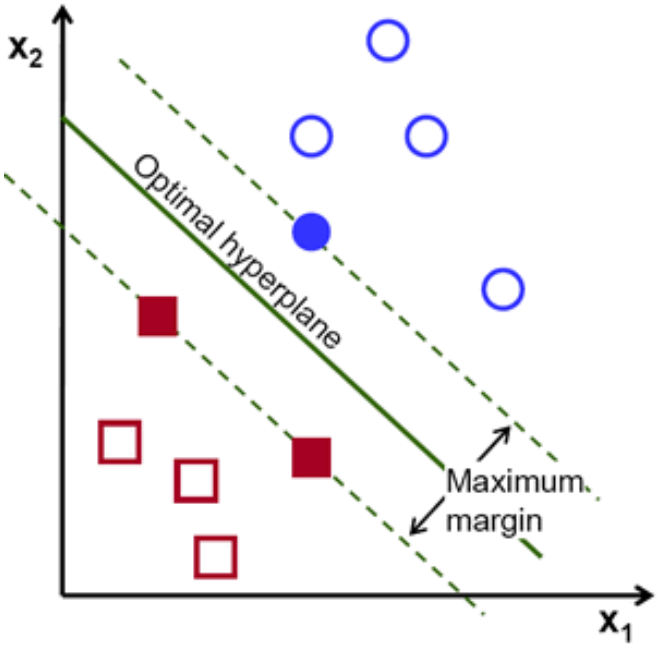
$$x = w_0 + w_1 a_1 + w_2 a_2 + \ldots + w_k a_k$$

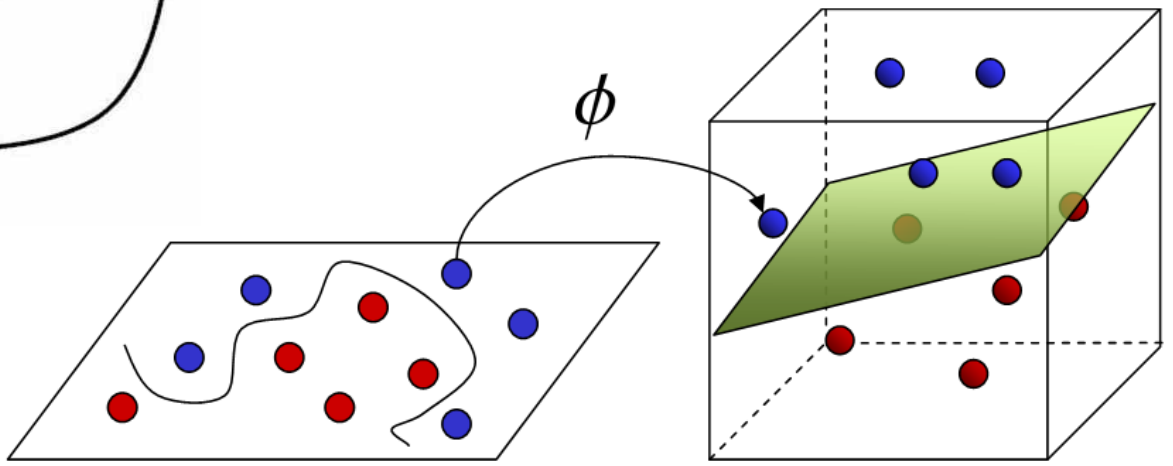


$$\sum_{i=1}^{n} \left(x^{(i)} - \sum_{j=0}^{k} w_j a_j^{(i)} \right)^2$$

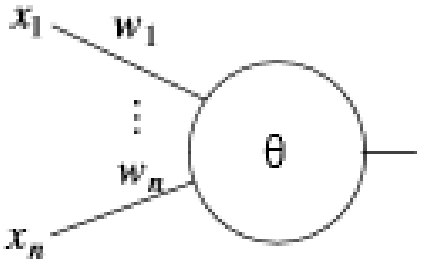
SVM


Support vectors
Support vectors
$\frac{-b}{\|w\|}$
$\frac{-\xi_i}{\|w\|}$


x_2
Optimal hyperplane
Maximum margin
x_1


ϕ
Input Space
Feature Space

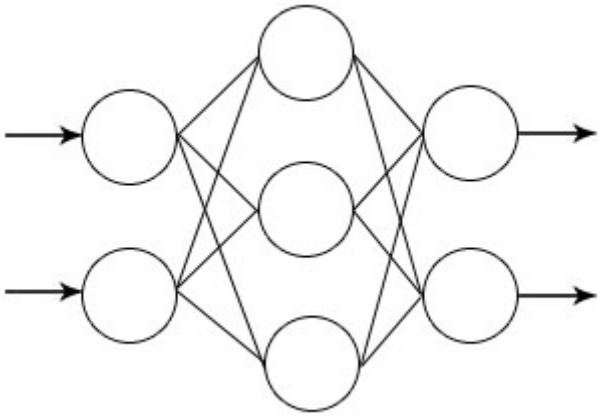
Νευρωνικά Δίκτυα



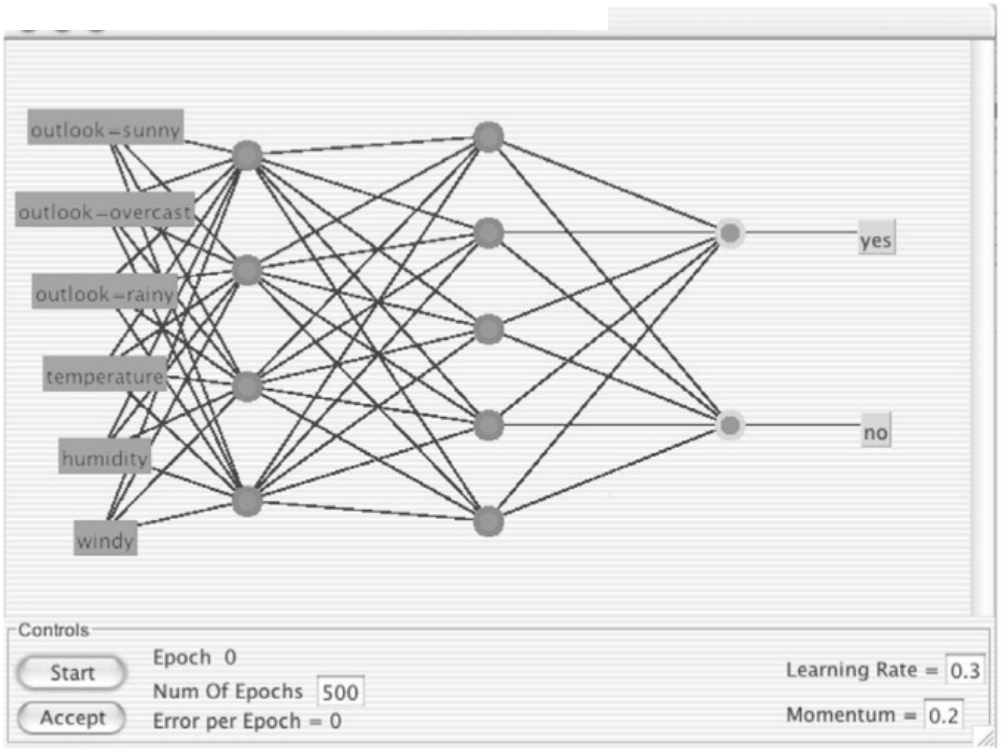
$$y = f\left(\sum_{i=1}^{n} w_i \cdot x_i\right) = \begin{cases} 1, \sum_{i=1}^{n} w_i \cdot x_i \geq \theta \\ 0, \sum_{i=1}^{n} w_i \cdot x_i < \theta \end{cases}$$

$$S(u) = \frac{1}{1 + e^{-cu}}$$

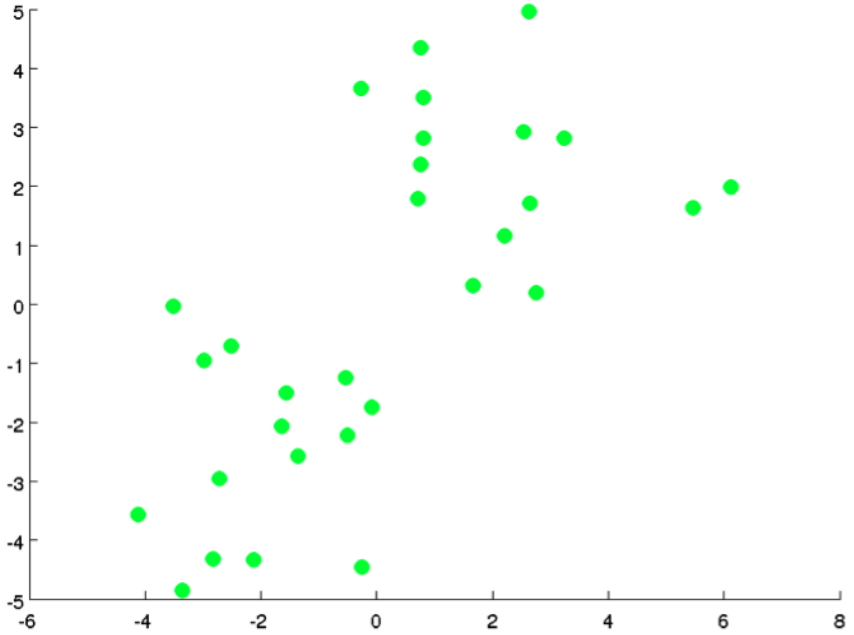
Backpropagation

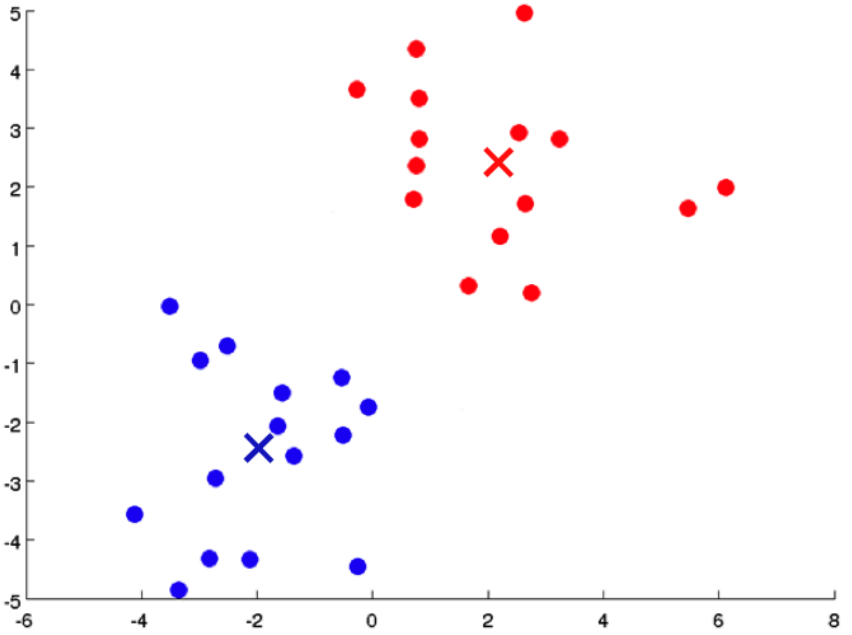


$$Total\_Error = \sum_{i=1} (T_i - O_i)^2$$

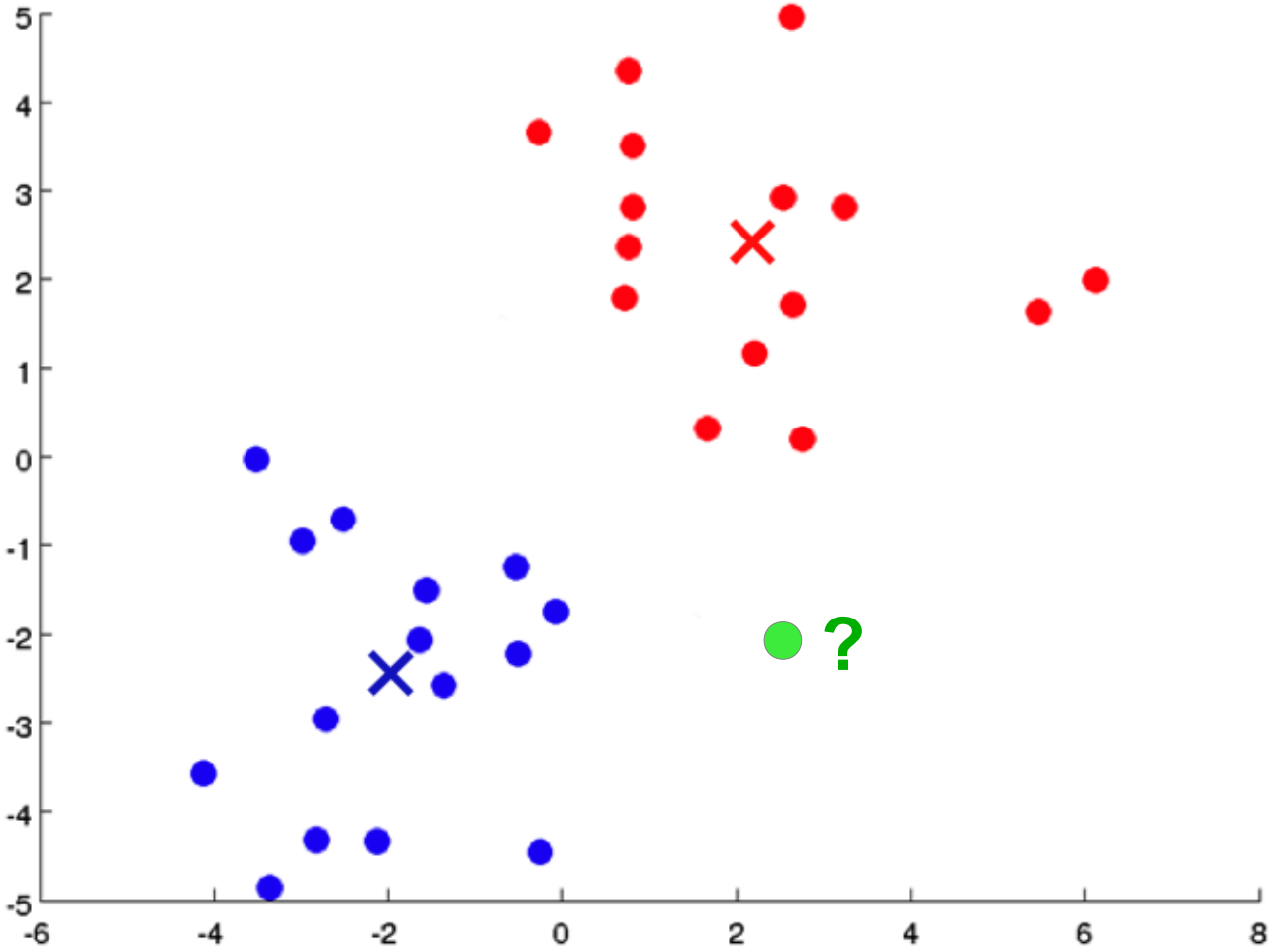


Αλγόριθμος K-Means

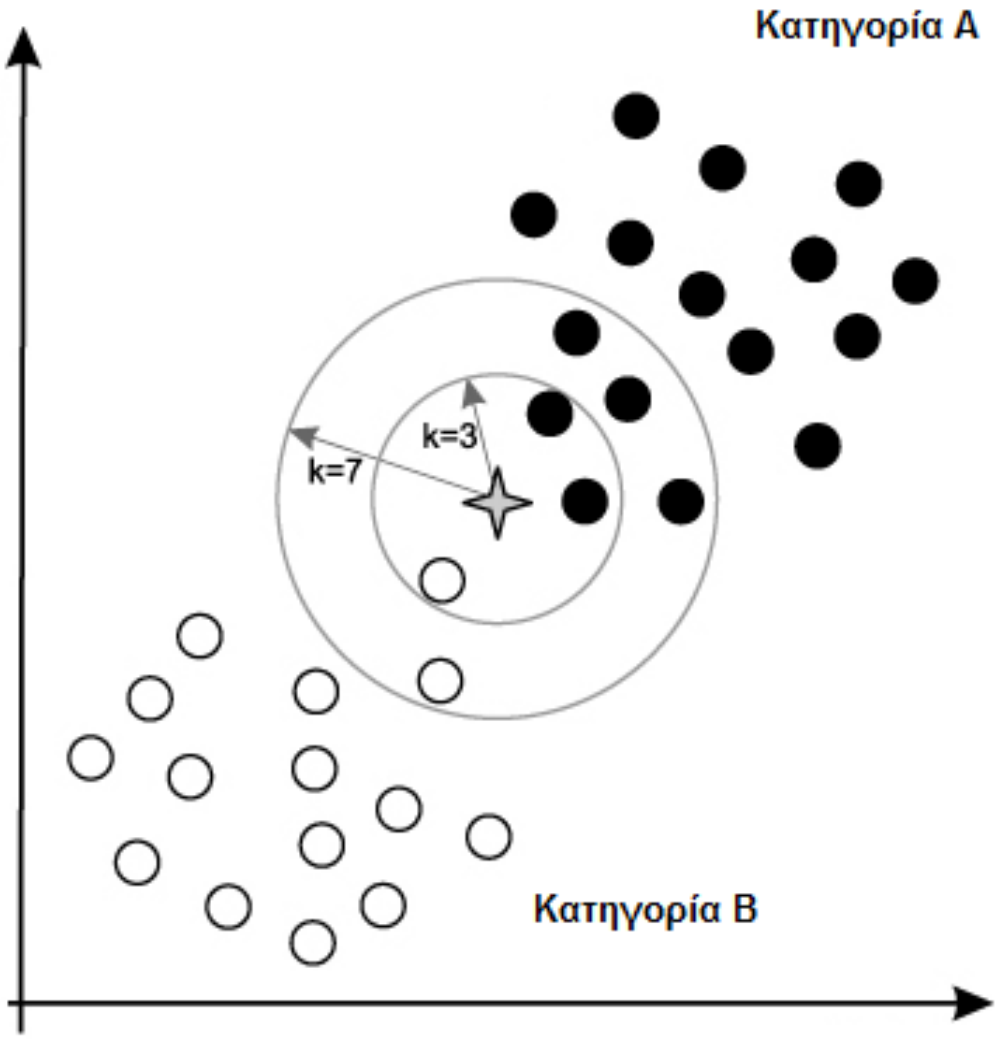




Instance-Based Learning



Αλγόριθμος kNN
(k Nearest Neighbour)



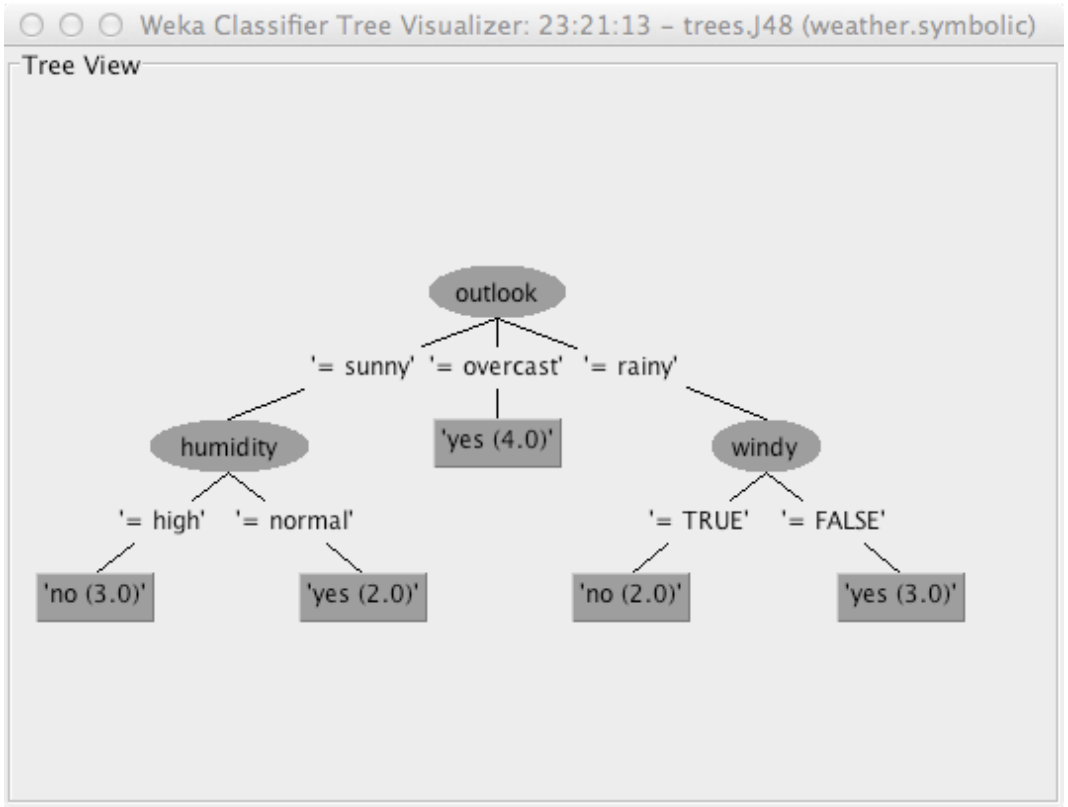
Αλγόριθμος C4.5 / J48

Viewer

Relation: weather.symbolic

No.	outlook Nominal	temperature Nominal	humidity Nominal	windy Nominal	play Nominal
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
3	overcast	hot	high	FALSE	yes
4	rainy	mild	high	FALSE	yes
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
7	overcast	cool	normal	TRUE	yes
8	sunny	mild	high	FALSE	no
9	sunny	cool	normal	FALSE	yes
10	rainy	mild	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
13	overcast	hot	normal	FALSE	yes
14	rainy	mild	high	TRUE	no

Undo OK Cancel



Αλγόριθμος Apriori

```
RULES:

 1. outlook=overcast 4 ==> play=yes 4    conf:(1)
 2. temperature=cool 4 ==> humidity=normal 4    conf:(1)
 3. humidity=normal windy=FALSE 4 ==> play=yes 4    conf:(1)
 4. outlook=sunny play=no 3 ==> humidity=high 3    conf:(1)
 5. outlook=sunny humidity=high 3 ==> play=no 3    conf:(1)
 6. outlook=rainy play=yes 3 ==> windy=FALSE 3    conf:(1)
 7. outlook=rainy windy=FALSE 3 ==> play=yes 3    conf:(1)
 8. temperature=cool play=yes 3 ==> humidity=normal 3    conf:(1)
 9. outlook=sunny temperature=hot 2 ==> humidity=high 2    conf:(1)
10. temperature=hot play=no 2 ==> outlook=sunny 2    conf:(1)
```

Relation: weather.symbolic

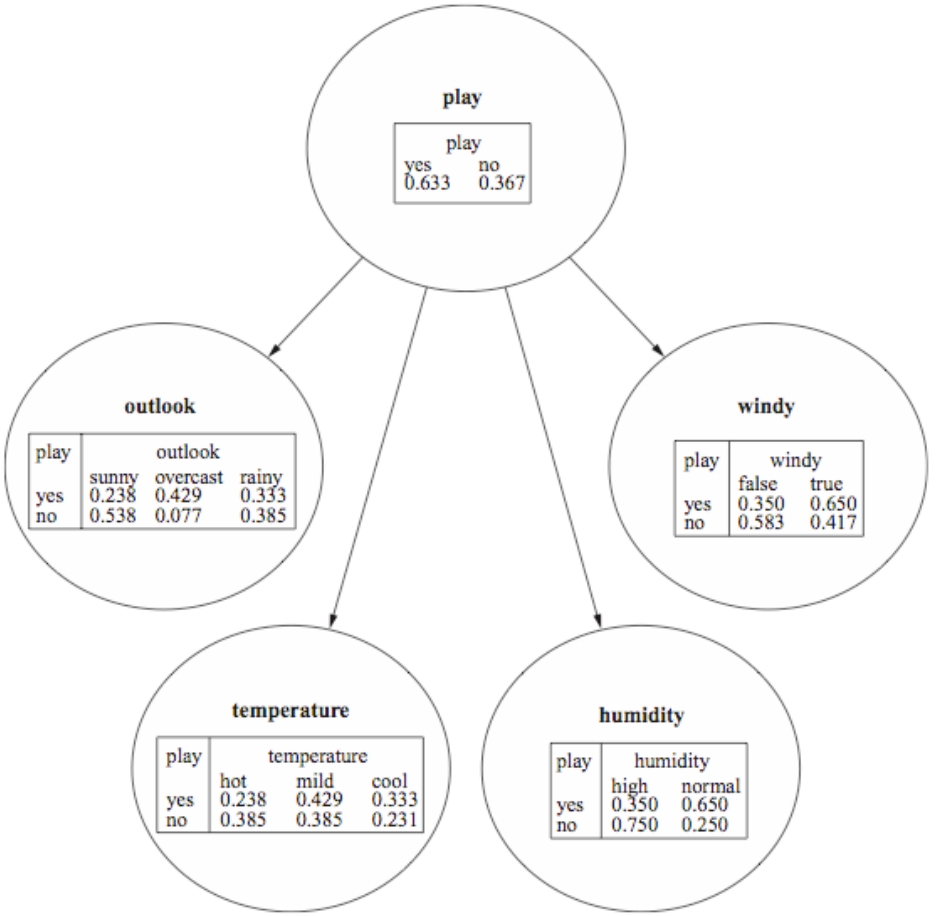
No.	outlook Nominal	temperature Nominal	humidity Nominal	windy Nominal	play Nominal
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
3	overcast	hot	high	FALSE	yes
4	rainy	mild	high	FALSE	yes
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
7	overcast	cool	normal	TRUE	yes
8	sunny	mild	high	FALSE	no
9	sunny	cool	normal	FALSE	yes
10	rainy	mild	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
	overcast	hot	normal	FALSE	yes
	rainy	mild	high	TRUE	no

Naive Bayes

$$P(Class\ A|Feature\ 1, Feature\ 2) = \frac{P(Feature\ 1|Class\ A) \cdot P(Feature\ 2|Class\ A) \cdot P(Class\ A)}{P(Feature\ 1) \cdot P(Feature\ 2)}$$

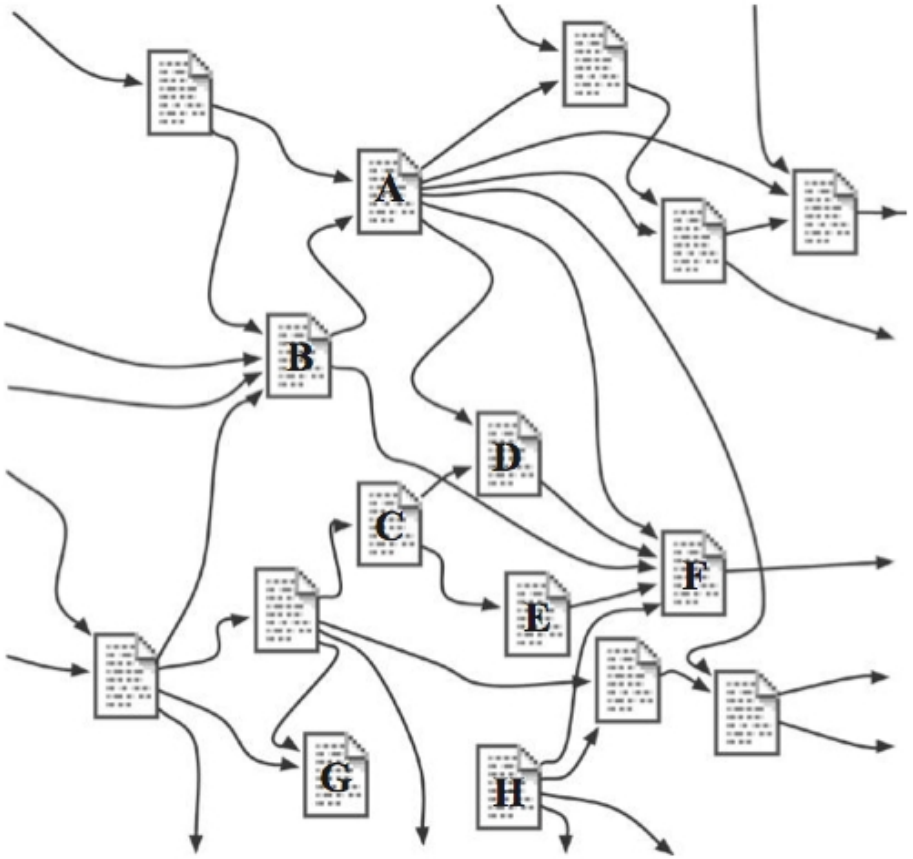
Relation: weather.symbolic

No.	outlook Nominal	temperature Nominal	humidity Nominal	windy Nominal	**play** Nominal
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
3	overcast	hot	high	FALSE	yes
4	rainy	mild	high	FALSE	yes
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
7	overcast	cool	normal	TRUE	yes
8	sunny	mild	high	FALSE	no
9	sunny	cool	normal	FALSE	yes
10	rainy	mild	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
13	overcast	hot	normal	FALSE	yes
14	rainy	mild	high	TRUE	no

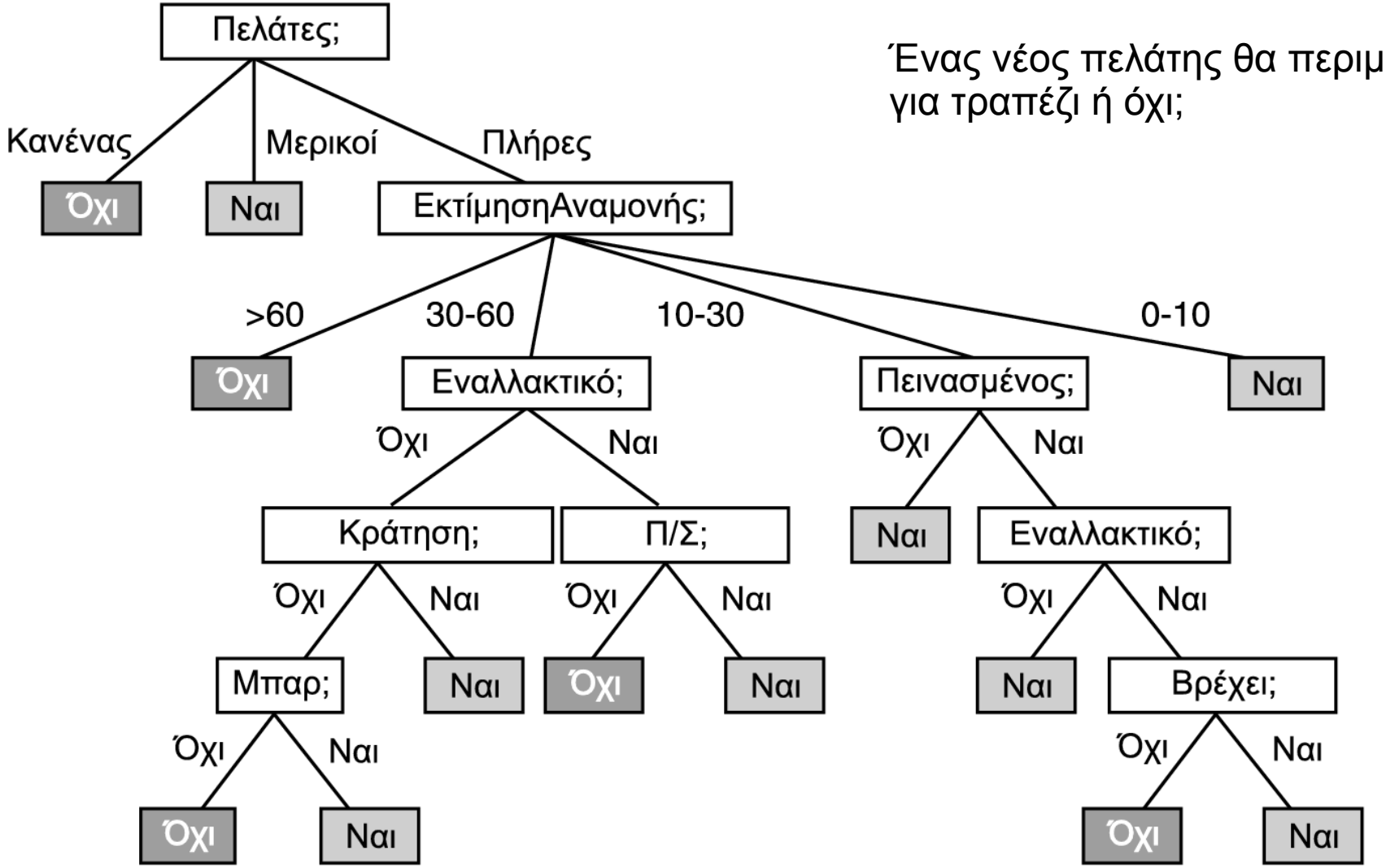


Αλγόριθμος PageRank

Website	PageRank
twitter.com	10
facebook.com	9
reddit.com	8
stackoverflow.com	7
tumblr.com	6
crucial.com	5
programmingzen.com	4
dearblogger.org	3



Δένδρα Αποφάσεων
Decision Trees

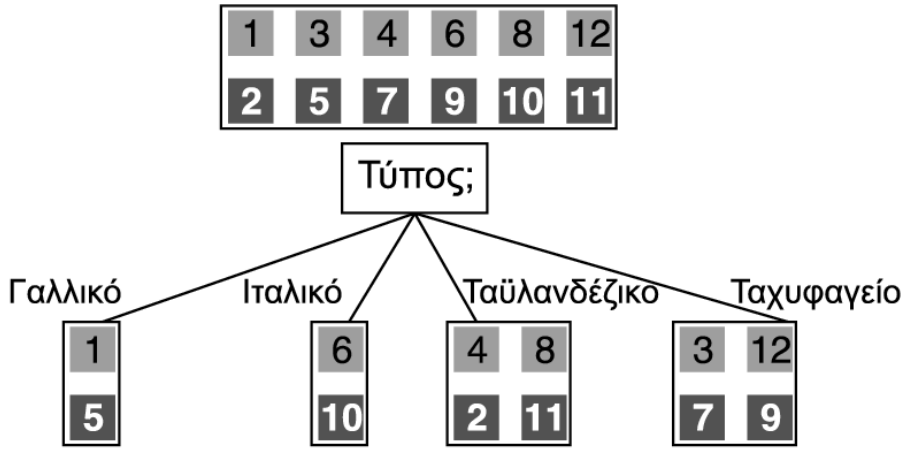


Ένας νέος πελάτης θα περιμένει για τραπέζι ή όχι;

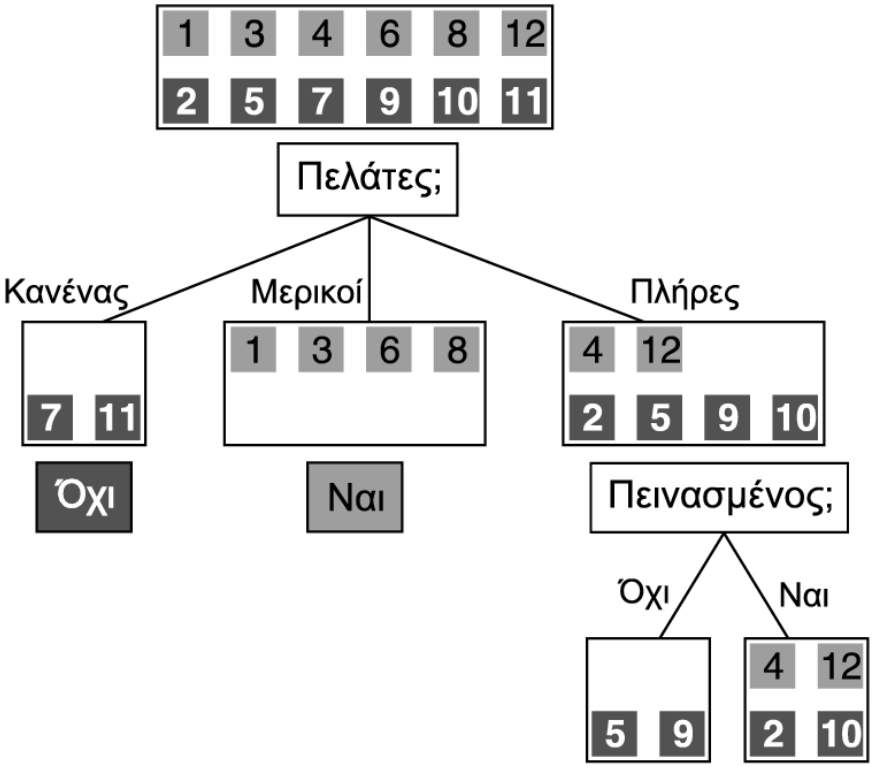
Πίνακας Δεδομένων Εκπαίδευσης

#	Εναλ	Μπαρ	Π/Σ	Πεινασμ	Πελατες	Τιμή	Βρέχει	Κράτηση	Τύπος	Εκτιμ	ΘαΠεριμένει
X_1	Ναι	Όχι	Όχι	Ναι	Μερικοί	$$$	Όχι	Ναι	Γαλλικό	0-10	**Ναι**
X_2	Ναι	Όχι	Όχι	Ναι	Πλήρες	$	Όχι	Όχι	Ταϋλ	30-60	**Όχι**
X_3	Όχι	Ναι	Όχι	Όχι	Μερικοί	$	Όχι	Όχι	Ταχυφ.	0-10	**Ναι**
X_4	Ναι	Όχι	Ναι	Ναι	Πλήρες	$	Ναι	Όχι	Ταϋλ	10-30	**Ναι**
X_5	Ναι	Όχι	Ναι	Όχι	Πλήρες	$$$	Όχι	Ναι	Γαλλικό	>60	**Όχι**
X_6	Όχι	Ναι	Όχι	Ναι	Μερικοί	$$	Ναι	Ναι	Ιταλικό	0-10	**Ναι**
X_7	Όχι	Ναι	Όχι	Όχι	Κανένας	$	Ναι	Όχι	Ταχυφ.	0-10	**Όχι**
X_8	Όχι	Όχι	Όχι	Ναι	Μερικοί	$$	Ναι	Ναι	Ταϋλ	0-10	**Ναι**
X_9	Όχι	Ναι	Ναι	Όχι	Πλήρες	$	Ναι	Όχι	Ταχυφ.	>60	**Όχι**
X_{10}	Ναι	Ναι	Ναι	Ναι	Πλήρες	$$$	Όχι	Ναι	Ιταλικό	10-30	**Όχι**
X_{11}	Όχι	Όχι	Όχι	Όχι	Κανένας	$	Όχι	Όχι	Ταϋλ	0-10	**Όχι**
X_{12}	Ναι	Ναι	Ναι	Ναι	Πλήρες	$	Όχι	Όχι	Ταχυφ.	30-60	**Ναι**

Επιλογή Χαρακτηριστικού



(α)



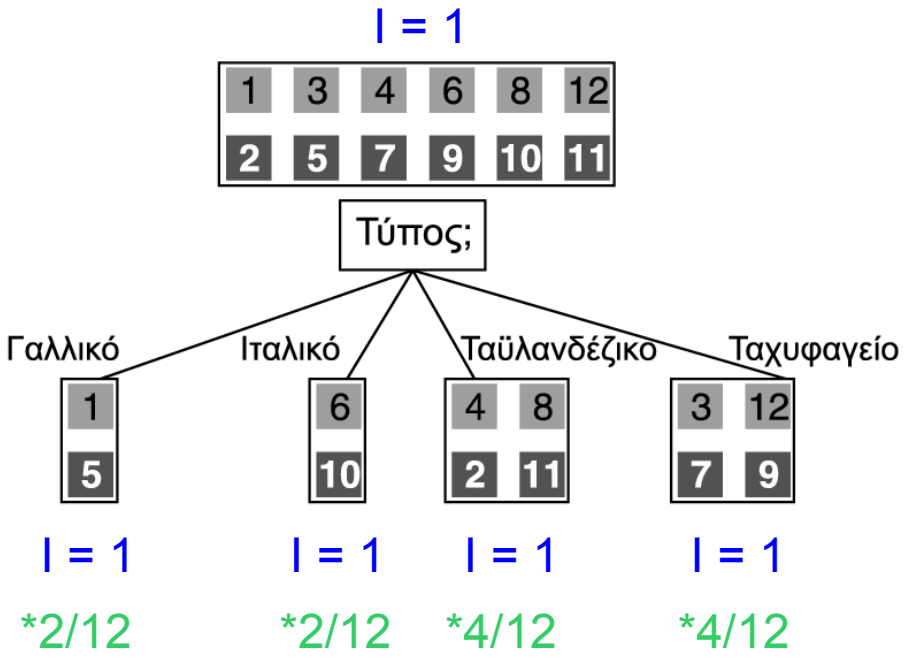
(β)

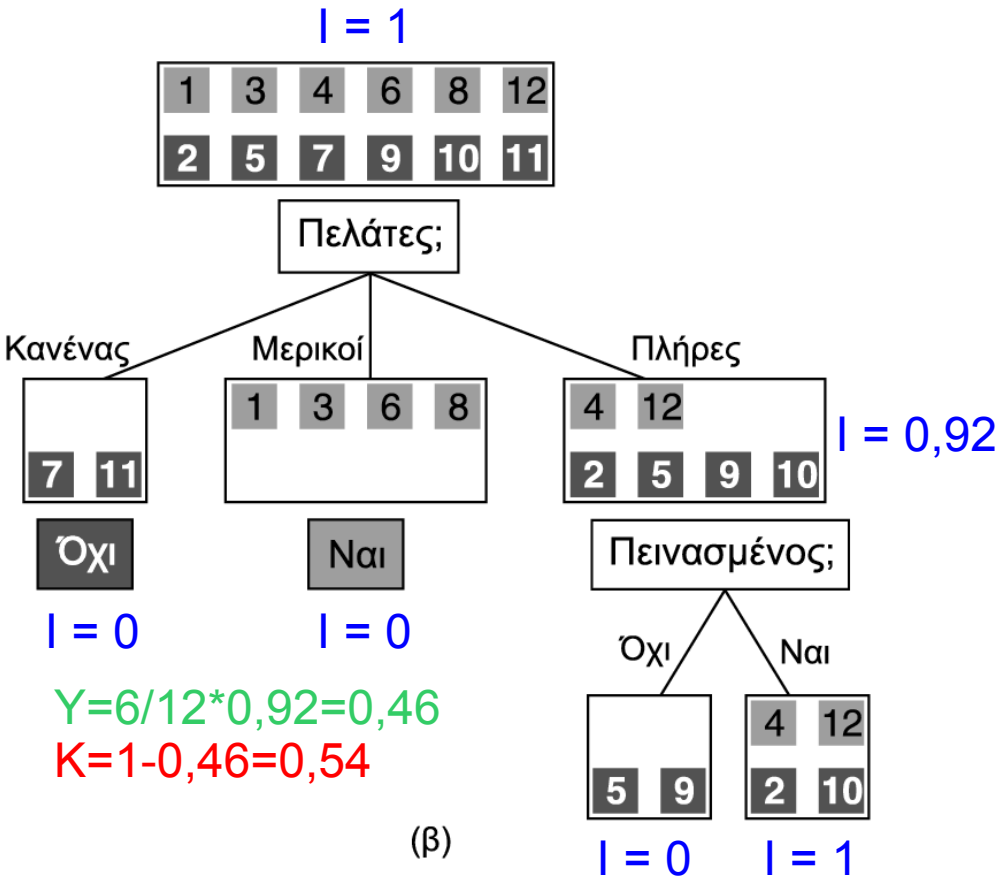
Κέρδος Πληροφορίας
Information Gain

$$I\left(\frac{p}{p+n},\frac{n}{p+n}\right)=-\frac{p}{p+n}\log_2\left(\frac{p}{p+n}\right)-\frac{n}{p+n}\log_2\left(\frac{n}{p+n}\right)$$

$$Υπόλοιπο(A)=\sum_{i=1}^{v}\frac{p_i+n_i}{p+n}I\left(\frac{p_i}{p_i+n_i},\frac{n_i}{p_i+n_i}\right)$$

$$Κέρδος(A)=I\left(\frac{p}{p+n},\frac{n}{p+n}\right)-Υπόλοιπο(A)$$



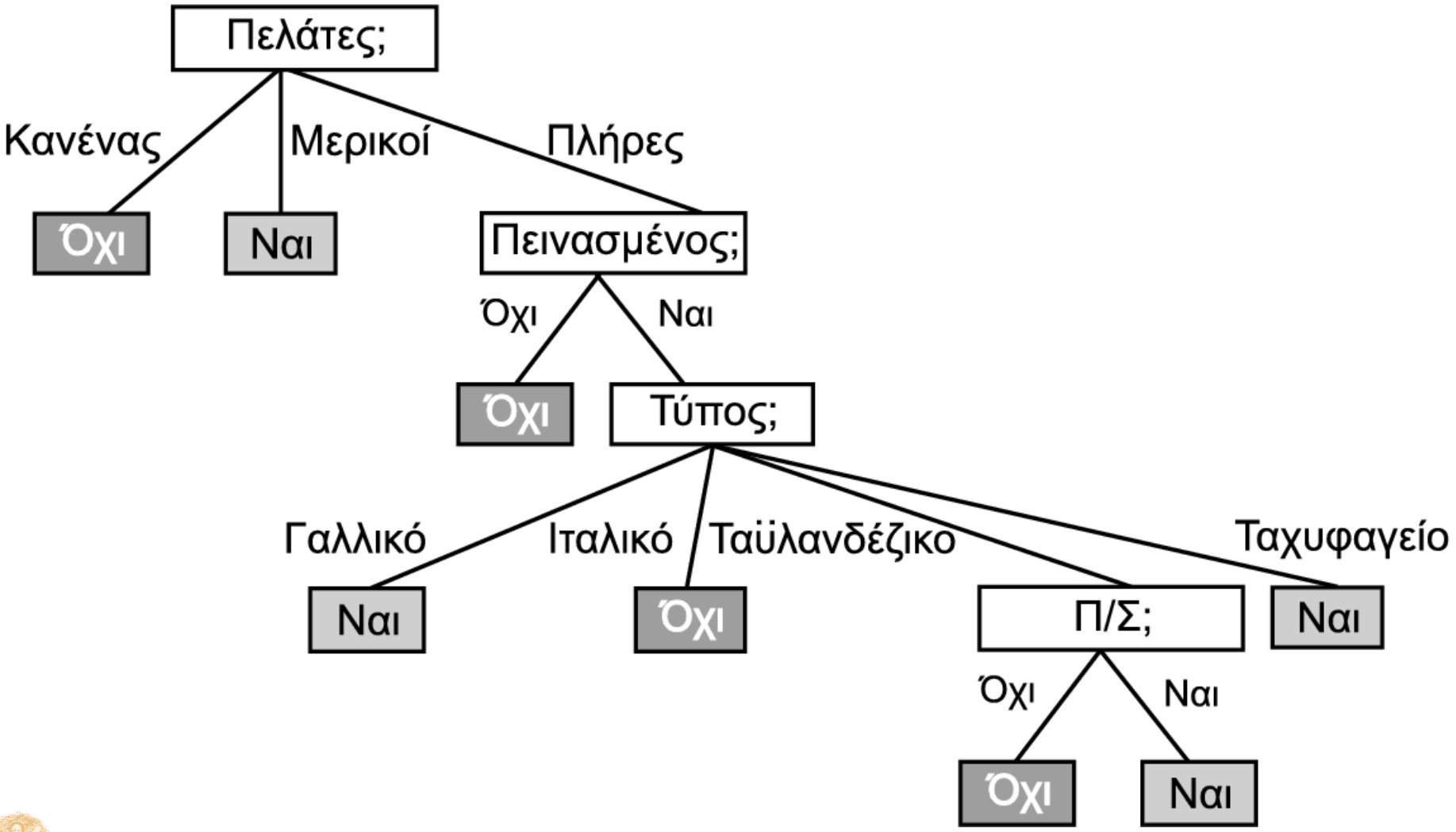


$$I\left(\frac{p}{p+n},\frac{n}{p+n}\right)=-\frac{p}{p+n}\log_2\left(\frac{p}{p+n}\right)-\frac{n}{p+n}\log_2\left(\frac{n}{p+n}\right)$$

$$Υπόλοιπο(A)=\sum_{i=1}^{v}\frac{p_i+n_i}{p+n}I\left(\frac{p_i}{p_i+n_i},\frac{n_i}{p_i+n_i}\right)$$

$$Κέρδος(A)=I\left(\frac{p}{p+n},\frac{n}{p+n}\right)-Υπόλοιπο(A)$$

Τελικά


Πελάτες;
Κανένας
Μερικοί
Πλήρες
Όχι
Ναι
Πεινασμένος;
Όχι
Ναι
Όχι
Τύπος;
Γαλλικό
Ιταλικό
Ταϋλανδέζικο
Ταχυφαγείο
Ναι
Όχι
Π/Σ;
Ναι
Όχι
Ναι
Όχι
Ναι

play?

No	outlook	temperature	humidity	windy	play
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
3	overcast	hot	high	FALSE	yes
4	rainy	mild	high	FALSE	yes
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
7	overcast	cool	normal	TRUE	yes
8	sunny	mild	high	FALSE	no
9	sunny	cool	normal	FALSE	yes
10	rainy	mild	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
13	overcast	hot	normal	FALSE	yes
14	rainy	mild	high	TRUE	no

$$I\left(\frac{p}{p+n},\frac{n}{p+n}\right) = -\frac{p}{p+n}\log_2\left(\frac{p}{p+n}\right) - \frac{n}{p+n}\log_2\left(\frac{n}{p+n}\right)$$

$$I\left(\frac{9}{14},\frac{5}{14}\right) = -\frac{9}{14}\log_2\left(\frac{9}{14}\right) - \frac{5}{14}\log_2\left(\frac{5}{14}\right) = 0.94$$

1. outlook

$$I\left(\frac{9}{14},\frac{5}{14}\right)=0.94$$

outlook

$$I\left(\frac{2}{5},\frac{3}{5}\right)=0.97$$

No	outlook	temperature	humidity	windy	play
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
8	sunny	mild	high	FALSE	no
9	sunny	cool	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes

$$I\left(\frac{3}{5},\frac{2}{5}\right)=0.97$$

No	outlook	temperature	humidity	windy	play
4	rainy	mild	high	FALSE	yes
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
10	rainy	mild	normal	FALSE	yes
14	rainy	mild	high	TRUE	no

No	outlook	temperature	humidity	windy	play
3	overcast	hot	high	FALSE	yes
7	overcast	cool	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
13	overcast	hot	normal	FALSE	yes

$$I\left(\frac{4}{4},\frac{0}{4}\right)=$$

$$\frac{-4}{4}\log_2\left(\frac{4}{4}\right)-\frac{0}{4}\log_2\left(\frac{0}{4}\right)=0$$

$= 0$ $\quad$ $= 0$

$$Υπόλοιπο(outlook)=\sum_{i=1}^{v}\frac{p_i+n_i}{p+n}I\left(\frac{p_i}{p_i+n_i},\frac{n_i}{p_i+n_i}\right)=\frac{5}{14}I\left(\frac{2}{5},\frac{3}{5}\right)+\frac{4}{14}I\left(\frac{4}{4},\frac{0}{4}\right)+\frac{5}{14}I\left(\frac{3}{5},\frac{2}{5}\right)=0.69$$

$$Κέρδος(outlook)=I\left(\frac{p}{p+n},\frac{n}{p+n}\right)-Υπόλοιπο(outlook)=0.94-0.69=0.25$$

2. temperature

$$I\left(\frac{9}{14},\frac{5}{14}\right)=0.94$$

temperature

$$I\left(\frac{2}{4},\frac{2}{4}\right)=1$$

No	outlook	temperature	humidity	windy	play
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
3	overcast	hot	high	FALSE	yes
13	overcast	hot	normal	FALSE	yes

$$I\left(\frac{3}{4},\frac{1}{4}\right)=0.81$$

No	outlook	temperature	humidity	windy	play
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
7	overcast	cool	normal	TRUE	yes
9	sunny	cool	normal	FALSE	yes

No	outlook	temperature	humidity	windy	play
4	rainy	mild	high	FALSE	yes
8	sunny	mild	high	FALSE	no
10	rainy	mild	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
14	rainy	mild	high	TRUE	no

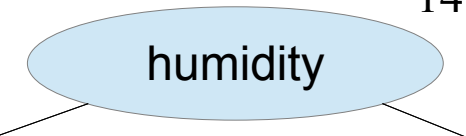
$$I\left(\frac{4}{6},\frac{2}{6}\right)=0.92$$

$$Υπόλοιπο(temperature)=\sum_{i=1}^{v}\frac{p_i+n_i}{p+n}I\left(\frac{p_i}{p_i+n_i},\frac{n_i}{p_i+n_i}\right)=\frac{4}{14}I\left(\frac{2}{4},\frac{2}{4}\right)+\frac{6}{14}I\left(\frac{4}{6},\frac{2}{6}\right)+\frac{4}{14}I\left(\frac{3}{4},\frac{1}{4}\right)=0.90$$

$$Κέρδος(temperature)=I\left(\frac{p}{p+n},\frac{n}{p+n}\right)-Υπόλοιπο(temperature)=0.94-0.90=0.04$$

3. humidity

$$I\left(\frac{9}{14},\frac{5}{14}\right)=0.94$$



$$I\left(\frac{3}{7},\frac{4}{7}\right)=0.99$$

No	outlook	temperature	humidity	windy	play
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
3	overcast	hot	high	FALSE	yes
4	rainy	mild	high	FALSE	yes
8	sunny	mild	high	FALSE	no
12	overcast	mild	high	TRUE	yes
14	rainy	mild	high	TRUE	no

$$I\left(\frac{6}{7},\frac{1}{7}\right)=0.59$$

No	outlook	temperature	humidity	windy	play
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
7	overcast	cool	normal	TRUE	yes
9	sunny	cool	normal	FALSE	yes
10	rainy	mild	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes
13	overcast	hot	normal	FALSE	yes

$$Υπόλοιπο(humidity)=\sum_{i=1}^{v}\frac{p_i+n_i}{p+n}I\left(\frac{p_i}{p_i+n_i},\frac{n_i}{p_i+n_i}\right)=\frac{7}{14}I\left(\frac{3}{7},\frac{4}{7}\right)+\frac{7}{14}I\left(\frac{6}{7},\frac{1}{7}\right)=0.79$$

$$Κέρδος(humidity)=I\left(\frac{p}{p+n},\frac{n}{p+n}\right)-Υπόλοιπο(humidity)=0.94-0.79=0.15$$

4. windy

$$I\left(\frac{9}{14},\frac{5}{14}\right)=0.94$$

windy

$$I\left(\frac{3}{6},\frac{3}{6}\right)=1$$

$$I\left(\frac{6}{8},\frac{2}{8}\right)=0.81$$

No	outlook	temperature	humidity	windy	play
2	sunny	hot	high	TRUE	no
6	rainy	cool	normal	TRUE	no
7	overcast	cool	normal	TRUE	yes
11	sunny	mild	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
14	rainy	mild	high	TRUE	no

No	outlook	temperature	humidity	windy	play
1	sunny	hot	high	FALSE	no
3	overcast	hot	high	FALSE	yes
4	rainy	mild	high	FALSE	yes
5	rainy	cool	normal	FALSE	yes
8	sunny	mild	high	FALSE	no
9	sunny	cool	normal	FALSE	yes
10	rainy	mild	normal	FALSE	yes
13	overcast	hot	normal	FALSE	yes

$$Υπόλοιπο(windy)=\sum_{i=1}^{v}\frac{p_i+n_i}{p+n}I\left(\frac{p_i}{p_i+n_i},\frac{n_i}{p_i+n_i}\right)=\frac{6}{14}I\left(\frac{3}{6},\frac{3}{6}\right)+\frac{8}{14}I\left(\frac{6}{8},\frac{2}{8}\right)=0.89$$

$$Κέρδος(windy)=I\left(\frac{p}{p+n},\frac{n}{p+n}\right)-Υπόλοιπο(windy)=0.94-0.89=0.05$$

Τελικά

$$I\left(\frac{9}{14},\frac{5}{14}\right)=0.94$$

outlook

$$I\left(\frac{2}{5},\frac{3}{5}\right)=0.97$$

No	outlook	temperatur e	humidi ty	windy	play
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
8	sunny	mild	high	FALSE	no
9	sunny	cool	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes

$$I\left(\frac{3}{5},\frac{2}{5}\right)=0.97$$

No	outlook	temperatur e	humidi ty	windy	play
4	rainy	mild	high	FALSE	yes
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
10	rainy	mild	normal	FALSE	yes
14	rainy	mild	high	TRUE	no

No	outlook	temperatur e	humidi ty	windy	play
3	overcast	hot	high	FALSE	yes
7	overcast	cool	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
13	overcast	hot	normal	FALSE	yes

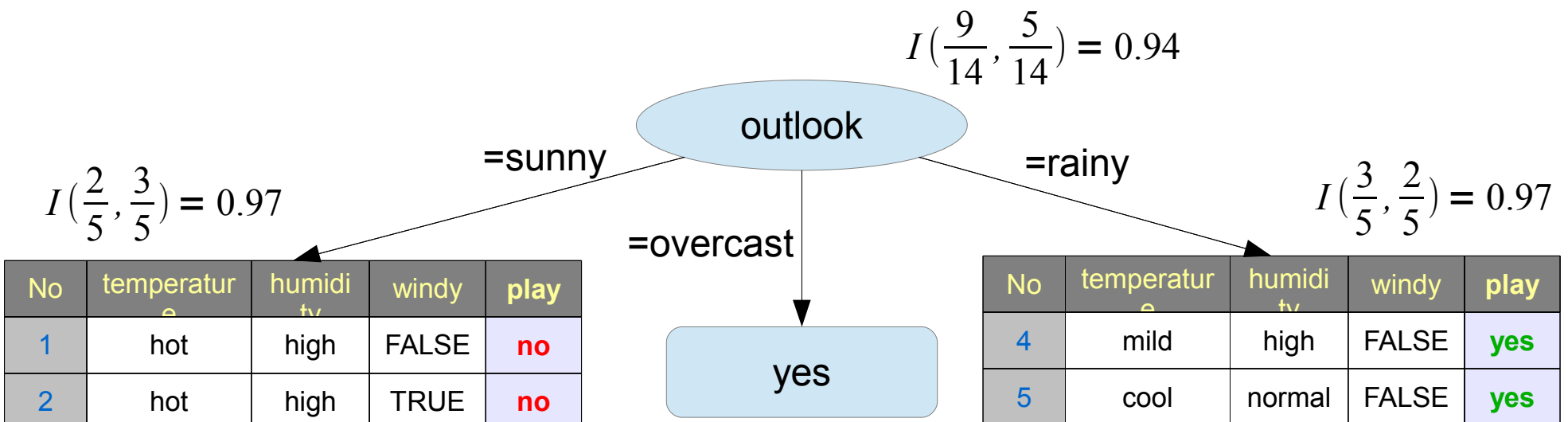
$$I\left(\frac{4}{4},\frac{0}{4}\right)=$$

$$\frac{-4}{4}\log_2\left(\frac{4}{4}\right)-\frac{0}{4}\log_2\left(\frac{0}{4}\right)=0$$

$= 0$ $\quad$ $= 0$

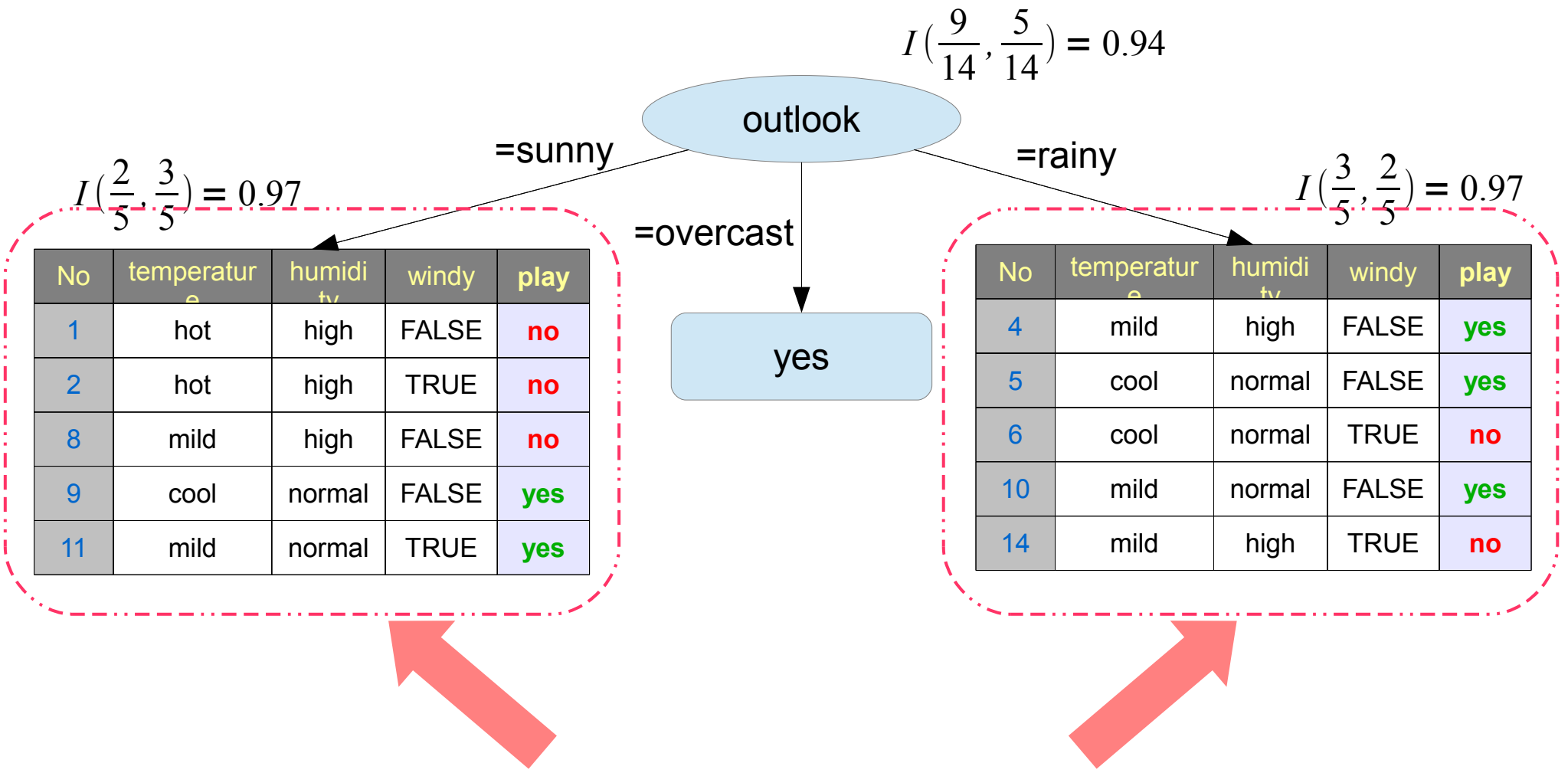
$$Υπόλοιπο(outlook)=\sum_{i=1}^{v}\frac{p_i+n_i}{p+n}I\left(\frac{p_i}{p_i+n_i},\frac{n_i}{p_i+n_i}\right)=\frac{5}{14}I\left(\frac{2}{5},\frac{3}{5}\right)+\frac{4}{14}I\left(\frac{4}{4},\frac{0}{4}\right)+\frac{5}{14}I\left(\frac{3}{5},\frac{2}{5}\right)=0.69$$

$$Κέρδος(outlook)=I\left(\frac{p}{p+n},\frac{n}{p+n}\right)-Υπόλοιπο(outlook)=0.94-0.69=0.25$$



No	temperatur e	humidi ty	windy	play
1	hot	high	FALSE	no
2	hot	high	TRUE	no
8	mild	high	FALSE	no
9	cool	normal	FALSE	yes
11	mild	normal	TRUE	yes

No	temperatur e	humidi ty	windy	play
4	mild	high	FALSE	yes
5	cool	normal	FALSE	yes
6	cool	normal	TRUE	no
10	mild	normal	FALSE	yes
14	mild	high	TRUE	no

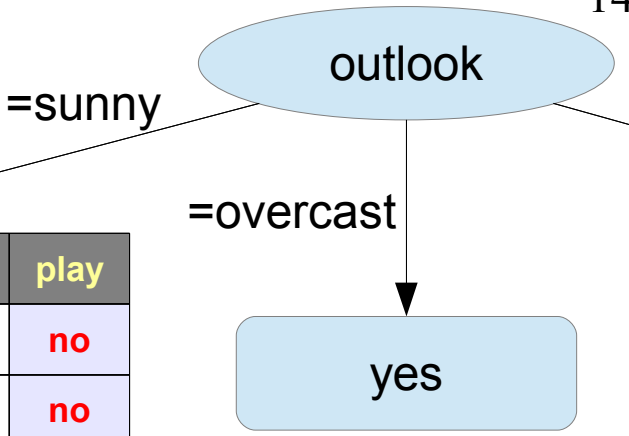


No	temperature	humidity	windy	play
1	hot	high	FALSE	no
2	hot	high	TRUE	no
8	mild	high	FALSE	no
9	cool	normal	FALSE	yes
11	mild	normal	TRUE	yes

No	temperature	humidity	windy	play
4	mild	high	FALSE	yes
5	cool	normal	FALSE	yes
6	cool	normal	TRUE	no
10	mild	normal	FALSE	yes
14	mild	high	TRUE	no

Και επαναλαμβάνουμε
για κάθε υποδέντρο

$$I\left(\frac{9}{14},\frac{5}{14}\right)=0.94$$



$$I\left(\frac{2}{5},\frac{3}{5}\right)=0.97$$

No	temperatur e	humidi ty	windy	play
1	hot	high	FALSE	no
2	hot	high	TRUE	no
8	mild	high	FALSE	no
9	cool	normal	FALSE	yes
11	mild	normal	TRUE	yes

$$I\left(\frac{3}{5},\frac{2}{5}\right)=0.97$$

No	temperatur e	humidi ty	windy	play
4	mild	high	FALSE	yes
5	cool	normal	FALSE	yes
6	cool	normal	TRUE	no
10	mild	normal	FALSE	yes
14	mild	high	TRUE	no

No	temperatur e	humidi ty	windy	play
1	hot	high	FALSE	no
2	hot	high	TRUE	no

$I=0$

No	temperatur e	humidi ty	windy	play
8	mild	high	FALSE	no
11	mild	normal	TRUE	yes

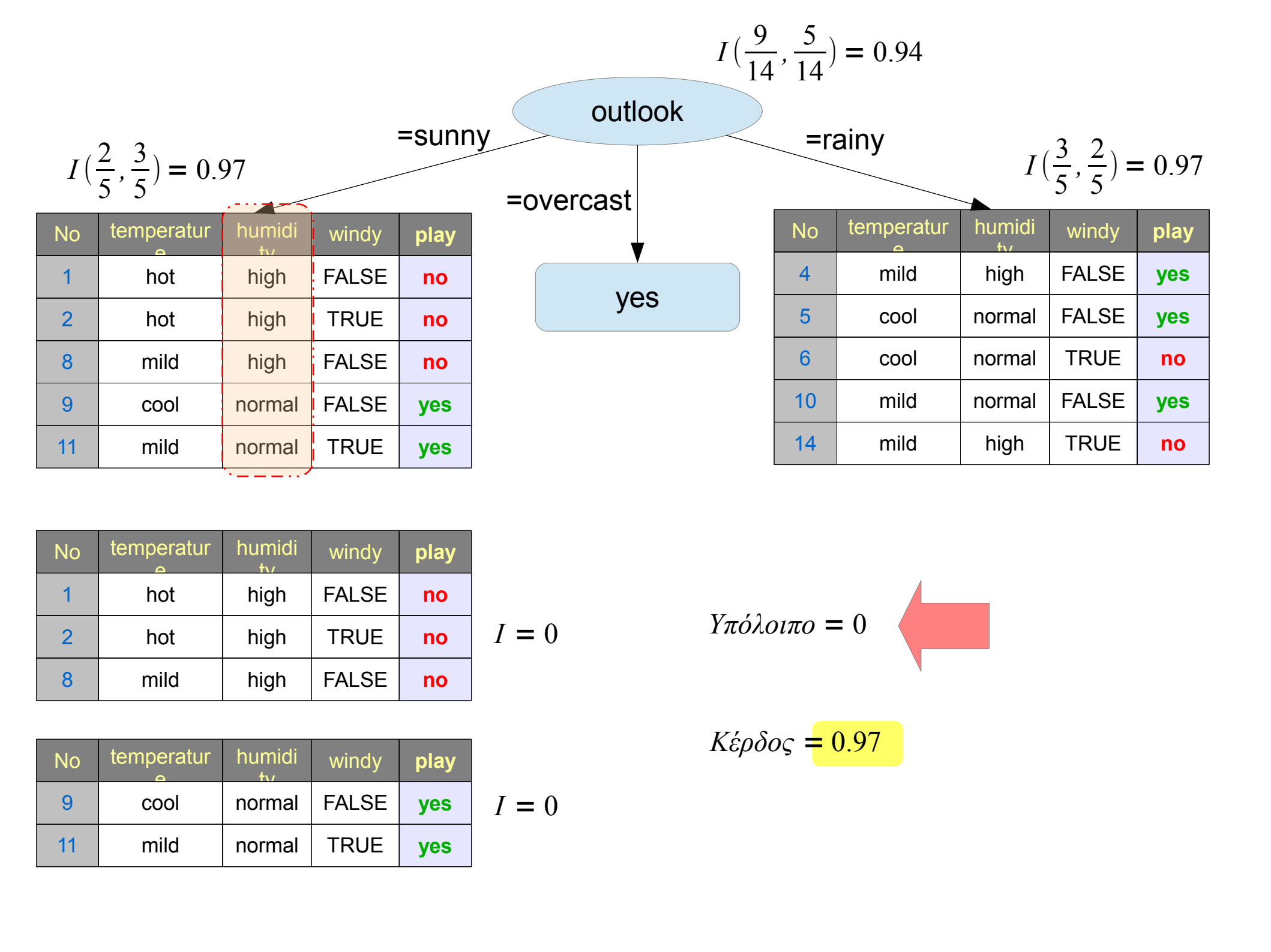
$I=1$

No	temperatur e	humidi ty	windy	play
9	cool	normal	FALSE	yes

$I=0$

$$Υπόλοιπο=\frac{2}{5}0+\frac{2}{5}1+\frac{1}{5}0=0.4$$

$$Κέρδος=0.97-0.4=0.57$$



No	temperatur e	humidi ty	windy	play
1	hot	high	FALSE	no
2	hot	high	TRUE	no
8	mild	high	FALSE	no
9	cool	normal	FALSE	yes
11	mild	normal	TRUE	yes

No	temperatur e	humidi ty	windy	play
4	mild	high	FALSE	yes
5	cool	normal	FALSE	yes
6	cool	normal	TRUE	no
10	mild	normal	FALSE	yes
14	mild	high	TRUE	no

No	temperatur e	humidi ty	windy	play
1	hot	high	FALSE	no
2	hot	high	TRUE	no
8	mild	high	FALSE	no

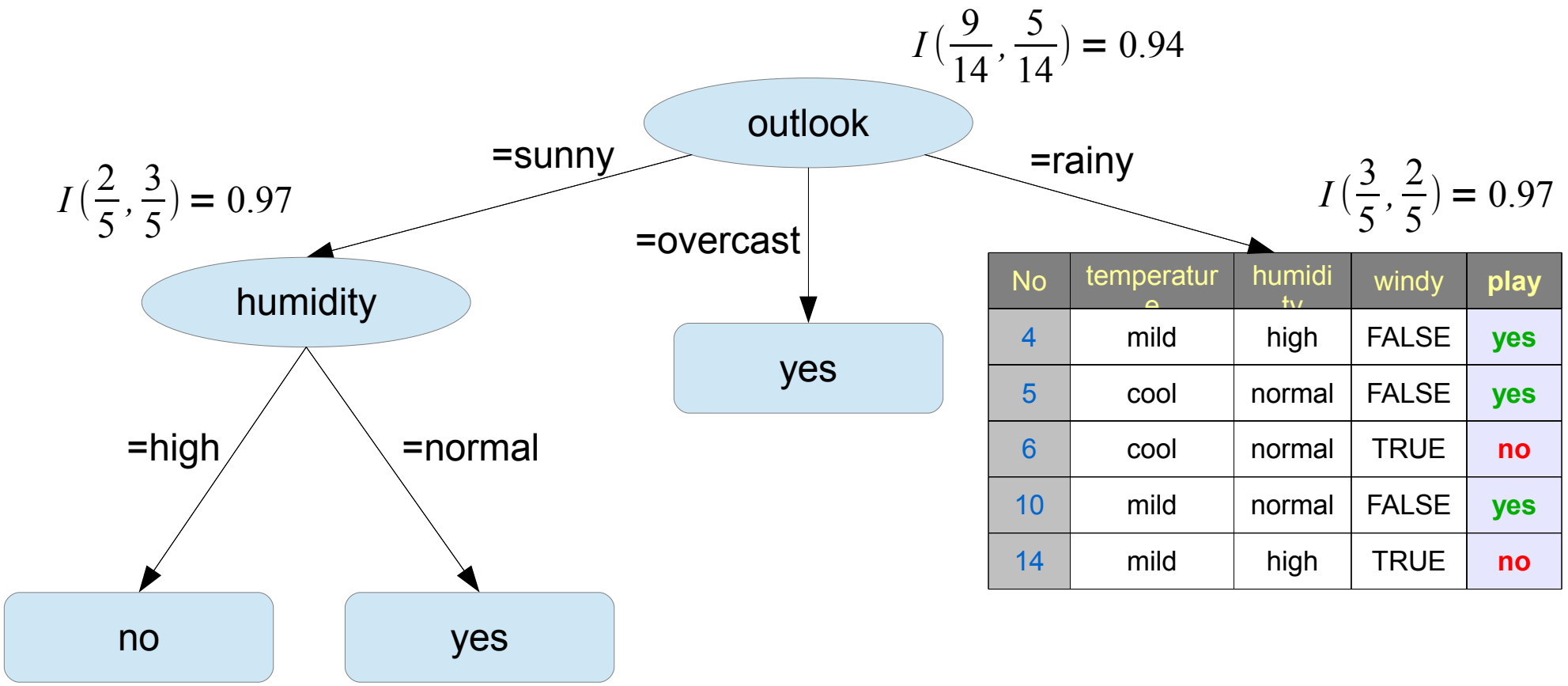
$I = 0$

No	temperatur e	humidi ty	windy	play
9	cool	normal	FALSE	yes
11	mild	normal	TRUE	yes

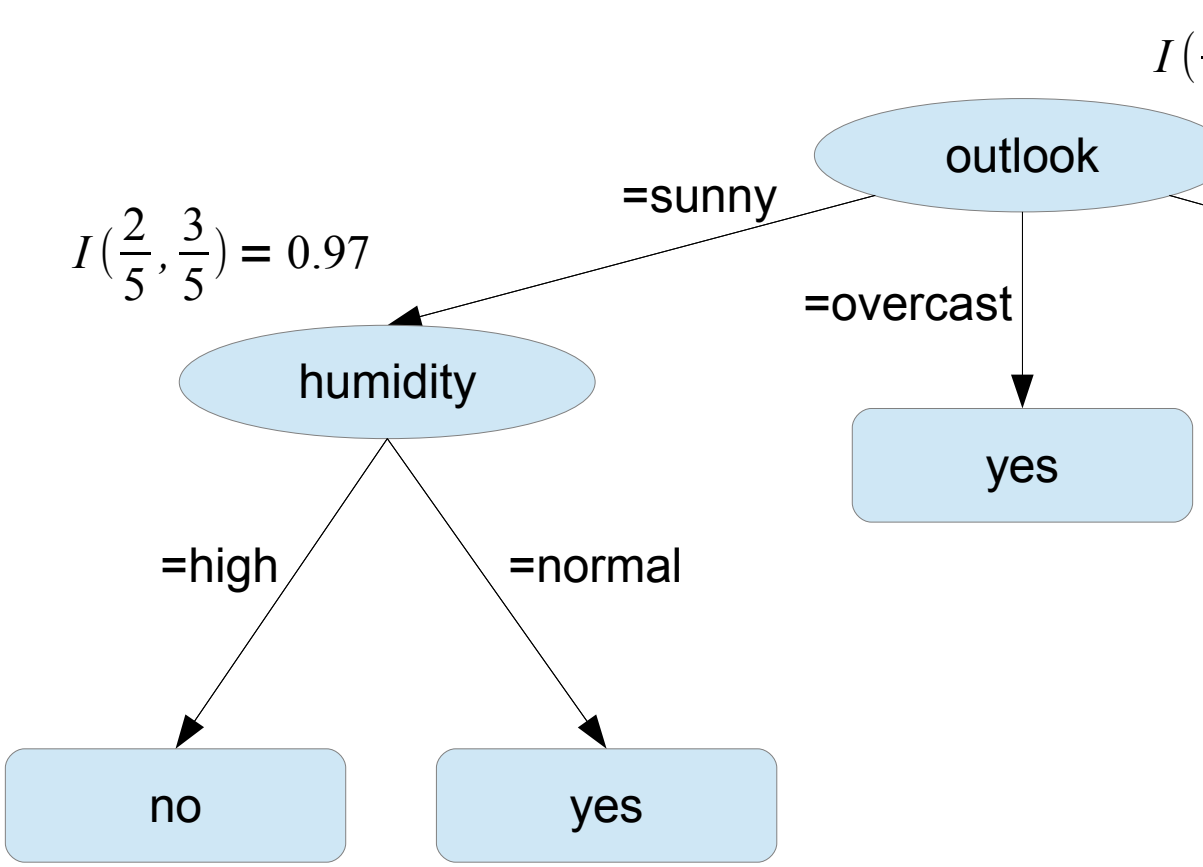
$I = 0$

$Υπόλοιπο = 0$

$Κέρδος = 0.97$



No	temperatur e	humidi ty	windy	play
4	mild	high	FALSE	yes
5	cool	normal	FALSE	yes
6	cool	normal	TRUE	no
10	mild	normal	FALSE	yes
14	mild	high	TRUE	no

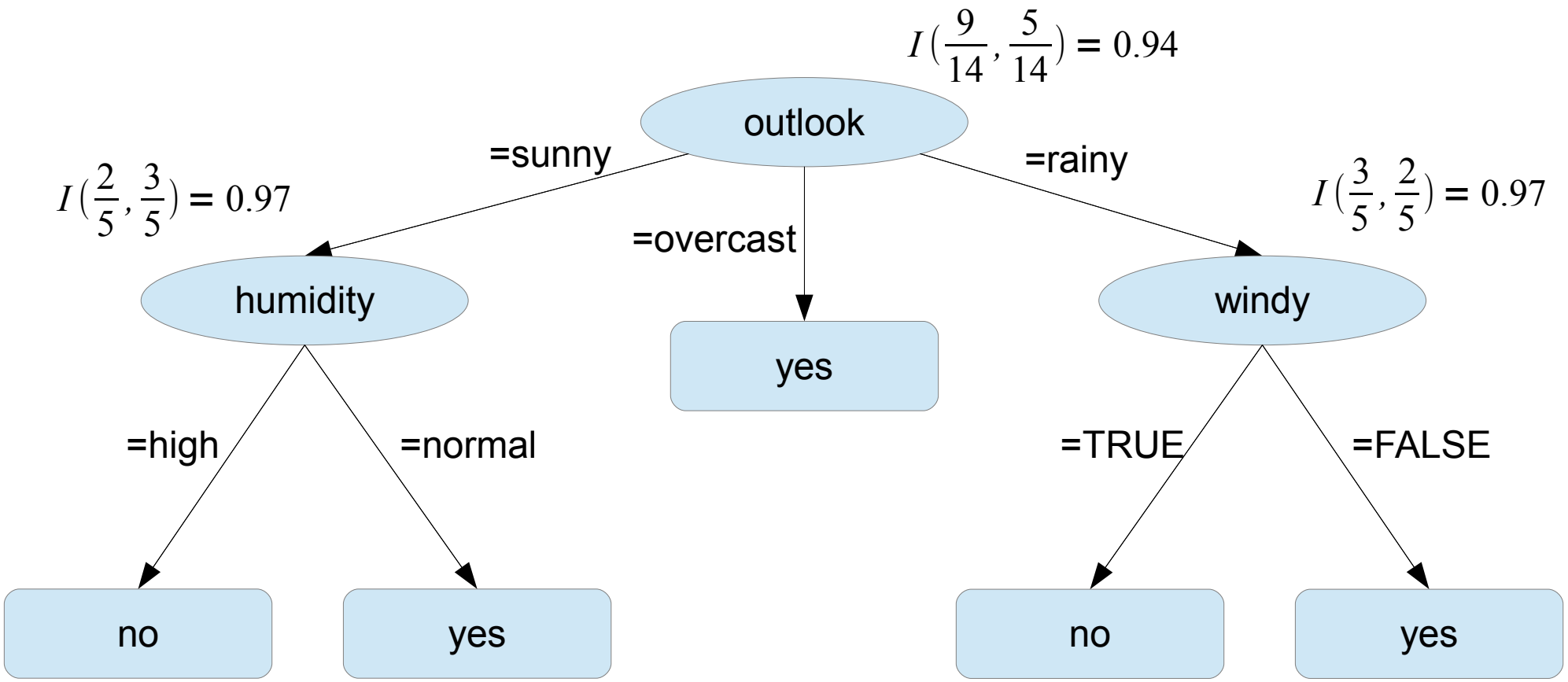


$I(\frac{3}{5}, \frac{2}{5}) = 0.97$

No	temperature	humidity	windy	play
4	mild	high	FALSE	yes
5	cool	normal	FALSE	yes
6	cool	normal	TRUE	no
10	mild	normal	FALSE	yes
14	mild	high	TRUE	no

No	temperature	humidity	windy	play
6	cool	normal	TRUE	no
14	mild	high	TRUE	no

No	temperature	humidity	windy	play
4	mild	high	FALSE	yes
5	cool	normal	FALSE	yes
10	mild	normal	FALSE	yes


$I(\frac{9}{14}, \frac{5}{14}) = 0.94$
outlook
=sunny
=overcast
=rainy
$I(\frac{2}{5}, \frac{3}{5}) = 0.97$
$I(\frac{3}{5}, \frac{2}{5}) = 0.97$
humidity
yes
windy
=high
=normal
=TRUE
=FALSE
no
yes
no
yes

$I(\frac{9}{14}, \frac{5}{14}) = 0.94$

outlook

=sunny

=overcast

=rainy

$I(\frac{2}{5}, \frac{3}{5}) = 0.97$

$I(\frac{3}{5}, \frac{2}{5}) = 0.97$

humidity

yes

windy

=high

=normal

=TRUE

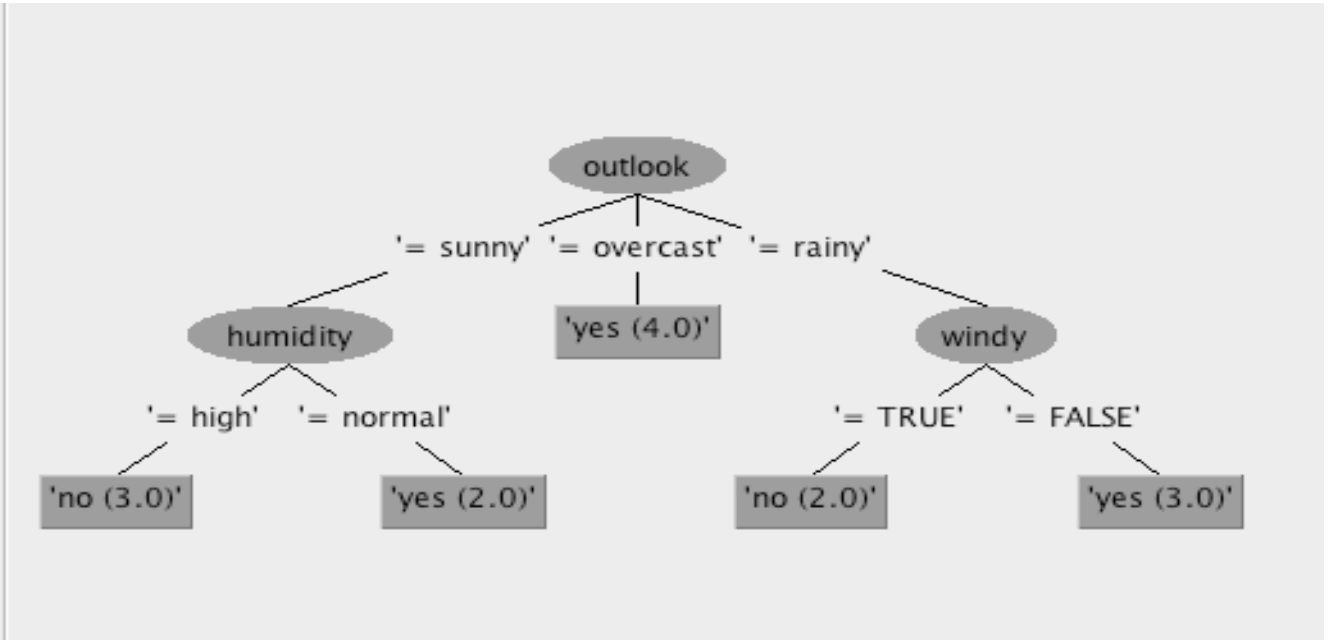
=FALSE

no

yes

no

yes



Κέρδος Πληροφορίας
(Δυαδική Κλάση)

$$I\left(\frac{p}{p+n},\frac{n}{p+n}\right)=-\frac{p}{p+n}\log_2\left(\frac{p}{p+n}\right)-\frac{n}{p+n}\log_2\left(\frac{n}{p+n}\right)$$

$$Υπόλοιπο\left(A\right)=\sum_{i=1}^{v}\frac{p_i+n_i}{p+n}I\left(\frac{p_i}{p_i+n_i},\frac{n_i}{p_i+n_i}\right)$$

$$Κέρδος\left(A\right)=I\left(\frac{p}{p+n},\frac{n}{p+n}\right)-Υπόλοιπο\left(A\right)$$

Κέρδος Πληροφορίας
(Γενική Περίπτωση)

$$I\left(\frac{c_1}{\sum c_i}, \frac{c_2}{\sum c_i}, \ldots, \frac{c_n}{\sum c_i}\right) =$$

$$= -\frac{c_1}{\sum c_i}\log_2\left(\frac{c_1}{\sum c_i}\right) - \frac{c_2}{\sum c_i}\log_2\left(\frac{c_2}{\sum c_i}\right) \ldots - \frac{c_n}{\sum c_i}\log_2\left(\frac{c_n}{\sum c_i}\right)$$

$$= -\sum_n \frac{c_n}{\sum c_i}\log_2\left(\frac{c_n}{\sum c_i}\right)$$

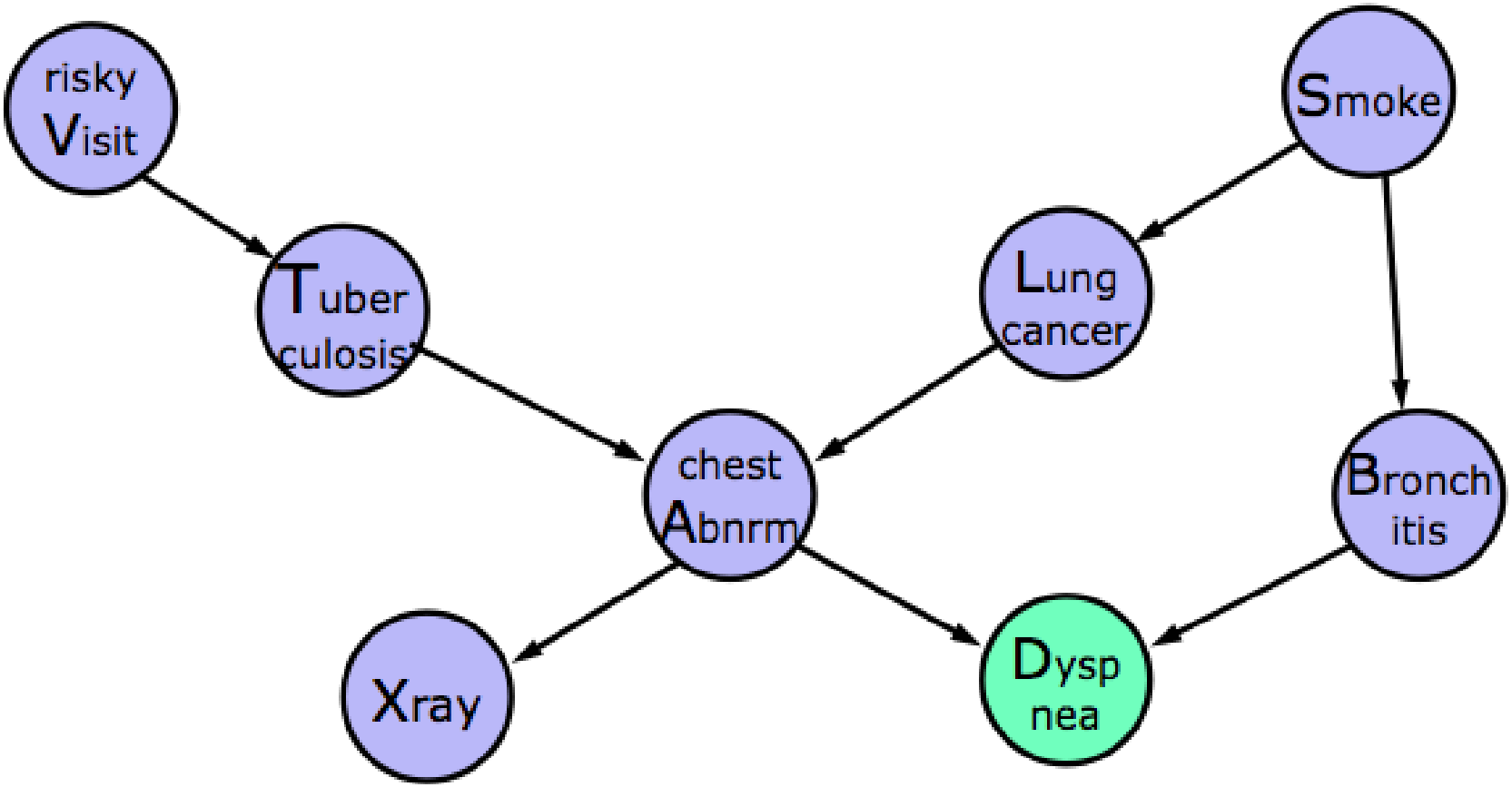
$$Υπόλοιπο(A) = \sum_n \frac{\sum c_n}{\sum c_i} I_n$$

$$Κέρδος(A) = I - Υπόλοιπο(A)$$

Εκφραστικότητα Δένδρων Αποφάσεων

- Ισοδύναμα με κανόνες if-then-else
- Ισοδύναμα με την προτασιακή λογική.
- Υπερβολικά μεγάλα δένδρα σε κάποιες περιπτώσεις.
  - Κλάδεμα δένδρου

Δίκτυο Bayes

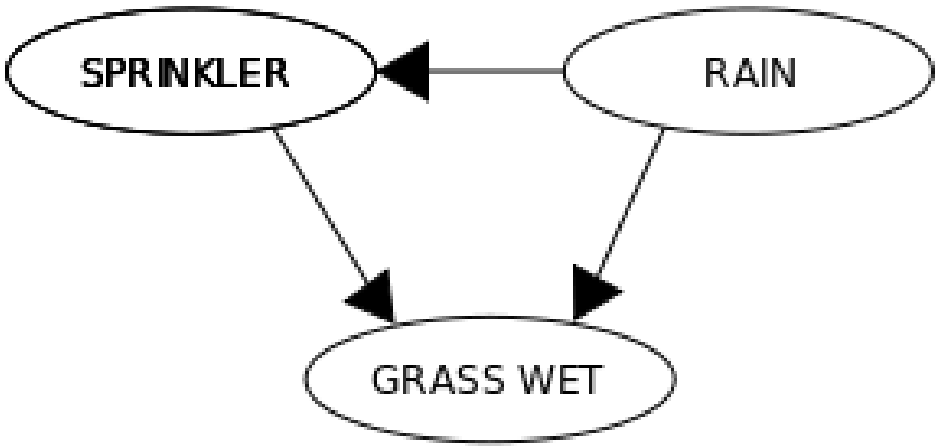


$$\Pr(d) = \sum_{A,B,L,T,S,X,V} \Pr(d \mid a,b)\Pr(a \mid t,l)\Pr(b \mid s)\Pr(l \mid s)\Pr(s)\ \Pr(x \mid a)\Pr(t \mid v)\Pr(v)$$

Δίκτυο Bayes

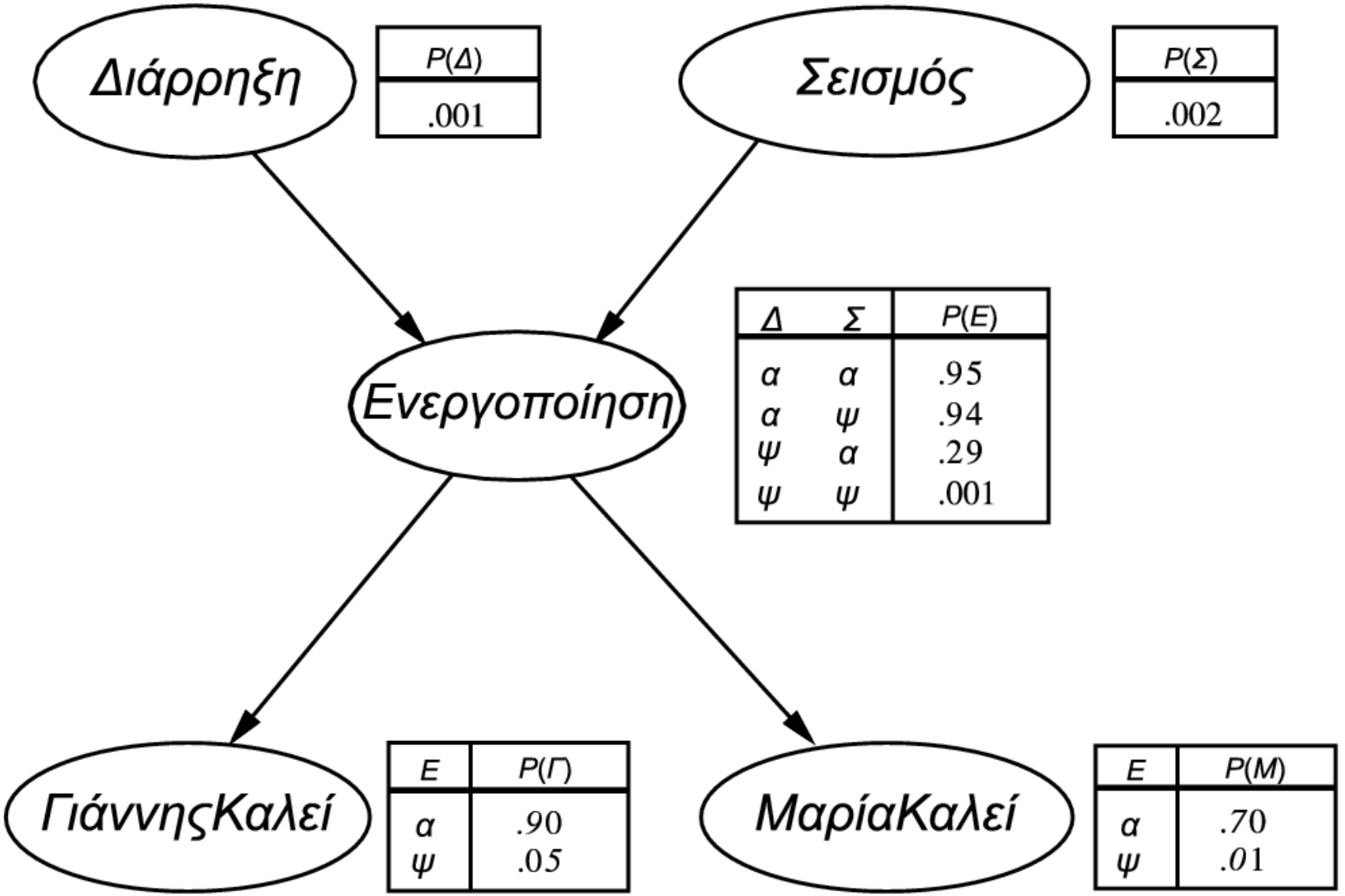
1. Ένα σύνολο τυχαίων μεταβλητών (διακριτών ή συνεχών) σχηματίζει τους κόμβους του γραφήματος.
2. Ένα σύνολο κατευθυνόμενων συνδέσμων ή βελών συνδέει ζευγάρια κόμβων. Αν υπάρχει βέλος από τον κόμβο *X* στον κόμβο *Y*, τότε λέμε ότι ο *X* είναι *γονέας* του *Y.*
3. Ο κάθε κόμβος X_i έχει μια υπό συνθήκη κατανομή πιθανότητας $P(X_i|\mathit{Γονείς}(X_i))$.
4. Το γράφημα δεν περιέχει κύκλους (DAG = Directed Acyclic Graph)

	SPRINKLER	
RAIN	T	F
F	0.4	0.6
T	0.01	0.99



	RAIN	
T	F	
0.2	0.8	

		GRASS WET	
SPRINKLER	RAIN	T	F
F	F	0.0	1.0
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.99	0.01


Διάρρηξη
P(Δ)
.001
Σεισμός
P(Σ)
.002
Ενεργοποίηση
Δ Σ P(E)
α α .95
α ψ .94
ψ α .29
ψ ψ .001
ΓιάννηςΚαλεί
E P(Γ)
α .90
ψ .05
ΜαρίαΚαλεί
E P(M)
α .70
ψ .01

Πιθανότητες

$P(A \wedge A) = P(A \vee A) = P(A)$
$P(A \wedge \sim A) = 0$
$P(A \vee \sim A) = 1$
$P(\sim A) = 1 - P(A)$

$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$
$P(A \wedge B) = P(A|B)*P(B) = P(B|A)*P(A)$ (Bayes)
$P(A \wedge B) = P(A)*P(B)$ (A,B ανεξάρτητα - independent)

Ανεξαρτησία υπό συνθήκη

Τα ενδεχόμενα A και B είναι ανεξάρτητα υπό συνθήκη (conditionally independent) ως προς το C, αν και μόνο αν:

$P(A \mid B,C) = P(A \mid C)$
$P(B \mid A,C) = P(B \mid C)$

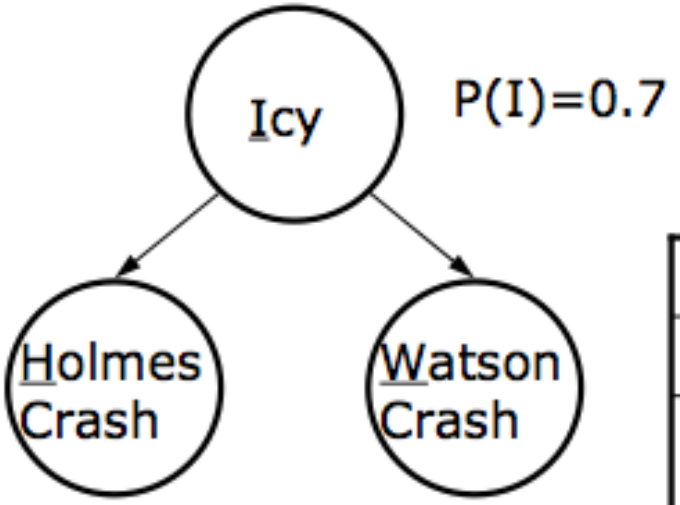
και τότε:

$P(A \wedge B \mid C) = P(A \mid C) * P(B \mid C)$

Συλλογισμός με Δίκτυο Bayes

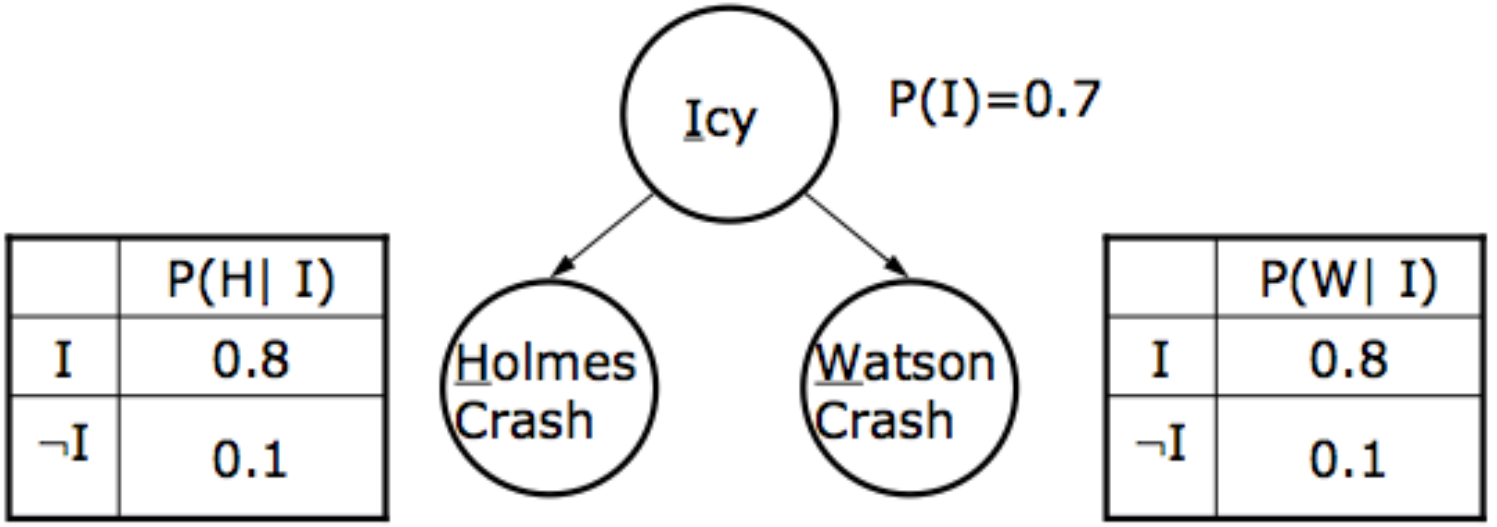
Inspector Smith is waiting for Holmes and Watson, who are driving (separately) to meet him. It is winter. His secretary tells him that Watson has had an accident. He says, "It must be that the roads are icy. I bet that Holmes will have an accident too. I should go to lunch." But, his secretary says, "No, the roads are not icy, look at the window." So, he says, "I guess I better wait for Holmes."

(Icy Roads)



	P(H| I)
I	0.8
¬I	0.1

	P(W| I)
I	0.8
¬I	0.1



$P(W) = P(W|I)*P(I)+P(W|\sim I)*P(\sim I) = 0.8*0.7+0.1*0.3 = 0.59$

επίσης P(H)

$P(I|W) = P(W|I)*P(I)/P(W) = 0.8*0.7/0.59 = 0.95$

μάλλον Icy

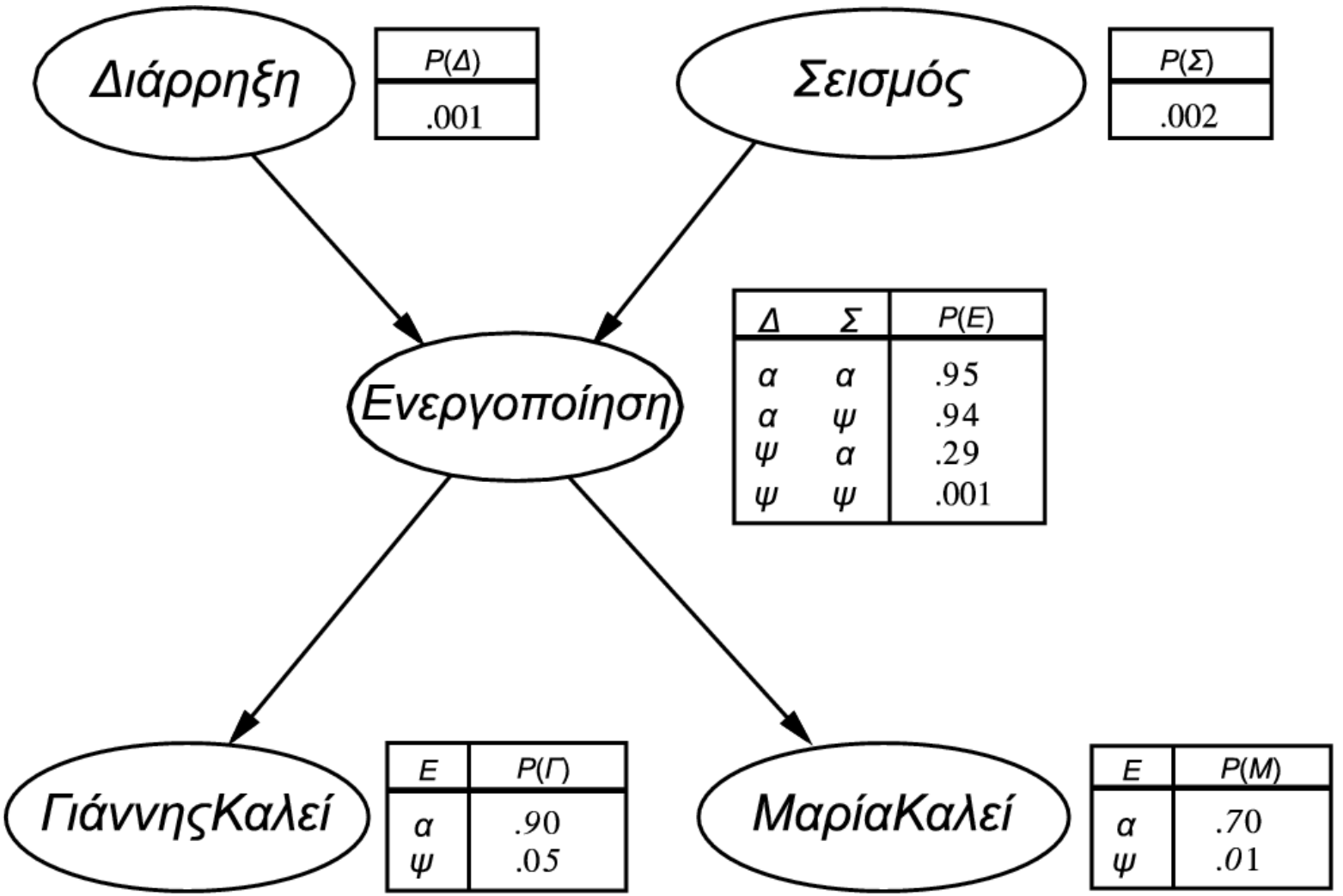
$P(H|W) = P(H|W,I)*P(I|W)+P(H|W,\sim I)*P(\sim I|W) =$

$= P(H|I)*P(I|W)+P(H|\sim I)*P(\sim I|W) =$

$= 0.8*0.95+0.1*0.05 = 0.765$ (>0.59) μάλλον H-Crash

$P(H|W,\sim I) = P(H|\sim I) = 0.1$ το H-Crash είναι λιγότερο πιθανό

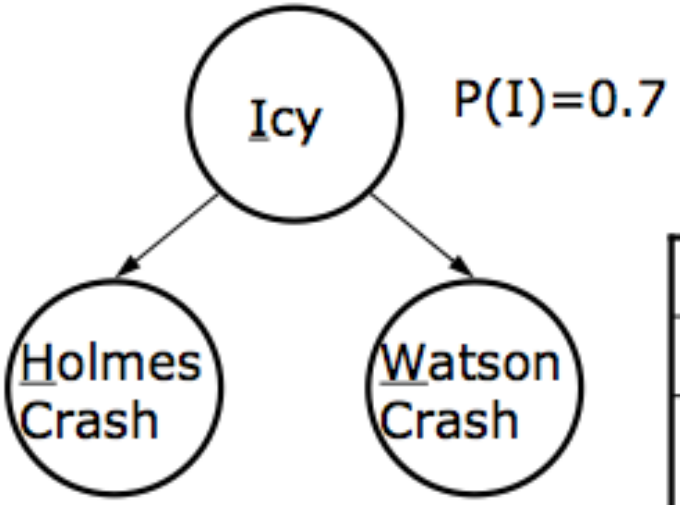
Δίκτυα Bayes



Συλλογισμός με Δίκτυο Bayes

Inspector Smith is waiting for Holmes and Watson, who are driving (separately) to meet him. It is winter. His secretary tells him that Watson has had an accident. He says, "It must be that the roads are icy. I bet that Holmes will have an accident too. I should go to lunch." But, his secretary says, "No, the roads are not icy, look at the window." So, he says, "I guess I better wait for Holmes."

(Icy Roads)

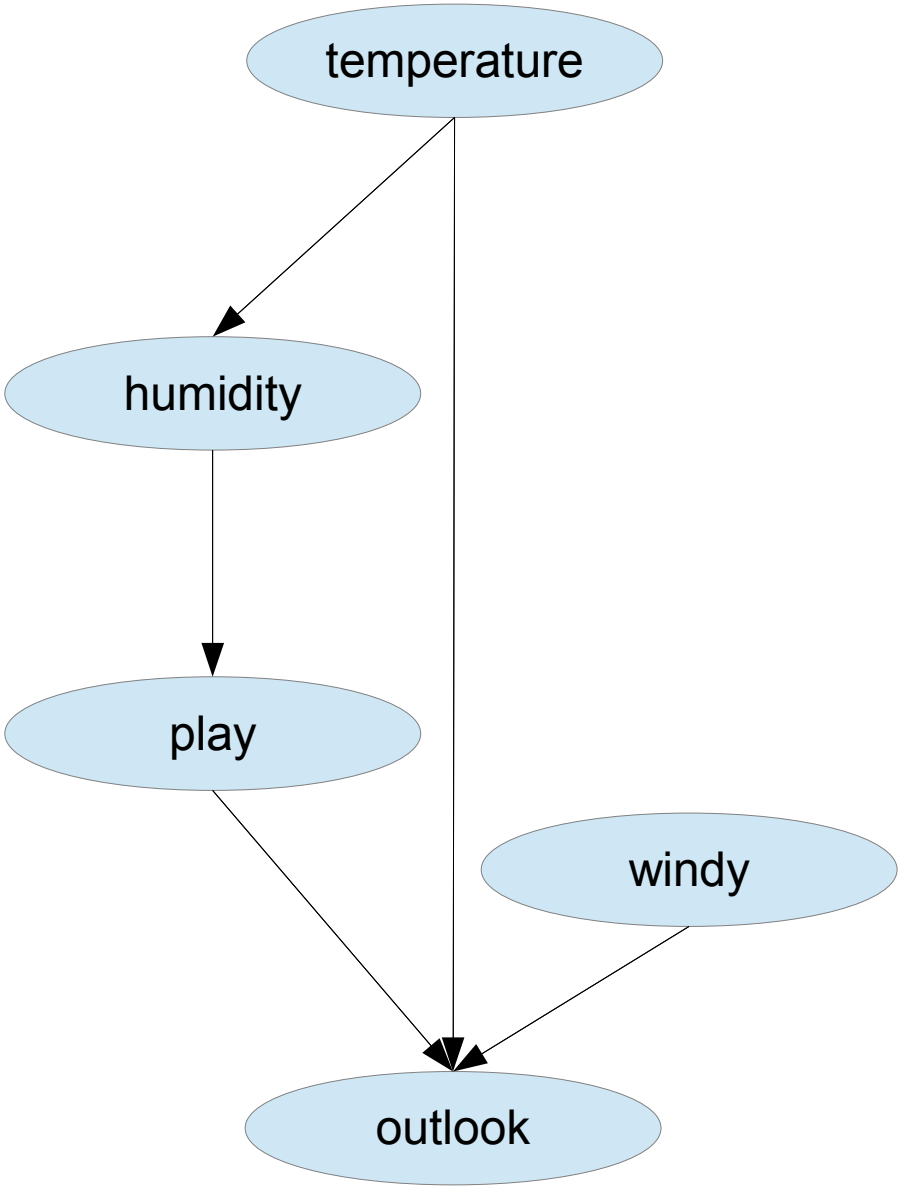


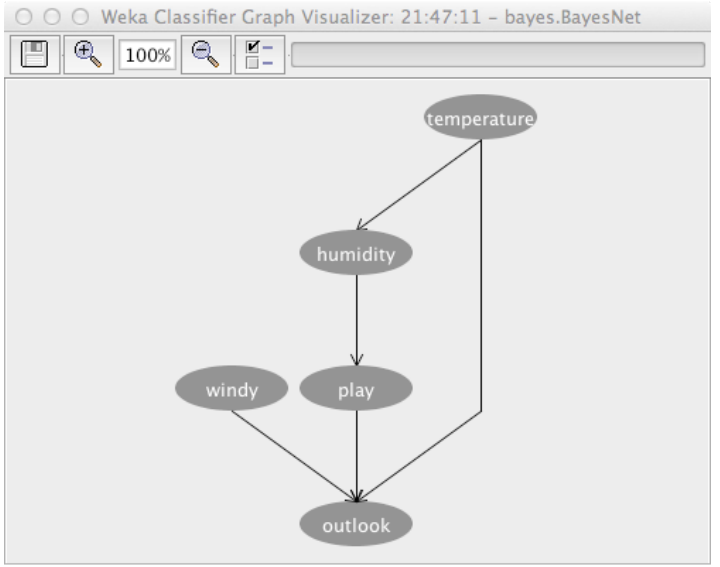
	P(H| I)
I	0.8
¬I	0.1

	P(W| I)
I	0.8
¬I	0.1

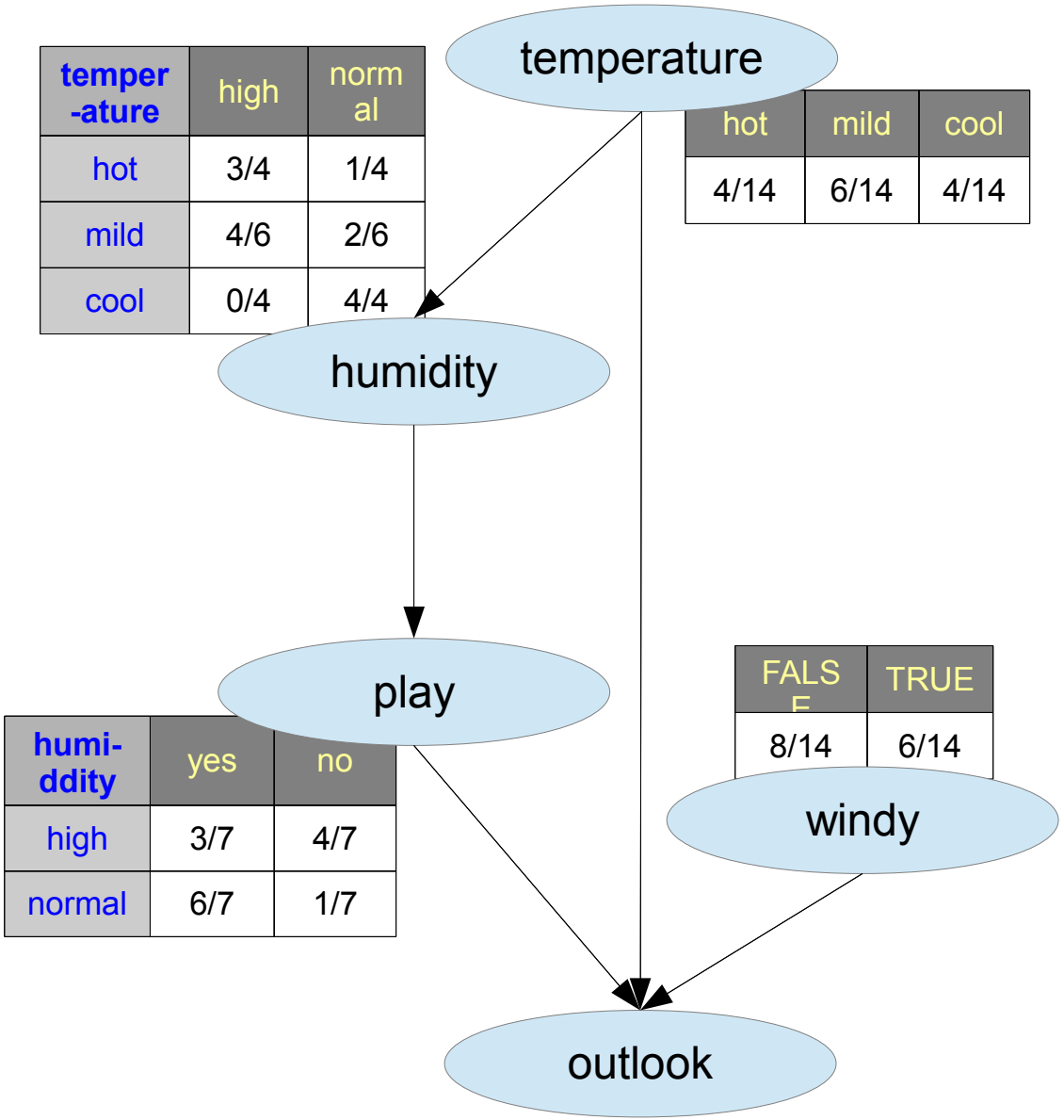
play?

No	outlook	temperature	humidity	windy	play
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
3	overcast	hot	high	FALSE	yes
4	rainy	mild	high	FALSE	yes
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
7	overcast	cool	normal	TRUE	yes
8	sunny	mild	high	FALSE	no
9	sunny	cool	normal	FALSE	yes
10	rainy	mild	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
13	overcast	hot	normal	FALSE	yes
14	rainy	mild	high	TRUE	no


temperature
humidity
play
windy
outlook

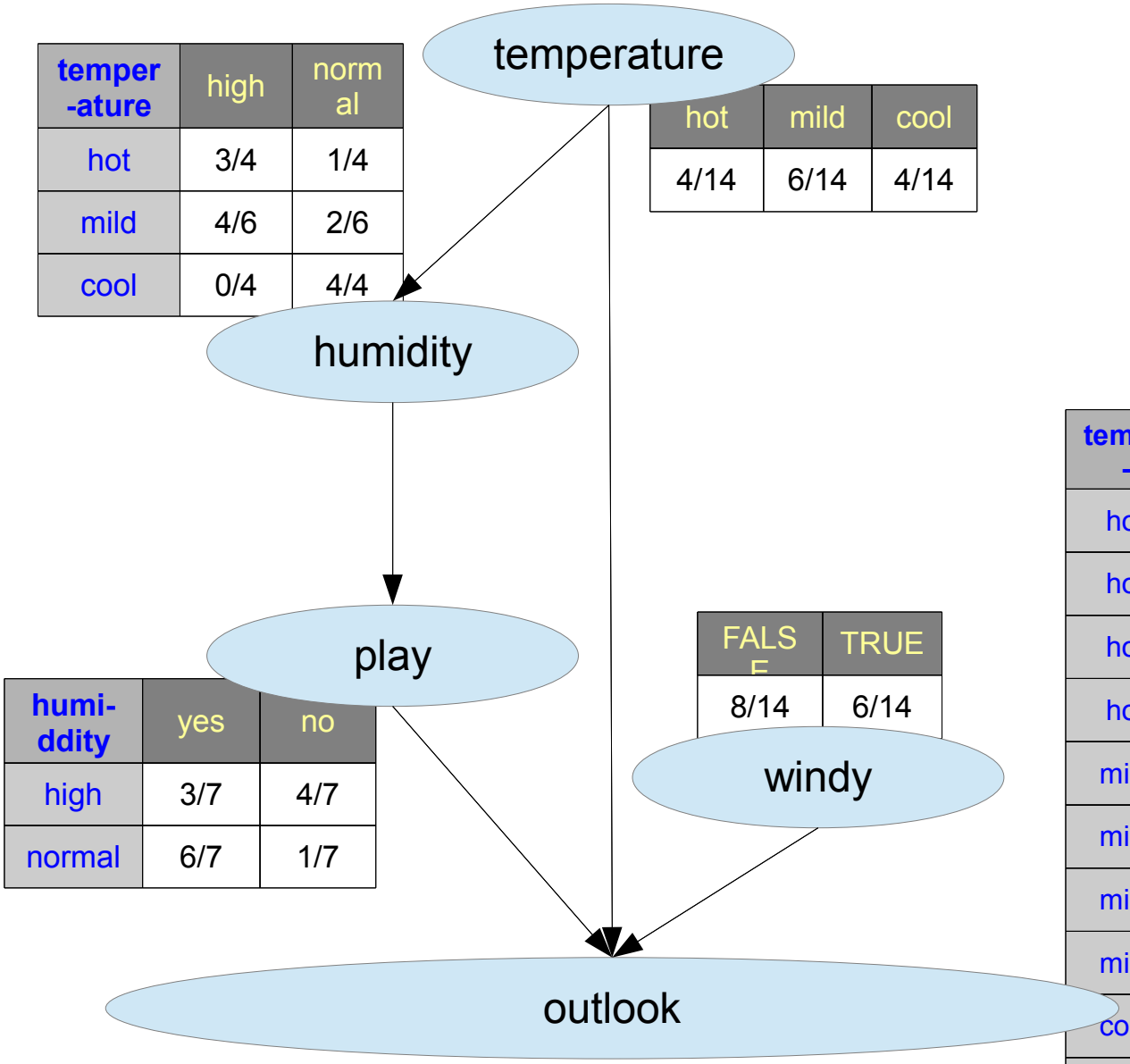

Weka Classifier Graph Visualizer: 21:47:11 - bayes.BayesNet
100%
temperature
humidity
windy
play
outlook

No	outlook	temperature	humidity	windy	play
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
3	overcast	hot	high	FALSE	yes
4	rainy	mild	high	FALSE	yes
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
7	overcast	cool	normal	TRUE	yes
8	sunny	mild	high	FALSE	no
9	sunny	cool	normal	FALSE	yes
10	rainy	mild	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
13	overcast	hot	normal	FALSE	yes
14	rainy	mild	high	TRUE	no



No	outlook	temperature	humidity	windy	play
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
3	overcast	hot	high	FALSE	yes
4	rainy	mild	high	FALSE	yes
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
7	overcast	cool	normal	TRUE	yes
8	sunny	mild	high	FALSE	no
9	sunny	cool	normal	FALSE	yes
10	rainy	mild	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
13	overcast	hot	normal	FALSE	yes
14	rainy	mild	high	TRUE	no

tempe-	windy	play	sunny	overcast	rainy
hot	TRUE	yes	-	-	-
hot	TRUE	no	1/1	0/1	0/1
hot	FALSE	yes	0/2	2/2	0/2
hot	FALSE	no	1/1	0/1	0/1
mild	TRUE	yes	1/2	1/2	0/2
mild	TRUE	no	0/1	0/1	1/1
mild	FALSE	yes	0/2	0/2	2/2
mild	FALSE	no	1/1	0/1	0/1
cool	TRUE	yes	0/1	1/1	0/1
cool	TRUE	no	0/1	0/1	1/1
cool	FALSE	yes	1/2	0/2	1/2
cool	FALSE	no	-	-	-



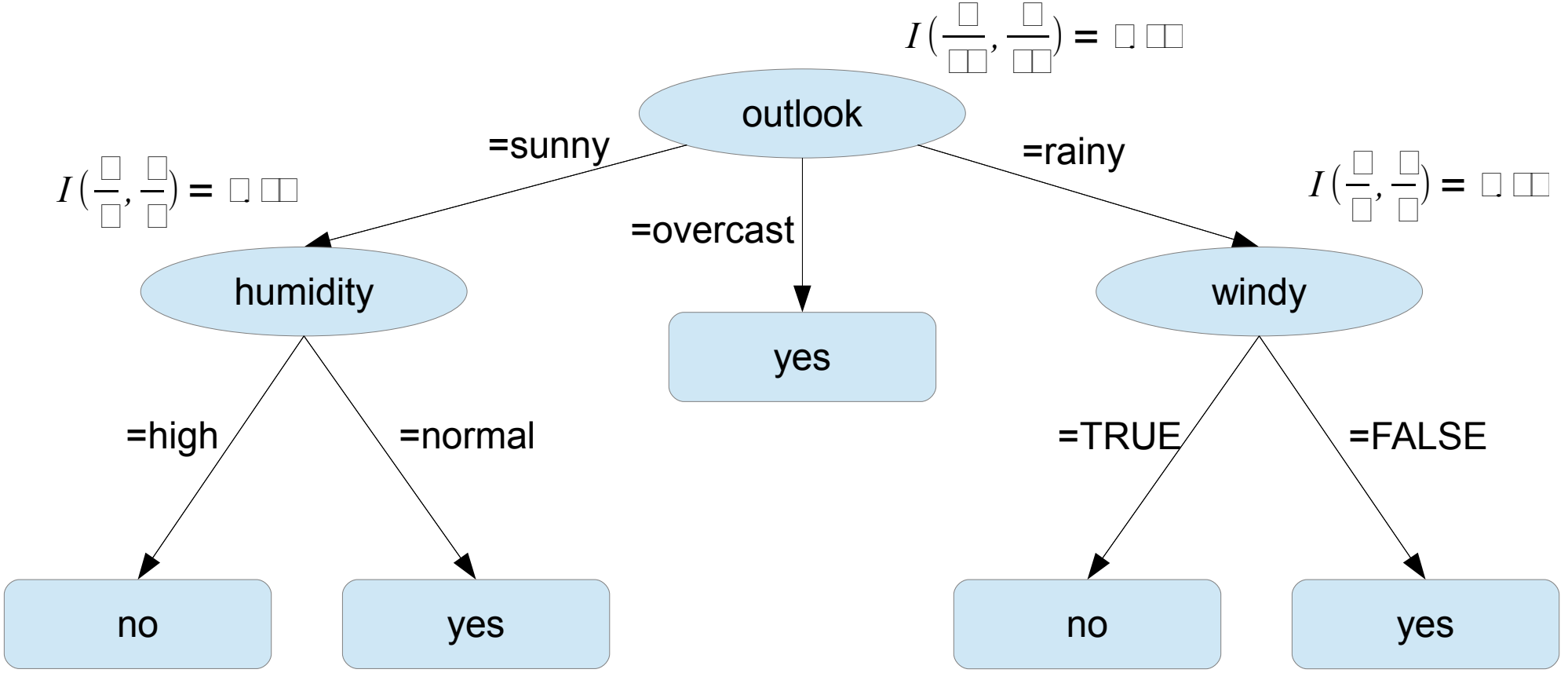
temper-ature	high	normal
hot	3/4	1/4
mild	4/6	2/6
cool	0/4	4/4

hot	mild	cool
4/14	6/14	4/14

humi-ddity	yes	no
high	3/7	4/7
normal	6/7	1/7

FALSE	TRUE
8/14	6/14

tempe-	windy	play	sunny	overcast	rainy
hot	TRUE	yes	-	-	-
hot	TRUE	no	1/1	0/1	0/1
hot	FALSE	yes	0/2	2/2	0/2
hot	FALSE	no	1/1	0/1	0/1
mild	TRUE	yes	1/2	1/2	0/2
mild	TRUE	no	0/1	0/1	1/1
mild	FALSE	yes	0/2	0/2	2/2
mild	FALSE	no	1/1	0/1	0/1
cool	TRUE	yes	0/1	1/1	0/1
cool	TRUE	no	0/1	0/1	1/1
cool	FALSE	yes	1/2	0/2	1/2
cool	FALSE	no	-	-	-



P(play=yes | outlook=overcast) = 1

temper -ature	high	normal
hot	3/4	1/4
mild	4/6	2/6
cool	0/4	4/4

Probability Dist...

temperature	high	normal
hot	0.7	0.3
mild	0.643	0.357
cool	0.1	0.9

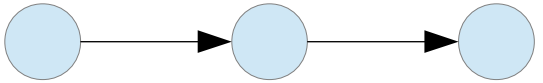
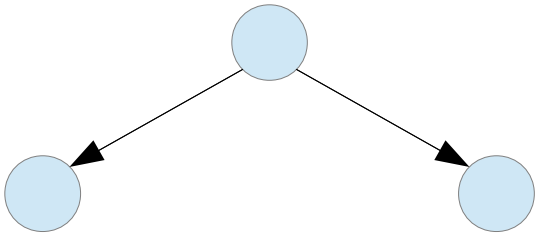
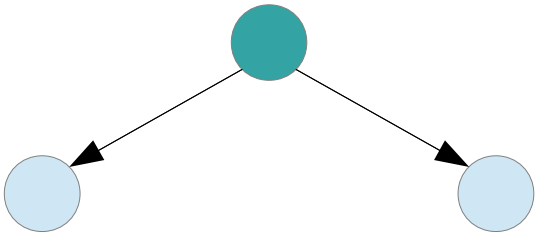
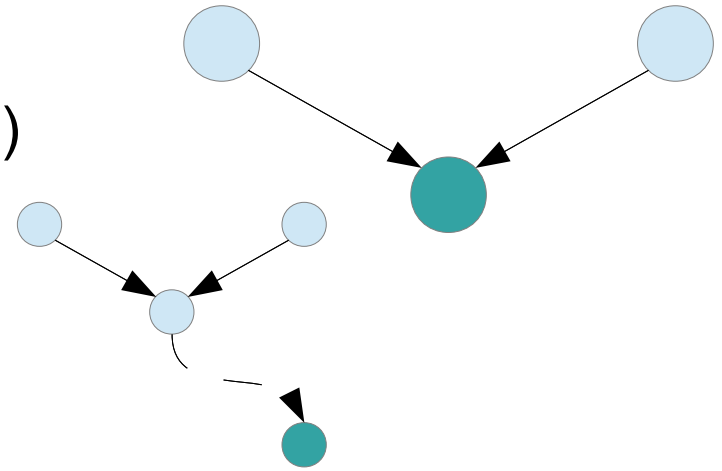
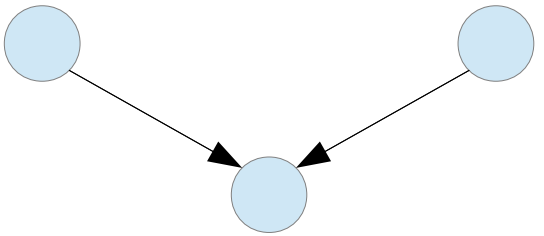
tempe -rature	windy	play	sunny	overcast	rainy
hot	TRUE	yes	-	-	-
hot	TRUE	no	1/1	0/1	0/1
hot	FALSE	yes	0/2	2/2	0/2
hot	FALSE	no	1/1	0/1	0/1
mild	TRUE	yes	1/2	1/2	0/2
mild	TRUE	no	0/1	0/1	1/1
mild	FALSE	yes	0/2	0/2	2/2
mild	FALSE	no	1/1	0/1	0/1
cool	TRUE	yes	0/1	1/1	0/1
cool	TRUE	no	0/1	0/1	1/1
cool	FALSE	yes	1/2	0/2	1/2
cool	FALSE	no	-	-	-

Probability Distribution Table For ou...

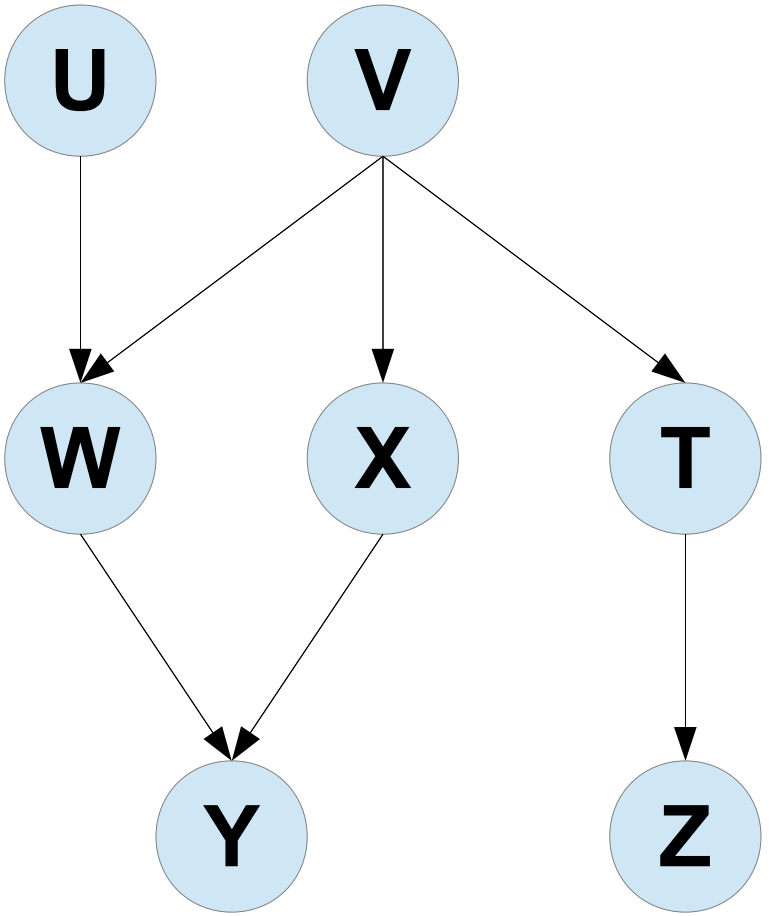
temperature	windy	play	sunny	overcast	rainy
hot	TRUE	yes	0.333	0.333	0.333
hot	TRUE	no	0.6	0.2	0.2
hot	FALSE	yes	0.143	0.714	0.143
hot	FALSE	no	0.6	0.2	0.2
mild	TRUE	yes	0.429	0.429	0.143
mild	TRUE	no	0.2	0.2	0.6
mild	FALSE	yes	0.143	0.143	0.714
mild	FALSE	no	0.6	0.2	0.2
cool	TRUE	yes	0.2	0.6	0.2
cool	TRUE	no	0.2	0.2	0.6
cool	FALSE	yes	0.429	0.143	0.429
cool	FALSE	no	0.333	0.333	0.333

Laplace Smoothing

D-Separation

	Active / Dependent (Ενεργή / Εξαρτημένες)	Inactive / Independent (Ανενεργή / Μη-εξαρτημένες)
causal chain (αλυσίδα αιτιότητας)		
common cause (κοινή αιτία)		
common effect (κοινό αποτέλεσμα)		

Παράδειγμα



V _||_ Z

V _||_ Z | T

U _||_ V

U _||_ V | W

U _||_ V | X

U _||_ V | Y

U _||_ V | Z

W _||_ X

X _||_ T | V

X _||_ W | U

Y _||_ Z

Y _||_ Z | T

Y _||_ Z | X

Y _||_ Z | V

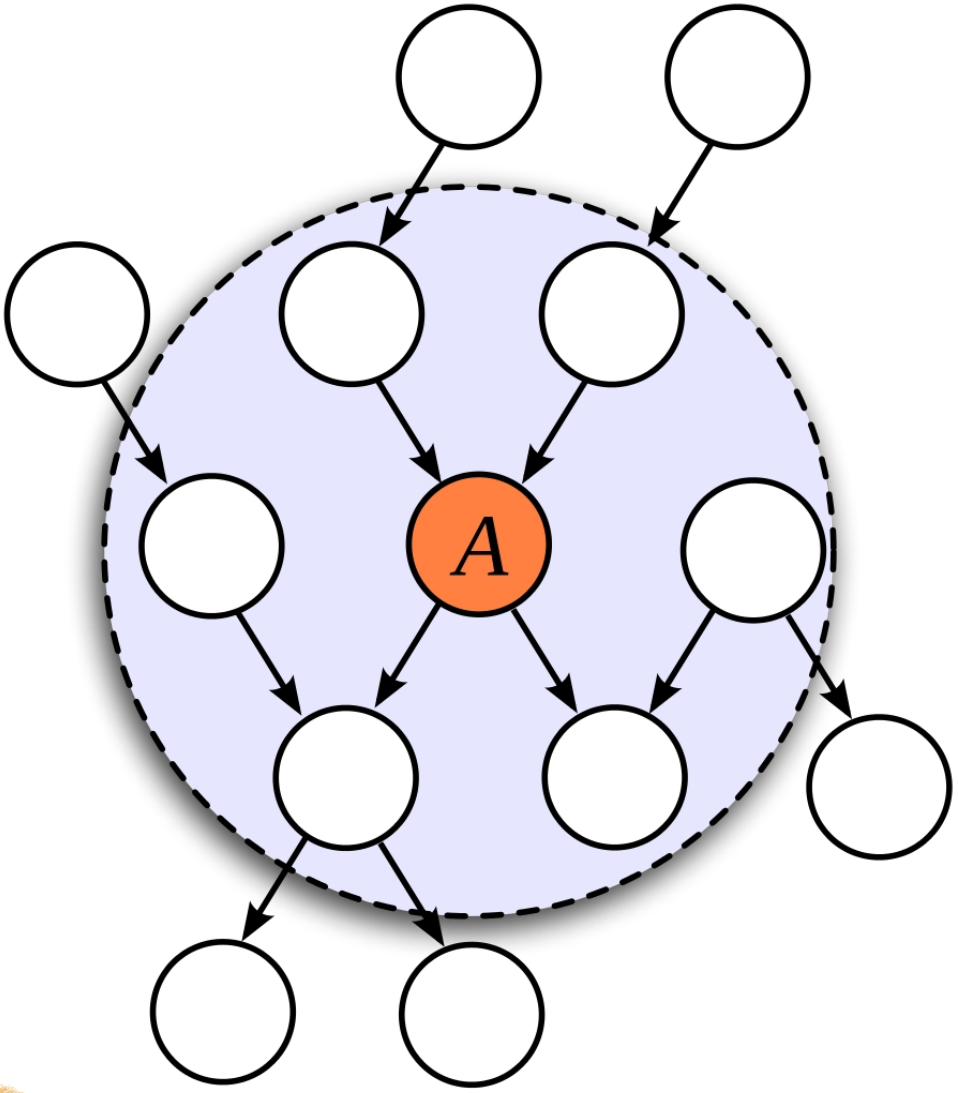
W _||_ Z | V

U _||_ Z

U _||_ Z | Y

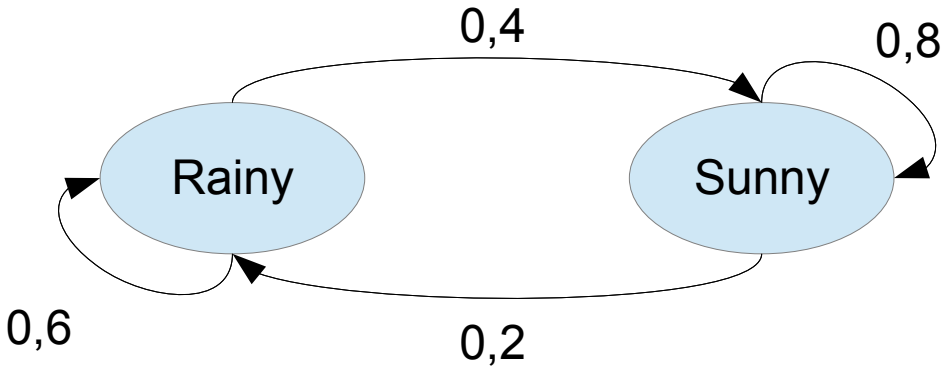
https://www.youtube.com/watch?v=yDs_q6jKHb0

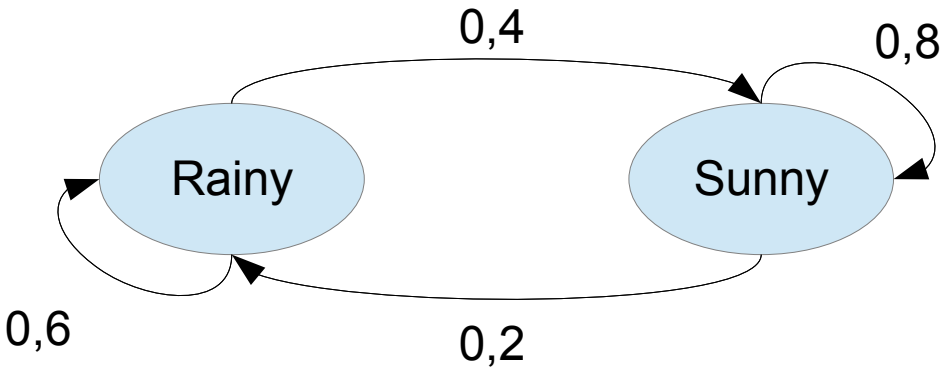
Markov Blanket



- Parents
- Children
- Other Parents of Children

Μοντέλο Markov
(Markov Model)



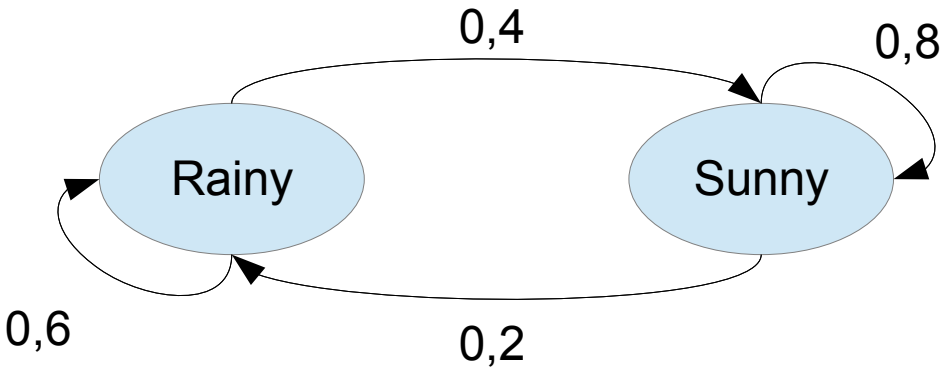


$$P(R_0) = 1$$

$$P(R_1) = ?$$

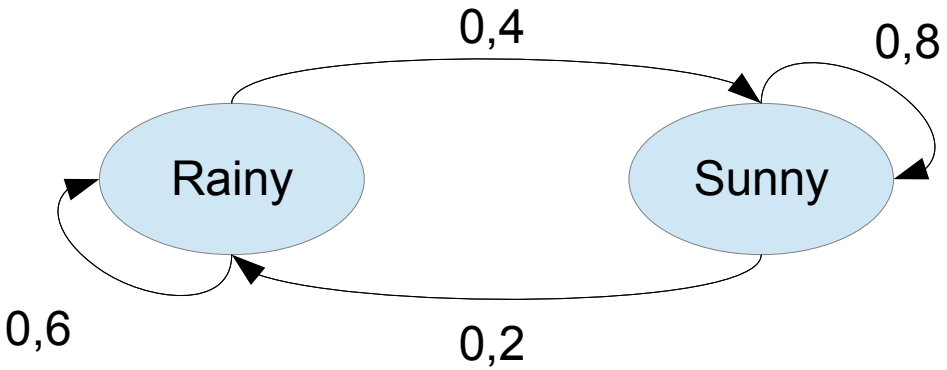
$$P(R_2) = ?$$

$$P(R_3) = ?$$



$$P(R_0) = 1$$

$$P(R_1) = P(R_1|R_0)P(R_0) + P(R_1|S_0)P(S_0) = 0{,}6$$

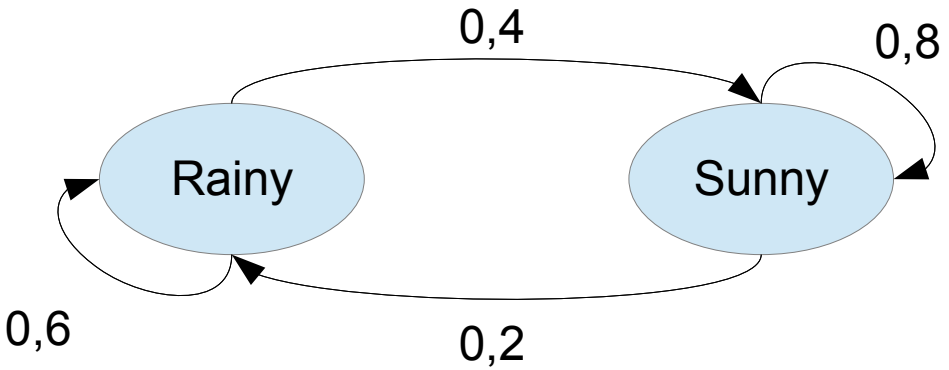


$$P(R_0) = 1$$

$$P(R_1) = P(R_1|R_0)P(R_0) + P(R_1|S_0)P(S_0) = 0{,}6$$

$$P(R_2) = P(R_2|R_1)P(R_1) + P(R_2|S_1)P(S_1) =$$

$$= 0{,}6 * 0{,}6 + 0{,}2 * 0{,}4 = 0{,}44$$



$$P(R_0) = 1$$

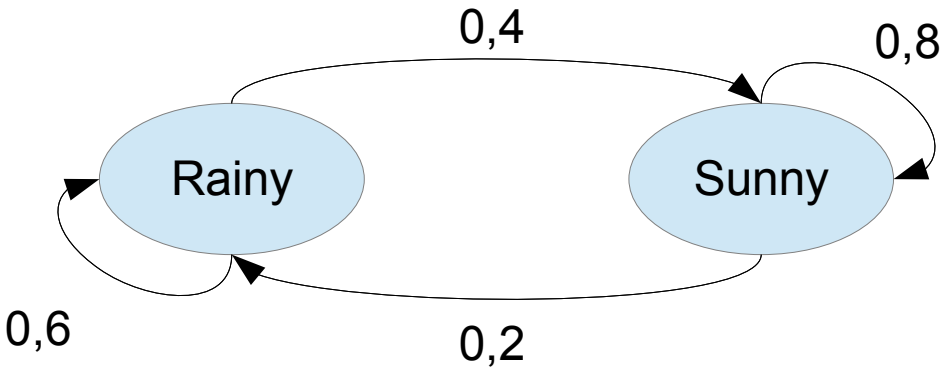
$$P(R_1) = P(R_1|R_0)P(R_0) + P(R_1|S_0)P(S_0) = 0,6$$

$$P(R_2) = P(R_2|R_1)P(R_1) + P(R_2|S_1)P(S_1) =$$

$$= 0,6*0,6 + 0,2*0,4 = 0,44$$

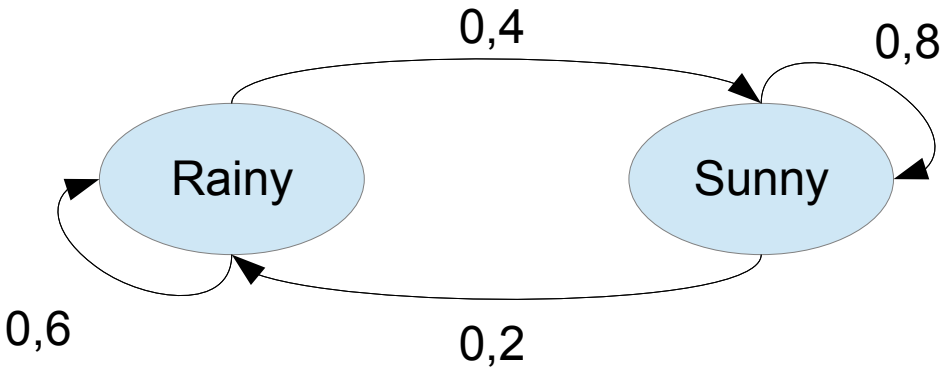
$$P(R_3) = P(R_3|R_2)P(R_2) + P(R_3|S_2)P(S_2) =$$

$$= 0,6*0,44 + 0,2*(1-0,44) = 0,376$$



$$P(R_0) = 1$$

$$P(R_\infty) = ?$$



$$P(R_0) = 1$$

$$P(R_\infty) = ?$$

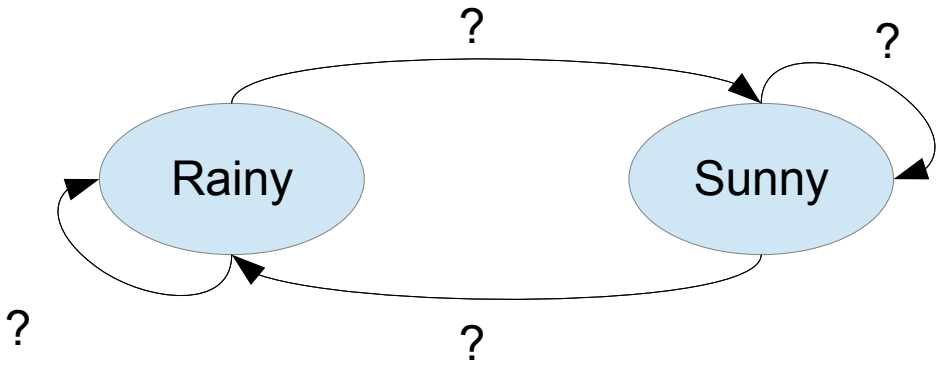
$$P(R_t) = P(R_t \mid R_{t-1})\,P(R_{t-1}) + P(R_t \mid S_{t-1})\,P(S_{t-1}) =$$

$$= 0{,}6\,P(R_{t-1}) + 0{,}2\,P(S_{t-1})$$

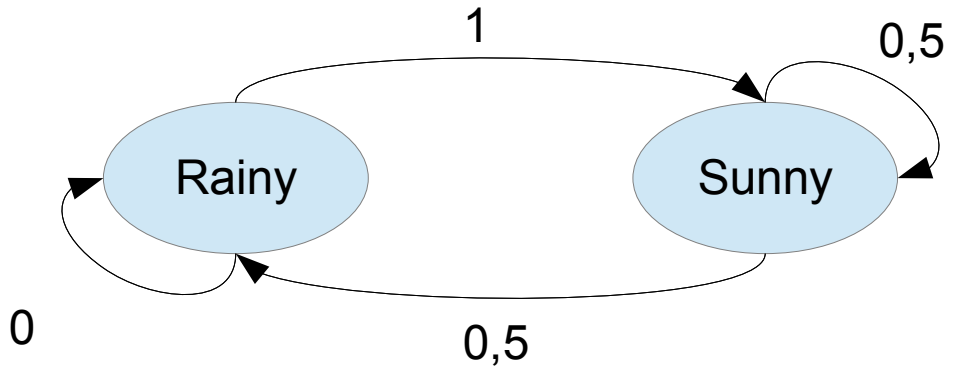
$$P(R_t) = P(R_{t-1}) = X$$

$$X = 0{,}6\,X + 0{,}2(1 - X)$$

$$X = \frac{1}{3} = P(R_\infty)$$



$$R\,S\,S\,S\,R\,S\,R$$



$$R\,S\,S\,S\,R\,S\,R$$

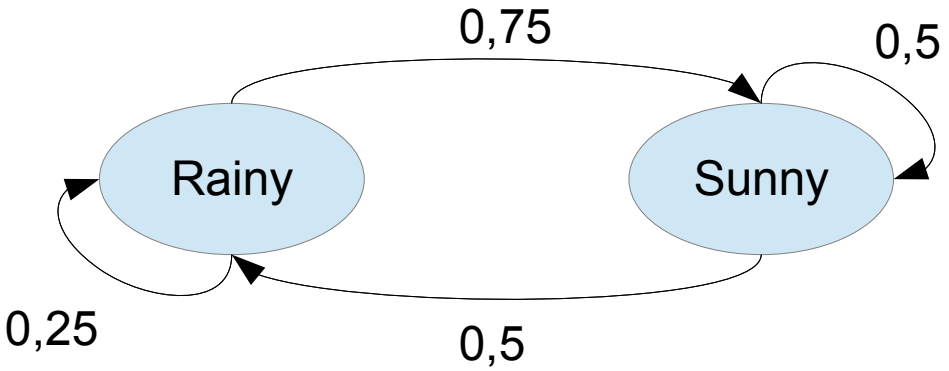
$$P(R_0) = 1$$

$$P(S|R) = 1$$

$$P(R|R) = 0$$

$$P(S|S) = 0{,}5$$

$$P(R|S) = 0{,}5$$



$$R\,S\,S\,S\,R\,S\,R$$

Laplacian Smoothing

$$P(R_0) = \frac{1+1}{1+2} = \frac{2}{3}$$

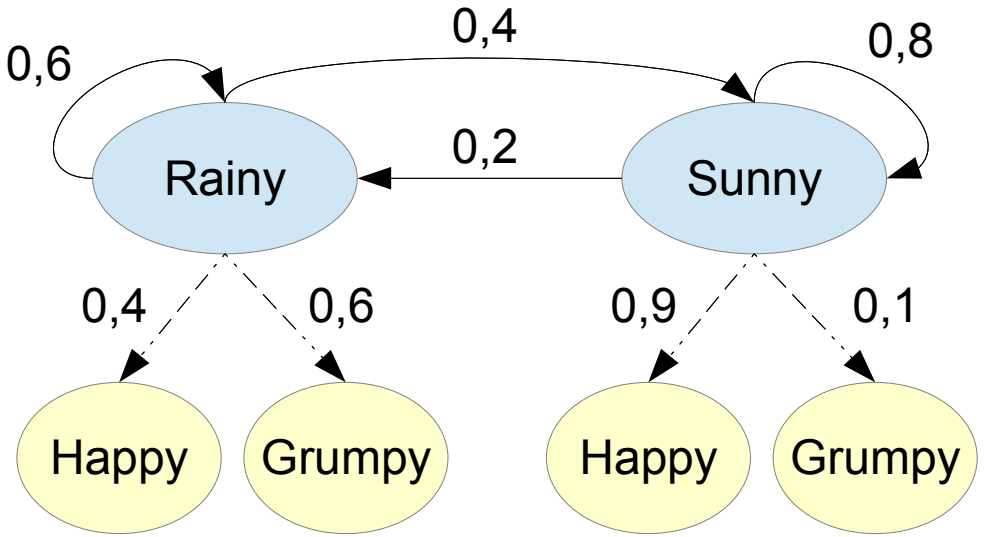
$$P(S|R) = \frac{2+1}{2+2} = \frac{3}{4}$$

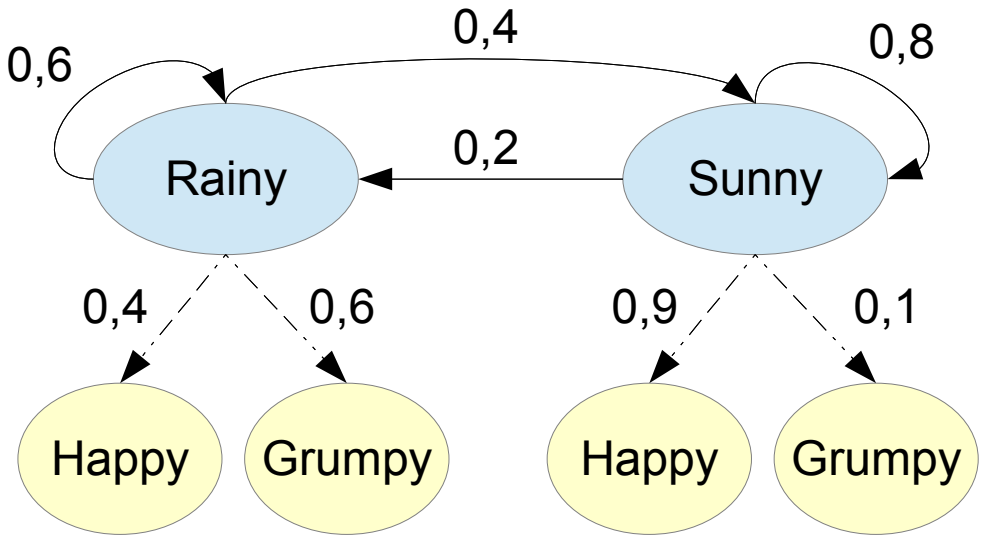
$$P(R|R) = \frac{0+1}{2+2} = \frac{1}{4}$$

$$P(S|S) = \frac{2+1}{4+2} = \frac{1}{2}$$

$$P(R|S) = \frac{2+1}{4+2} = \frac{1}{2}$$

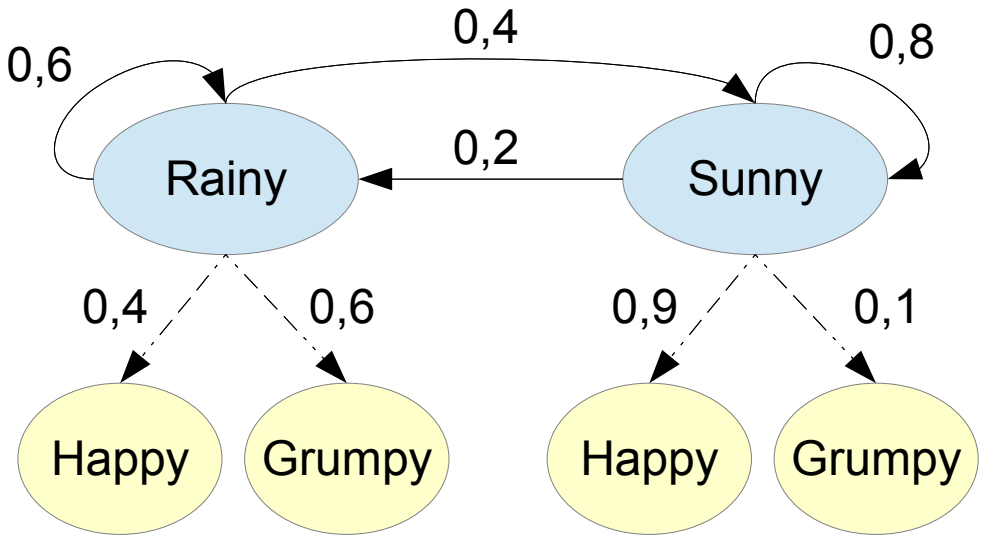
Κρυμμένο Μοντέλο Markov
(Hidden Markov Model = HMM)





$$P(R_0) = \frac{1}{2}$$

$$P(R_1 | H_1) = ?$$



$$P(R_0) = \frac{1}{2}$$

$$P(R_1|H_1) = \frac{P(H_1|R_1)P(R_1)}{P(H_1)} = \frac{0{,}4*0{,}4}{0{,}7} = 0{,}2286$$

$$P(R_1) = P(R_1|R_0)P(R_0) + P(R_1|S_0)P(S_0) = 0{,}4$$

$$P(H_1|R_1) = 0{,}4$$

$$P(H_1) = P(H_1|R_1)P(R_1) + P(H_1|S_1)P(S_1) =$$

$$= 0{,}4*0{,}4 + 0{,}9*(1-0{,}4) = 0{,}7$$

Εξεταστέα Ύλη
Μηχανικής Μάθησης

<u>Από Russell-Norvig:</u>

- 13.2-13.6 Πιθανότητες
- 14.1-14.2 Δίκτυα Bayes
- 15.1-15.3 Markov
- 18.1-18.4 Δένδρα αποφάσεων
- 20 Bayes, KNN, K-Means

<u>Από Βλαχάβα κλπ:</u>

- 13.1-13.2 Πιθανότητες
- 13.4.1 Δίκτυα Bayes
- 18.1.2 Δένδρα αποφάσεων
- 18.1.5 Μάθηση Bayes
- 18.2.2 K-Means

(Όλα τα παραπάνω αλλά με έμφαση σε όσα είπαμε στην αίθουσα)

Τέλος Ενότητας

Χρηματοδότηση

- Το παρόν εκπαιδευτικό υλικό έχει αναπτυχθεί στο πλαίσιο του εκπαιδευτικού έργου του διδάσκοντα
- Το έργο **«Ανοικτά Ακαδημαϊκά Μαθήματα στο Πανεπιστήμιο Αθηνών»** έχει χρηματοδοτήσει μόνο την αναδιαμόρφωση του εκπαιδευτικού υλικού
- Το έργο υλοποιείται στο πλαίσιο του Επιχειρησιακού Προγράμματος «Εκπαίδευση και Δια Βίου Μάθηση» και συγχρηματοδοτείται από την Ευρωπαϊκή Ένωση (Ευρωπαϊκό Κοινωνικό Ταμείο) και από εθνικούς πόρους.

Σημειώματα

Σημείωμα Ιστορικού Εκδόσεων Έργου

Το παρόν έργο αποτελεί την έκδοση 1.0.

Έχουν προηγηθεί οι κάτωθι εκδόσεις:

- Έκδοση 1.0 διαθέσιμη εδώ.

Σημείωμα Αναφοράς

Σημείωμα Αδειοδότησης

Διατήρηση Σημειωμάτων

Οποιαδήποτε αναπαραγωγή ή διασκευή του υλικού θα πρέπει να συμπεριλαμβάνει:

- το Σημείωμα Αναφοράς
- το Σημείωμα Αδειοδότησης
- τη δήλωση Διατήρησης Σημειωμάτων
- το Σημείωμα Χρήσης Έργων Τρίτων (εφόσον υπάρχει)

μαζί με τους συνοδευόμενους υπερσυνδέσμους.

Σημείωμα Χρήσης Έργων Τρίτων

Το Έργο αυτό κάνει χρήση των ακόλουθων έργων:

Εικόνες/Σχήματα/Διαγράμματα/Φωτογραφίες

Διαφάνεια 11: Εικόνα από Weka Explorer