ΣΥΣΤΗΜΑΤΑ ΑΝΟΙΚΤΟΥ ΒΡΟΧΟΥ

Σχέση $E(z)$ κ $E^*(s)$

$$E^*(s) \triangleq \sum_{n=0}^{\infty} e(nT)\, e^{-nTs}$$ → άρρητη συνάρτηση (όχι Σ, Μ)

→ αλλαγή μεταβλητών $z \triangleq e^{sT}$ $\left(s = \frac{1}{T}\ln z\right)$

$$\Rightarrow \quad E^*(s)\Big|_{s=\frac{1}{T}\ln z} \equiv \boxed{E(z) = \sum_{n=0}^{\infty} e(nT)\, z^{-n}}$$ → ρητή συνάρτηση

ΜΣ: $x(t) \rightarrow X(z) = \mathcal{Z}\{x(t)\}$, $x(t) \rightarrow x^*(t) \rightarrow X^*(s) \rightarrow X(z)$
$\rightarrow X(s) \rightarrow X^*(s)$

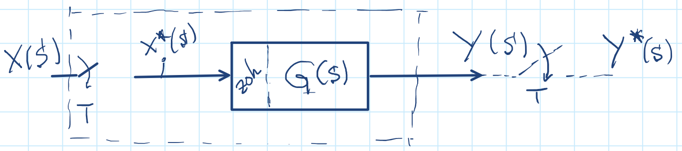
$$\boxed{X(z) \;:\; \text{ΜΟΝΟ} \;:\; t = kT}$$

Παράδειγμα: $E(s) = \dfrac{1}{(s+1)(s+2)}$ → $E^*(s) = \dfrac{1}{1-e^{-Ts}e^{-T}} - \dfrac{1}{1-e^{-Ts}e^{-2T}} = \dfrac{e^{-Ts}(e^{-T}-e^{-2T})}{(1-e^{-Ts}e^{-T})(1-e^{-Ts}e^{-2T})}$

$$E(z) \triangleq E^*(s)\Big|_{z=e^{sT}} = \frac{z^{-1}(e^{-T}-e^{-2T})}{(1-e^{-T}z^{-1})(1-e^{-2T}z^{-1})}$$

- □ $E^*(s)$ → άπειρους πόλους + μηδενικά s-επίπεδο
- □ $E(z)$ → 2 πόλους $z=e^{-T}$, $z=e^{-2T}$, 1 μηδενικό $z=0$

"ΠΑΛΜΙΚΗ" ΣΥΝΑΡΤΗΣΗ ΜΕΤΑΦΟΡΑΣ



(Εξισώσεις)

$Y(s) = G(s)\, X^*(s)$ (συνεχής έξοδος, ψηφιακή είσοδος)

ψηφιακή -"- -"-

$$\Rightarrow Y^*(s) = \left[G(s)\, X^*(s)\right]^*$$

$$= \frac{1}{T}\sum_{k=0}^{\infty}\left[G(s+jk\omega_s)\, \underbrace{X^*(s+jk\omega_s)}_{\equiv\, X^*(s)}\right]$$

$$\Rightarrow Y^*(s) = \frac{1}{T}\sum_{k=0}^{\infty}\left[G(s+jk\omega_s)\, X^*(s)\right] = \frac{1}{T}\sum_{k=0}^{\infty}\left[G(s+jk\omega_s)\right]\cdot X^*(s)$$

$$Y^*(s) = G^*(s)\cdot X^*(s)\Big|_{z=e^{sT}}$$

$$\Rightarrow Y(z) = G(z)\, X(z)$$

$$\Rightarrow \boxed{\frac{Y(z)}{X(z)} = G(z)}$$

$$\Rightarrow \boxed{\frac{Y(z)}{X(z)} = G(z)}$$

π.χ. $G(s) = \dfrac{1}{s+1}$ $\sim$

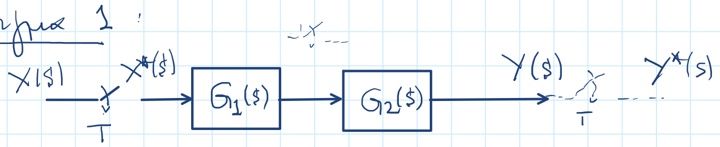
$$Y(z) = G(z)\,X(z) = ?$$

$$Y(s) = \underbrace{\left[\frac{1-e^{-sT}}{s}\cdot\frac{1}{s+1}\right]}_{G(s)}\cdot X^*(s) \quad , \quad G(z) = \mathcal{Z}\left[\frac{1-e^{-sT}}{s}\,\frac{1}{s+1}\right] = \frac{1-e^{-T}}{z-e^{-T}}$$

Βηματική Απόκριση : $X(z) = \mathcal{Z}\left[u(t)\right] = \dfrac{z}{z-1}$

$$Y(z) = \frac{z(1-e^{-T})}{(z-1)(z-e^{-T})} = \frac{z}{z-1} - \frac{z}{z-e^{-T}} \overset{\mathcal{Z}^{-1}}{\rightsquigarrow} \boxed{y(nT) = 1 - e^{-nT}}$$

Παράδειγμα 1 :



$Y^*(s)$? ολική παλμική Σ.Μ.

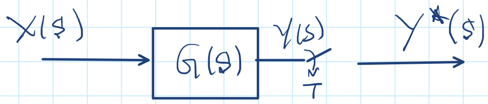
(Κανόνας) : $Y(s) = G_1(s)\,G_2(s)\,X^*(s) \Rightarrow$

$$Y^*(s) = \left[G_1(s)\,G_2(s)\,X^*(s)\right]^* = \left[G_1(s)\,G_2(s)\right]^* X^*(s)\Big|_{z=e^{sT}}$$

$$\Rightarrow Y(z) = \mathcal{Z}\left[G_1(s)G_2(s)\right]\cdot X(z)$$

$$\Rightarrow \boxed{\frac{Y(z)}{X(z)} = \mathcal{Z}\left[G_1(s)\,G_2(s)\right]} \neq G_1(z)\,G_2(z)$$

Παράδειγμα 2:

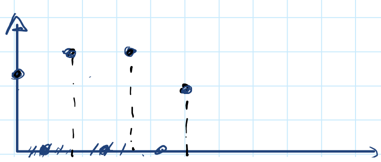


$$Y(s) = G(s)X(s) \Rightarrow Y^*(s) = \left[G(s)\,X(s)\right]^*\Big|_{z=e^{sT}}$$

$$\Rightarrow \boxed{Y(z) = \mathcal{Z}\left[G(s)\,X(s)\right]}$$

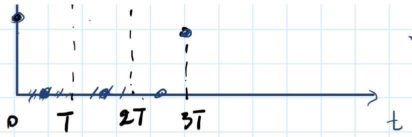
Σ.Μ. δεν ορίζεται

ΑΠΟΚΡΙΣΗ ΜΕΤΑΞΥ ΣΤΙΓΜΩΝ ΔΕΙΓΜΑΤΟΛΗΨΙΑΣ



$y(nT) = \dots\dots$, $y(nT/2) = ?$

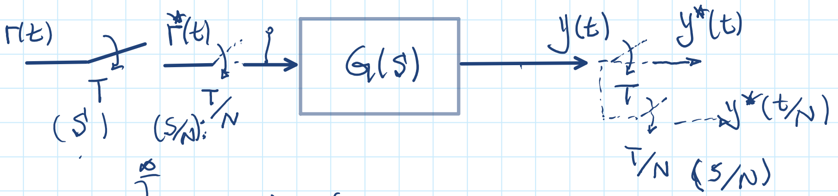
$y(nT/3) = ?$



$y(nT) = \dots\dots$, $y(nT/2) =$

$y(nT/3) = ?$

ΥΠΟΘΕΤΙΚΟΣ ΔΕΙΓΜΑΤΟΛΗΠΤΗΣ



$\rightarrow y(t) = \sum_{m=0}^{\infty} r(mT)\, g(t - mT)$

$\rightarrow y(kT/N) = \sum_{m=0}^{\infty} r(mT)\, g(kT/N - mT) \Rightarrow y^*(kT/N) = \sum_{k=0}^{\infty} y(kT/N)\,\delta(t - kT/N)$

Μ.Ζ. $\Rightarrow$ $Y(z)_N = \mathcal{Z}\{y^*(kT/N)\} = \sum_{k=0}^{\infty} y(kT/N)\, z^{-k/N} = \sum_{v=0}^{\infty} g(vT/N)\, z^{-v/N} \sum_{m=0}^{\infty} r(mT) z^{-m}$

$$\boxed{Y(z)_N \overset{\text{ορ.}}{=} Y(z)\Big|_{\substack{z = z^{1/N} \\ T = \frac{T}{N}}}} \quad \rightarrow \quad G(z)_N \triangleq \sum_{v=0}^{\infty} g(vT/N)\, z^{-v/N}\Big|_{\substack{z = z^{1/N} \\ T = \frac{T}{N}}} \qquad R(z)$$

$$\Rightarrow \boxed{Y(z)_N = G(z)_N\, R(z)}$$

Παρατήρηση: $(1 - z^{-1})\, \mathcal{Z}\left\{\frac{G(s)}{s}\right\}$

δεν επηρεάζεται

Παράδειγμα: $G(s) = \frac{1}{s+1}$ απόκριση $T/3$, $t = kT/3$: $0, 1/3, 2/3, T, \dots\dots$

$\boxed{N = 3}$

$Y(z)_3 = G(z)_3\, R(z)$ (Βηματική απόκριση). $R(z) = \frac{z}{z-1}$

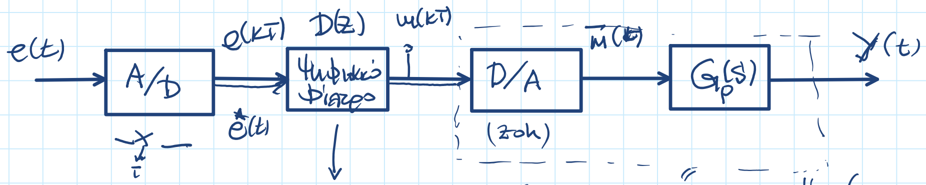
$G(z)_3 = G(z)\Big|_{\substack{z = z^{1/3} \\ T = T/3}} = (1 - z^{-1})\, \mathcal{Z}\left\{\frac{1}{s(s+1)}\right\} = \frac{z}{z - e^{-T}}\Big|_{\substack{z = z^{1/3} \\ T = \frac{T}{3}}}$

$\Rightarrow G(z)_3 = \frac{z^{1/3}}{z^{1/3} - e^{-T/3}} = \frac{z^{1/3}}{z^{1/3} - 0{,}717}\Big|_{T = 1sec}$ $\rightarrow$ αντικατάσταση: $z_3 \triangleq z^{1/3}$

$G(z)_3 = \frac{z_3}{z_3 - 0{,}717} \cdot \frac{z_3^3}{z_3^3 - 1} = \frac{z_3^4}{(z_3 - 0{,}717)(z_3^3 - 1)}$

$y(t)_3 = y(kT/3) : \underline{y(0)},\ y(T/3),\ y(2T/3),\ \underline{y(T)},\ \dots$

ΕΦΑΡΜΟΓΗ: ΣΥΣΤ. ΑΝΟΙΚΤΟΥ ΒΡΟΧΟΥ ΜΕ ΨΗΦΙΑΚΑ ΦΙΛΤΡΑ



$\rightarrow \quad M(z) = D(z)E(z)$: "επιλύει" (υλοποιεί) των εξ. διαφορών

$$\bar{M}(s) = \left[\frac{1-e^{-sT}}{s}\right] M^*(s)$$

$$Y(s) = G_p(s)\bar{M}(s) = G_p(s)\left[\frac{1-e^{-sT}}{s}\right] M^*(s)$$

$$= G_p(s)\left[\frac{1-e^{-sT}}{s}\right] D(z)\Big|_{z=e^{sT}} E^*(s)$$

ΜΖ $\Rightarrow$
$$Y(z) = Z\left\{G_p(s)\left(\frac{1-e^{-sT}}{s}\right)\right\} D(z)E(z)$$

$$\boxed{Y(z) = G(z)D(z)E(z)}$$

Π.χ. (βηματική απόκριση)

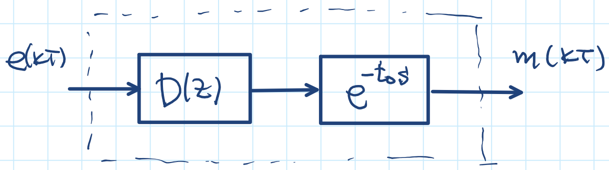
εξ. διαφορών του ψηφ. φίλτρου : $m(kT) = 2e(kT) - e[(k-1)T]$

$$D(z) = \frac{M(z)}{E(z)} = 2 - z^{-1} = \frac{2z-1}{z}$$

αν $G_p(s) = \dfrac{1}{s+1}$, $E(z) = \dfrac{z}{z-1}$ $\Rightarrow$

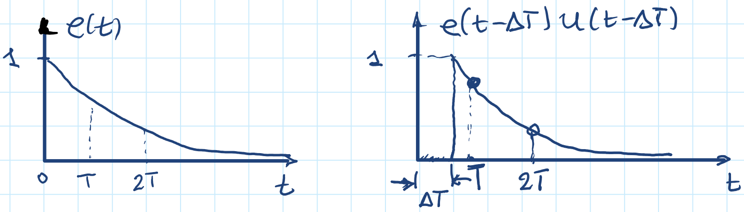
$$Y(z) = \left[\frac{2z-1}{z}\right] Z\left\{\frac{1-e^{-sT}}{s}\,\frac{1}{s+1}\right\}\left[\frac{z}{z-1}\right] = \dots$$

$$y(kT) = 1 + e^{-T(k-1)} - 2e^{-kT}, \qquad k \geq 1$$



t_0 = χρόνος υπολογισμού

ΤΡΟΠΟΠΟΙΗΜΕΝΟΣ Μ.Ζ.



$0 < \Delta \leq 1$

$$Z\left\{e(t-\Delta T)u(t-\Delta T)\right\} = Z\left\{E(s)e^{-\Delta T s}\right\} = \sum_{n=0}^{\infty} e(nT-\Delta T)z^{-n}$$

$$\mathcal{Z}\{e(t-\Delta T)u(t-\Delta T)\} = \mathcal{Z}\{E(s)e^{-\Delta Ts}\} = \sum_{n=0} e(nT-\Delta T)z^{-n}$$

$$E(z,\Delta) \triangleq \mathcal{Z}\{e(t-\Delta T)u(t-\Delta T)\} = \mathcal{Z}\{E(s)e^{-\Delta Ts}\}$$ "καθυστερημένος Μ.Ζ."

π.χ. $\Delta = 0,4$, $e(t) = e^{-\alpha t}u(t)$

$$E(z,\Delta) = e^{-0,6\alpha T}z^{-1} + e^{-1,6\alpha T}z^{-1} + e^{-2,6\alpha T}z^{-2} + \dots =$$

$$= \frac{e^{-0,6\alpha T}z^{-1}}{1-e^{-\alpha T}z^{-1}} = \frac{e^{-0,6\alpha T}}{z-e^{-\alpha T}}$$

- Τροποποιημένος ΜΖ : $\Delta T \triangleq T - mT \rightarrow \Delta = 1-m$

$$E(z,m) = E(z,\Delta)\big|_{\Delta=1-m} = \mathcal{Z}\{E(s)e^{-\Delta Ts}\}\big|_{\Delta=1-m} =$$

$$\Big(E(z,m) = e(mT)z^{-1} + e[(1+m)]Tz^{-2} + e[(2+m)]T z^{-3} + \dots$$

$$\Rightarrow E(z,m) = \mathcal{Z}\{E(s)e^{-(1-m)Ts}\} = z^{-1}\mathcal{Z}\{E(s)e^{mTs}\}\Big]$$

$$\boxed{E(z,m) = z^{-1}\sum \text{Res}\, E(\lambda)e^{mT\lambda}\frac{1}{1-z^{-1}e^{T\lambda}}\bigg|_{\text{πόλοι } E(\lambda)}}$$

π.χ. $e(t) = t \Rightarrow E(s) = \frac{1}{s^2}$

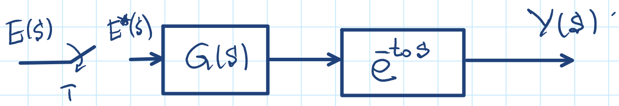
$$E(z,m) = z^{-1}\left[\frac{d}{d\lambda}\frac{e^{mT\lambda}}{1-z^{-1}e^{T\lambda}}\bigg|_{\lambda=0}\right] = \dots = \frac{mTz^{-1}}{1-z^{-1}} + \frac{Tz^{-2}}{(1-z^{-1})^2} = .$$

<u>Ιδιότητα Μ.Ζ. (Τροποποιημένου)</u>

$$\mathcal{Z}_m[E(s)] \equiv E(z,m) = \mathcal{Z}\{e^{-\Delta Ts}E(s)\big|_{\Delta=1-m}\}$$

μετατόπιση χρόνου $\Rightarrow$ $$\mathcal{Z}_m[e^{-kTs}E(s)] = z^{-k}\mathcal{Z}_m[E(s)] = z^{-k}E(z,m)$$

ΣΥΣΤΗΜΑΤΑ ΜΕ ΧΡΟΝΙΚΗ ΚΑΘΥΣΤΕΡΗΣΗ



$$Y(z) = \mathcal{Z}\{G(s)e^{-t_0 s}\}E(z)$$

$t_0 = kT + \Delta T$, $0 < \Delta < 1$

$$\Rightarrow \boxed{Y(z) = z^{-k}\mathcal{Z}\{G(s)e^{-\Delta Ts}\}E(z) = z^{-k}G(z,m)E(z)}$$

$$\Rightarrow \boxed{Y(z) = z^{-k}\, \mathcal{Z}\left\{ G(s)\, e^{-\Delta T s} \right\} E(z) = z^{-k}\, G(z,m)\, E(z)}$$

<u>Εφαρμογή</u>: (όταν χρονική καθυστέρηση ≡ χρόνος υπολογισμού)

"Ψηφ. Φίλτρο" : $m(k) = \alpha_0 e(k) + \alpha_1 e(k-1) + \dots + \alpha_n e(k-n) - \beta_1 m(k-1) - \dots - \beta_n m(k-n)$

↓

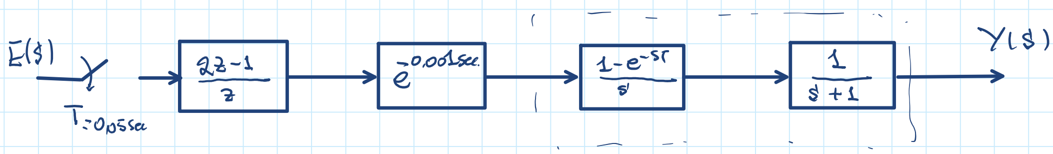
επαύω <u>κάθε</u> T sec

είσοδος $t=0$ ⇝ έξοδος ⇝ $t = T + t_0$

$$Y(z) = \mathcal{Z}\left\{ G(s) e^{-t_0 s} \right\} D(z) E(z)$$

$$= z^{-k} G(z,m) D(z) E(z)$$

Π.χ. $D(z) = \dfrac{2z-1}{z}$ $\quad t_0 = 1\,\text{msec}$, $\quad T = 0{,}05\,\text{sec}$ $\Rightarrow t_0 = 10^{-3}\,\text{sec}$



$$mT + \Delta T = T \Rightarrow mT = T - \Delta T = 0{,}05\,\text{sec} - 0.001 = 0.049$$

$$G(z,m) = \mathcal{Z}_m \left\{ \frac{1-e^{-sT}}{s(s+1)} \right\}\Bigg|_{mT = 0.049\,\text{sec}} = \frac{z-1}{z}\, \mathcal{Z}_m \left[\frac{1}{s(s+1)} \right]\Bigg|_{mT=0.049}$$

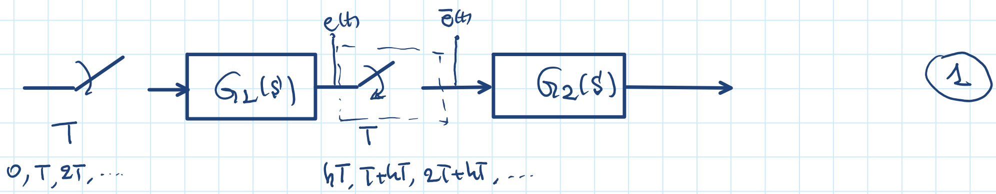
$$= \frac{z-1}{z} \left[\frac{z(1-e^{-0.049}) + (e^{-0.049} - e^{-0.05})}{(z-1)(z-e^{-0.05})} \right]$$

$R(z) = \dfrac{z}{z-1} \quad \Rightarrow$

$$Y(z) = G(z,m)\, D(z)\, E(z)$$

$$= \frac{z-1}{z} \left[\qquad \right] \left[\frac{2z-1}{z} \right] \left[\frac{z}{z-1} \right] = \dots$$

ΜΗ-ΣΥΓΧΡΟΝΙΣΜΕΝΟΙ ΔΕΙΓΜΑΤΟΛΗΠΤΕΣ



$$\bar{e}(t) = e(hT)\left[u(t-hT) - u(t-T-hT)\right] + e(T+hT)\left[u(t-T-hT) - u(t-2T-hT)\right] + \dots$$

$$+\,e(T+hT)\left[u(t-T-hT)-u(t-2T-hT)\right]+\dots$$

ML. $\Rightarrow \bar E(s) = e(hT)\left[\dfrac{e^{-hTs}}{s} - \dfrac{e^{-(T+hT)s}}{s}\right] + e(T+hT)\left[\dots\right] + \dots$

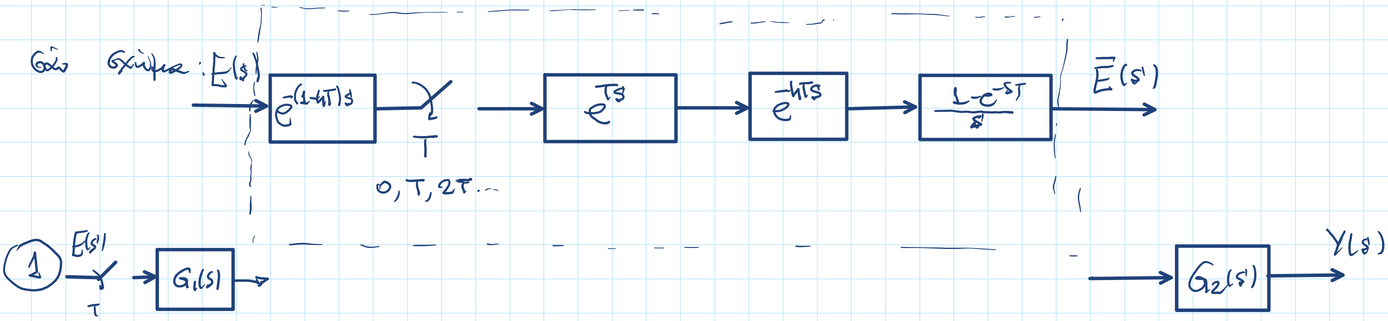
$$= \left[\frac{1-e^{-sT}}{s}\right] e^{-hTs}\left[e(hT) + e(T+hT)e^{-Ts} + e(2T+hT)e^{-2Ts} + \dots\right]$$

$$= \left[\frac{1-e^{-sT}}{s}\right] e^{Ts} e^{-hTs}\left[e(hT)e^{-Ts} + e(T+hT)e^{-2Ts} + \dots\right]$$

$$\Rightarrow \boxed{\bar E(s) = \left[\frac{1-e^{-sT}}{s}\right] e^{sT} e^{-hTs} E(z,m)\Big|_{\substack{m=h\\ z=e^{sT}}}}$$

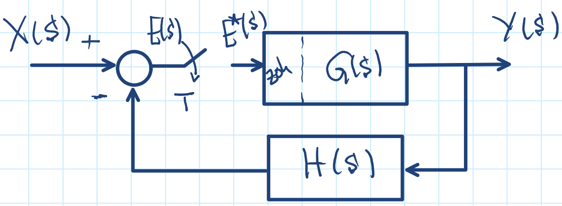
$$E(z,m) = \mathcal{Z}\left[E(s)e^{-\Delta Ts}\Big|_{\Delta=1-m}\right]$$

Ισοδ. σχήμα: $E(s)$



$$\boxed{Y(z) = z\,E(z)\,G_1(z,m)\Big|_{m=h}\,G_2(z,m)\Big|_{m=1-h}}$$

ΣΥΣΤΗΜΑΤΑ ΚΛΕΙΣΤΟΥ ΒΡΟΧΟΥ



Γράφω εξισώσεις → έξοδο

Κανόνας 1: αριθμ. ανεξάρτητων εξισώσεων ≡ = αριθμ. δειγματοληπτών + εξίσωση εξόδου

Κανόνας 2: είσοδοι δειγματοληπτών ≡ έξοδοι συστ.
έξοδοι –//– ≡ είσοδοι –//–

Π.χ.

① $Y(s) = G(s)E^*(s)$

② $E(s) = X(s) - H(s)Y(s)$

$\Rightarrow E(s) = X(s) - H(s)G(s)E^*(s)$

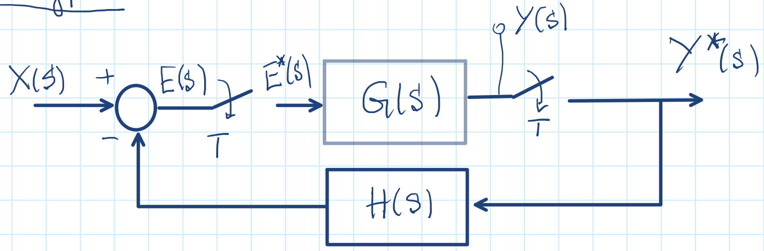
$\Rightarrow E^*(s) = X^*(s) - [H(s)G(s)]^* E^*(s) \Rightarrow E^*(s) = \dfrac{X^*(s)}{\dots}$

$$\Rightarrow E^*(s) = X^*(s) - [H(s)G(s)]^* E^*(s) \Rightarrow E^*(s) = \frac{X^*(s)}{1+[H(s)G(s)]^*}$$

$$1 \Rightarrow Y^*(s) = G^*(s) E^*(s) = \frac{G^*(s)}{1+[H(s)G(s)]^*} \cdot X^*(s) \Bigg|_{z=e^{sT}} =$$

$$\Rightarrow Y(z) = \left(\frac{G(z)}{1+Z[H(s)G(s)]}\right) \cdot X(z) \Rightarrow \boxed{\frac{Y(z)}{X(z)} = \frac{G(z)}{1+\underline{GH(z)}}}$$

Παράδειγμα 2:



$$\frac{Y(z)}{X(z)} = ?$$

$$Y(s) = G(s) E^*(s) \quad 1 \quad \Rightarrow Y^*(s) = G(s)^* E^*(s)$$

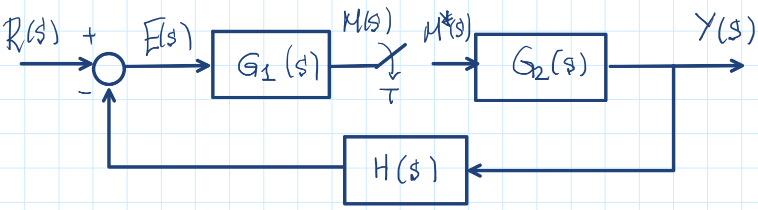
$$E(s) = X(s) - H(s) Y^*(s) \quad 2 \quad \Rightarrow E(s) = [X(s) - H(s) G^*(s) E^*(s)]^*$$

$$\Rightarrow E^*(s) = X^*(s) - [H(s) G^*(s) E^*(s)]^*$$

$$X^*(s) - H^*(s) G^*(s) E^*(s) \Rightarrow E^*(s) = \frac{X^*(s)}{1 + H^*(s) G^*(s)}$$

$$1 \quad Y^*(s) = \frac{G^*(s)}{1 + H^*(s)G^*(s)} X^*(s) \Rightarrow \boxed{\frac{Y(z)}{X(z)} = \frac{G(z)}{1 + H(z)G(z)}}$$

Παράδειγμα 3:



$$Y(z) = ?$$

$$Y(s) = G_2(s) M^*(s) \quad 1$$

$$M(s) = G_1(s) E(s) \quad 2$$

$$E(s) = R(s) - H(s) Y(s) \quad 3$$

$$2, 3 \Rightarrow M(s) = G_1(s) [R(s) - H(s) Y(s)] = G_1(s) R(s) - G_1(s) H(s) Y(s)$$

$$\overset{1}{\Rightarrow} [M(s)]^* = [G_1(s) R(s) - G_1(s) H(s) G_2(s) M^*(s)]^*$$

$$\Rightarrow M^*(s) = [G_1(s) R(s)]^* - [G_1(s) H(s) G_2(s)]^* M^*(s)$$

$$\Rightarrow M^*(s) = [G_1(s)R(s)]^* - [G_1(s)H(s)G_2(s)]^* M^*(s)$$

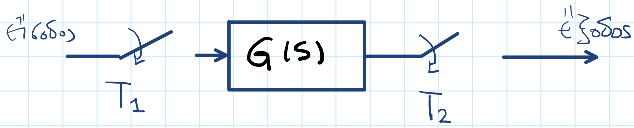
$$\Rightarrow M^*(s) = \frac{[G_1(s)R(s)]^*}{1+[G_1(s)G_2(s)H(s)]^*}$$

(1) $\Rightarrow Y^*(s) = G_2^*(s)\, M^*(s)$

$$\Rightarrow Y^*(s) = \frac{G_2^*(s)\,[G_1(s)R(s)]^*}{1+[G_1(\,)G_2(\,)H(s)]^*}\Bigg|_{z=e^{sT}}$$

$$\boxed{Y(z) = \frac{Z[G_1(s)R(s)]}{1+Z[G_1(s)G_2(s)H(s)]}\,G_2(z)}$$

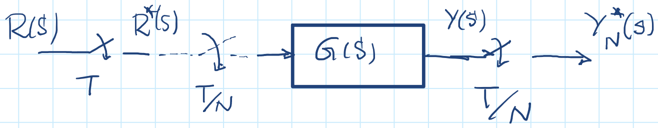
ΣΥΣΤΗΜΑΤΑ ΜΕ ΠΟΛΛΑΠΛΟΥΣ ΡΥΘΜΟΥΣ ΔΕΙΓΜΑΤΟΛΗΨΙΑΣ



"Αργός", "Γρήγορος" $\qquad T_1 = N T_2, \quad N>1$

1. Περίπτωση Αργό - Γρήγορο

1α. Μέθοδος Υποθετικού Δειγματολήπτη



$$Y_N(z) = G_N(z)\,R(z)$$

$$G_N(z) = \sum_{k=0}^{\infty} g(kT/N)\, z^{-k/N}$$

$$\equiv G(z)\Big|_{\substack{z=z^{1/N}\\ T=T/N}}$$

π.χ. $G(s) = \frac{1}{s(s+1)}$, $N=3$

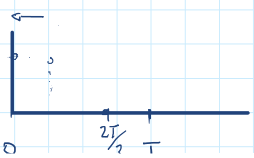
$$G(z)_3 = G(z)\Big|_{\substack{z=z^{1/3}\\ T=\frac{T}{3}}} = Z\left\{\frac{1}{s(s+1)}\right\}\Bigg|_{\substack{z=z^{1/N}\\ T=}} = Z\left\{\frac{1}{s} - \frac{1}{s+1}\right\} =$$

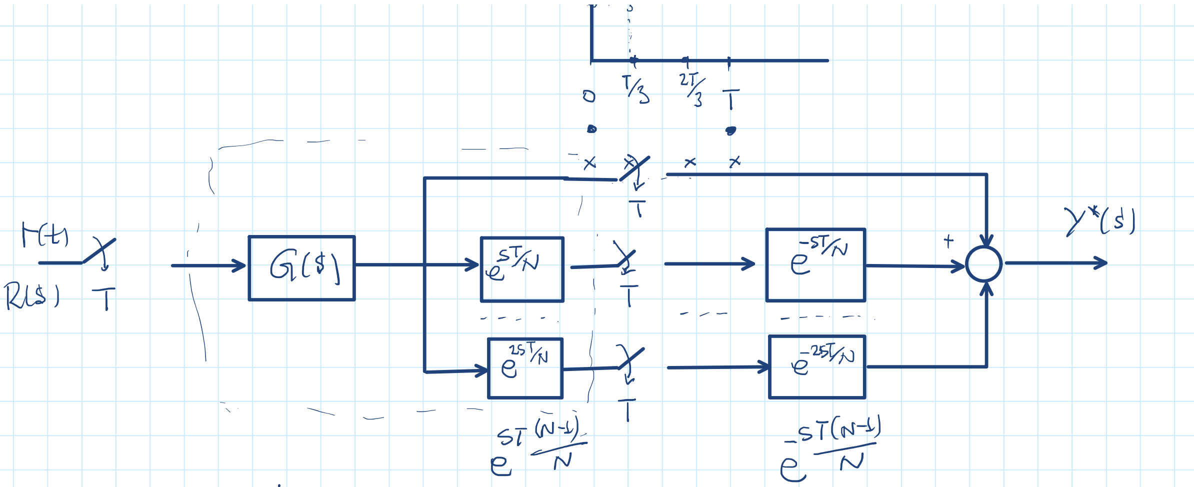
$$= \frac{1}{1-z^{1/3}} - \frac{1}{1-z^{-1/3}e^{-1/3}}$$

άρα $Y(z)_3 = \left[\frac{1}{1-z^{-1/3}} - \frac{1}{1-z^{-1/3}e^{-1/3}}\right]\left[\frac{1}{1-z^{-1}}\right] = \dots$

1β. Διάσπαση σε Παράλληλους Δειγματολήπτες

T, T/N





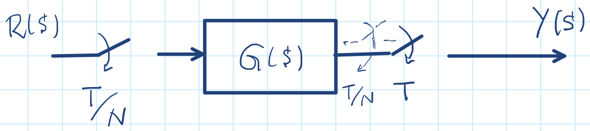
$$Y(z)_N = \sum_{k=0}^{N-1} z^{-k/N}\, Z\left\{ e^{skT/N}\, G(s) \right\} R(z)$$

$$Y(z)_N = \sum_{k=0}^{N-1} z^{-k/N}\, G\left(z, \tfrac{k}{N}\right) R(z)$$

γενικά, $Z\left\{ e^{skT/N}\, G(s) \right\} = z\, G\left(z, \tfrac{k}{N}\right) \equiv z\, G(z, \omega)\Big|_{\omega = k/N}$

$$\Rightarrow \boxed{G(z)_N = \sum_{k=0}^{N-1} z^{1-k/N}\, G\left(z, k/N\right)}$$

2. Περίπτωση Γρήγορο - Αργό



2α. Υποστικός δειγματολήπτης

$$Y(z)_N = G(z)_N\, R(z)_N \quad , \quad G(z)_N = \sum_{k=0}^{\infty} g(kT/N)\, z^{-k/N} \equiv G(z)\Big|_{\substack{z = z^{1/N} \\ T = T/N}}$$

$Y(z) = ?$ άρα $Y(z)$ από των $Y(z)_N$:

$$Y(z)_N = \sum_{k=0}^{\infty} y(kT/N)\, z^{-k/N}$$

$$\overset{MZ^{-1}}{\Rightarrow} \quad y(kT/N) = \frac{1}{2\pi j} \oint_{\Gamma} Y(z_N)\, z_N^{k-1}\, dz \Big|_{z_N = z^{1/N}}$$

$$Y(z) = \sum_{n=0}^{\infty} y(nT)\, z^{-n} \qquad \text{Θέτουμε } n = \frac{k}{N}$$

$$\Rightarrow Y(z) = \sum_{n=0}^{\infty} \frac{1}{2\pi j} \oint_{\Gamma} Y(z_N)\, z_N^{nN-1}\, dz_N\; z^{-n} =$$

$$= \frac{1}{\ } \oint Y(z_N) \left(\sum^{\infty} z^{nN}\, z^{-n} \right) dz_N$$

$$= \frac{1}{2\pi j}\oint_\Gamma Y(z_N)\left(\sum_{n=0}^{\infty} z_N^{nN} z^{-n}\right)\frac{dz_N}{z_N}$$

$$= \frac{1}{2\pi j}\oint_\Gamma Y(z_N)\left(\frac{1}{1-z_N^N z^{-1}}\right)\frac{dz_N}{z_N}$$ → Θ. ολοκληρωτικών υπολοίπων

Άρα,

$$\boxed{Y(z) = \sum \operatorname{Res} Y(z_N)\,\frac{1}{1-z_N^N z^{-1}}\cdot z_N^{-1}\,\Bigg|_{\text{πόλους της } Y(z_N)\,z_N^{-1}}}$$

π.χ. $G(s) = \frac{1}{s+1}$, $R(z)_N = \frac{z_N}{z_N-1}$ (βηματική)

$$\Rightarrow G(z)_N = \frac{z_N}{z_N - e^{-T/N}}$$

$$\Rightarrow Y(z)_N = G(z)_N R(z)_N = \frac{z_N^2}{(z_N-1)(z_N-e^{-T/N})}$$ = έξοδος υποθετικού δειγματολήπτη

Τελικά, $$Y(z) = \sum \operatorname{Res}\left[Y(z_N)\,\frac{z_N^{-1}}{1-z_N^N z^{-1}}\right]\Bigg|_{\text{πόλους}} = \ldots$$

2β. Ανάπτυξη σε Παράλληλους Δειγματολήπτες

$$Y(z) = \sum_{k=0}^{N-1} \mathcal{Z}\left\{R(s)\,e^{\frac{kTs}{N}}\right\}\mathcal{Z}\left\{G(s)\,e^{-\frac{kTs}{N}}\right\}$$

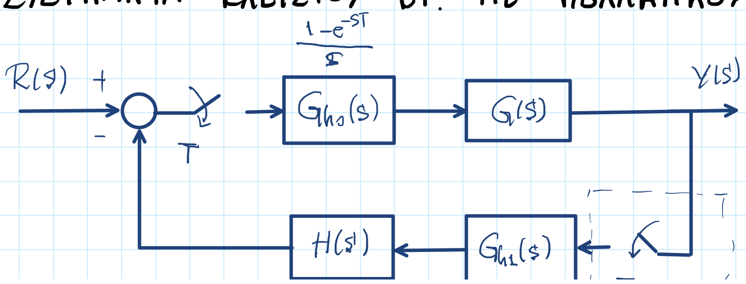
$$\mathcal{Z}\left\{R(s)\,e^{\frac{kTs}{N}}\right\} = z\,R\left(z,\frac{k}{N}\right)$$

$$\mathcal{Z}\left\{G(s)\,e^{-\frac{kTs}{N}}\right\} = G(z,m)\Big|_{m=1-\frac{k}{N}}$$

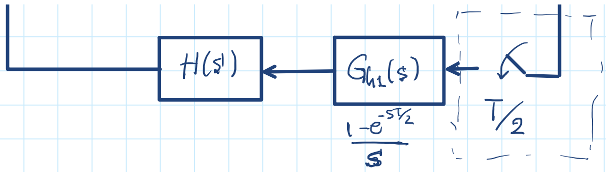
Άρα,

$$\boxed{Y(z) = R(z)G(z) + \sum_{k=0}^{N-1} z\,R\left(z,\frac{k}{N}\right)G\left(z,1-\frac{k}{N}\right)}$$

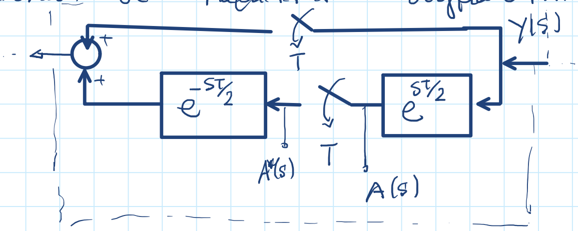
ΣΥΣΤΗΜΑΤΑ ΚΛΕΙΣΤΟΥ ΒΡ. ΜΕ ΠΟΛΛΑΠΛΟΥΣ ΡΥΘΜΟΥΣ ΔΕΙΓΜΑΤΟΛΗΨΙΑΣ



$N = 2$



Γιά παράθυρα : Διάσπαση σέ παράλληλους δειγματολήπτες



$$S_1: \quad E(s) = R(s) - H(s)\, G_{h1}(s) \left[Y^*(s) + e^{-sT/2} A^*(s) \right]$$

$$S_2: \quad Y(s) = G(s)\, G_{h0}(s)\, E^*(s) \quad \Rightarrow \quad Y^*(s) = \left[G_{h0}(s) G(s) \right]^* E^*(s)$$

$$S_3: \quad A(s) = \left[e^{sT/2} G(s) G_{h0}(s) \right] E^*(s) \quad \Rightarrow \quad A^*(s) = \left[e^{sT/2} G(s) G_{h0}(s) \right]^* E^*(s)$$

$$\Rightarrow A^*(s) = \frac{\left[e^{sT/2} G(s) G_{h0}(s) \right]^*}{\left[G(s) G_{h0}(s) \right]^*} \cdot Y^*(s)$$

$$E^*(s) = R^*(s) - \left[H(s) G_{h1}(s) \right]^* Y^*(s) - \left[H(s)\, G_{h1}(s)\, e^{-sT/2} \right]^* A^*(s)$$

$$\Rightarrow Y^*(s) = \ldots\ldots$$