Ρομποτικά Συστήματα

Ενότητα 14: Area Coverage control techniques

Αντώνιος Τζές – Ευάγγελος Δερματάς

Σχολή Πολυτεχνική

Τμήμα ΗΜ&ΤΥ

Σκοποί ενότητας

- Σκοπός της ενότητας είναι η παρουσίαση και εξοικείωση με τα ακόλουθα στοιχεία ρομποτικών συστημάτων:

- Area Coverage control techniques

Περιεχόμενα ενότητας

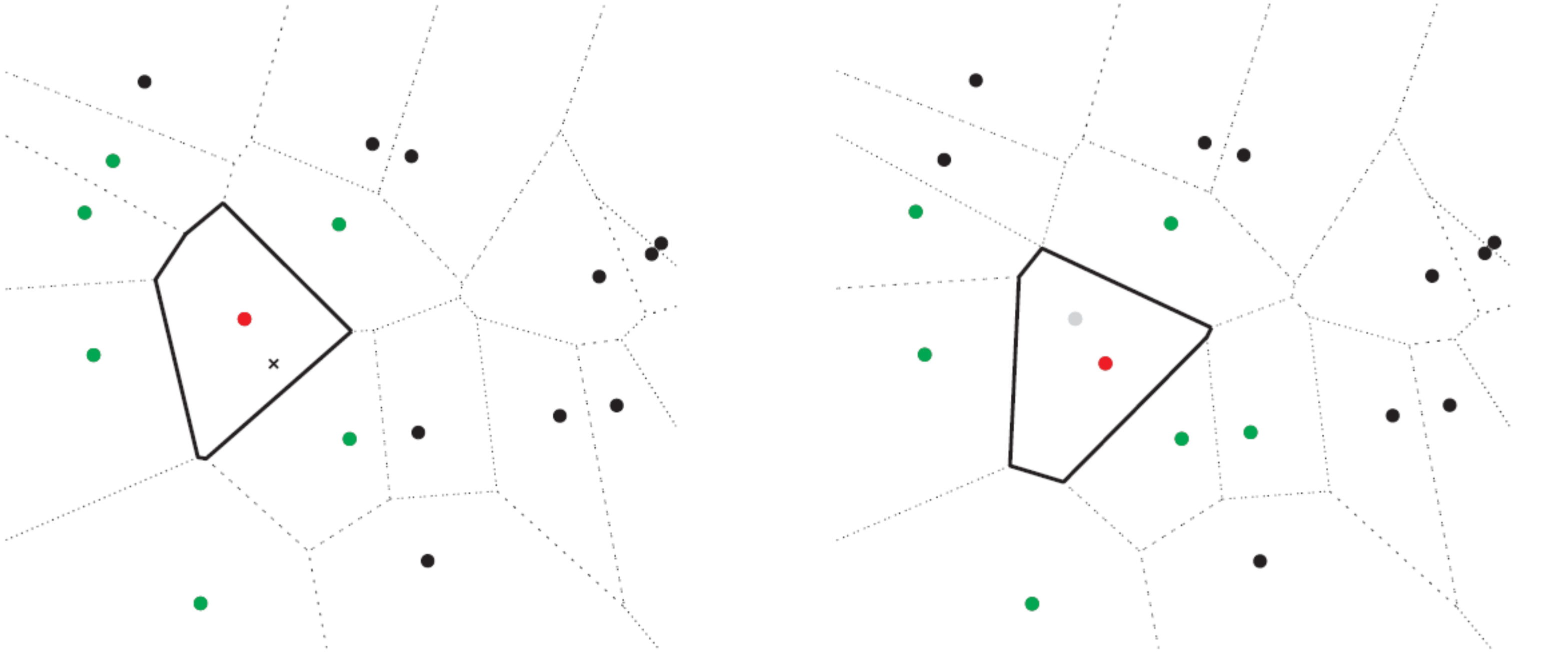
- Area Coverage control techniques

Coordination Scheme

- Move nodes one-by-one: the total coverage area $\mathcal{H}$ of Ω is non-decreasing
- Node-to-move should have sufficient spatial information of the Voronoi cells that will be affected
- Assume node i is the node-to-move in time-step k
- Thus, the only affected Voronoi cells are those in the neighborhood of $i \hookrightarrow$ Delaunay neighbors $\hookrightarrow \mathcal{N}_i^k$

Coordination Scheme

Πηγή: Y. Stergiopoulos, “Cooperative Control of Networked Robotic Systems”, *Ph.D. Dissertation*, September 2014, URL: http://hdl.handle.net/10889/8238



Εικόνα 1: Alteration of the Delaunay neighbors of a node caused by the motion of the latter

Coordination Scheme

- Node $j \in \mathcal{N}_i^k$ leaves set: $j \in \mathcal{N}_i^k \setminus \mathcal{N}_i^{k+1}$, the edge of the moving node's Voronoi cell that corresponds to that node degenerates into a vertex
- Node $j \notin \mathcal{N}_i^k$ enters set: $j \in \mathcal{N}_i^{k+1} \setminus \mathcal{N}_i^k$, then a vertex of V_i will evolve into two vertices after its motion, adding Δ_{ij} into the set of its Voronoi edges
- Suppose the possible motion of i @ step k restricted in convex compact $\mathcal{W}_i^k \subset \mathcal{V}_i^k$: $x_i^k, x_i^{k+1} \in \mathcal{W}_i^k$. We define the set containing union of k step Delaunay neighbors and node-to-move and $k+1$ step possible Delaunay neighbors as:

$$\mathcal{F}_i^k = \mathcal{N}_i^k \cup \{i\} \bigcup_{x_i^{k+1} \in W_i^k} \mathcal{N}_i^{k+1}$$

Coordination Scheme

- Lemma 1: The set $\mathcal{F}_i^k$ contains the nodes of the network whose Voronoi cell is possibly affected by the motion of node i, given the restriction of $x_i^{k+1} \in \mathcal{W}_i^k$
- Communication assumption: The node–to–move i is able to exchange information at each time–step k with the nodes in the set $\mathcal{F}_i^k$ for a given subset $\mathcal{W}_i^k \subset \mathcal{V}_i^k$
- Let $q^\ell \mid_I$ the evaluation (by node-to-move) of arbitrary variable q at step ℓ based on information from nodes in set $I \subseteq I_n$
- The r –limited Voronoi cell of node j $\in \mathcal{F}_i^k$ evaluated by i at step k is denoted $V_j^r \mid_{\mathcal{F}_i^k}$

Coordination Scheme

- <u>Coverage optimization</u>: Node i should move at a point x_i^{k+1} such that network's coverage will be increased at the maximum possible rate $\hookrightarrow$ $\mathcal{H}^{k+1}\,|_{\mathcal{F}_i^k} - \mathcal{H}^k\,|_{\mathcal{F}_i^k}$

Red x: New position
Green: Delaunay neighbors at time k
Yellow: Future Delaunay neighbors at time k+1

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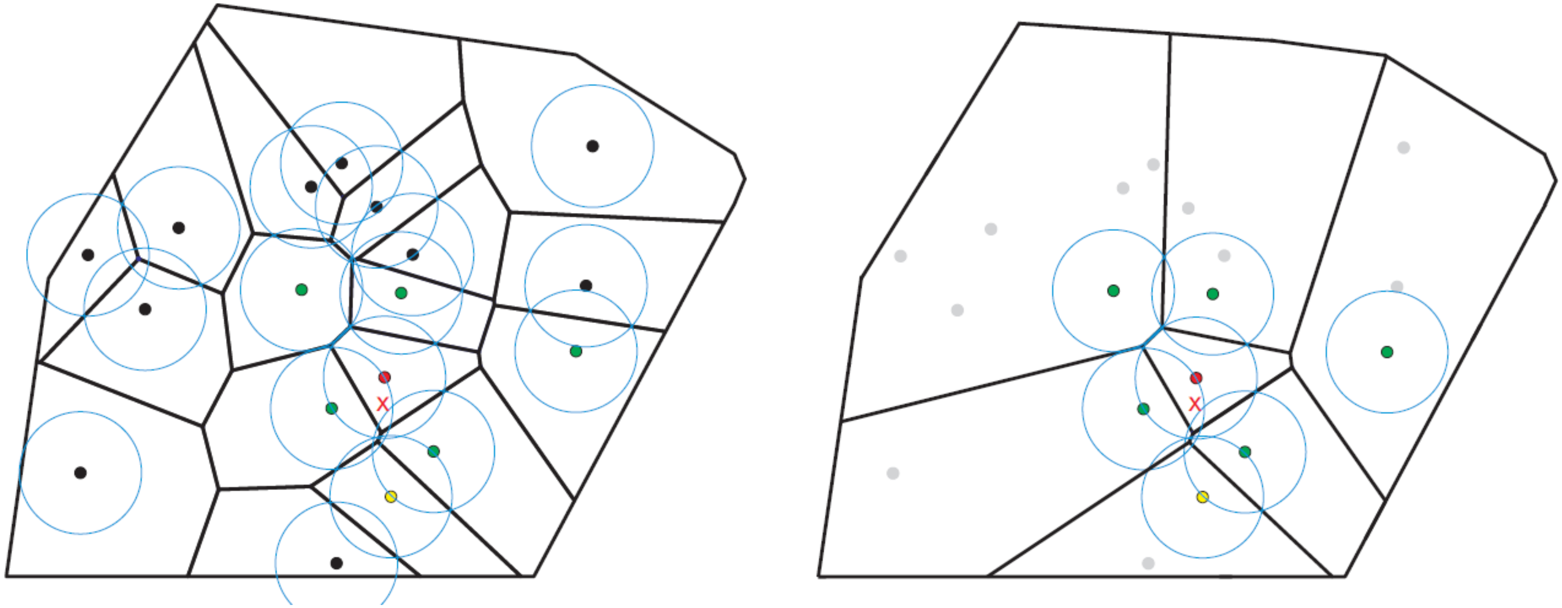


Εικόνα 2: Illustrative example for definition of $V_j^r\,|_{\mathcal{F}_i^k}$ sets

Coordination Scheme

- Lemma 2: For each node $j \in \mathcal{F}_i^k$, it holds that

$$\mathcal{A}\left(V_j^{r,\ell}\right) = \mathcal{A}\left(V_j^{r,\ell} \mid_{\mathcal{F}_i^k}\right) - \sum_{m \in \mathcal{N}_j^{\ell} \setminus \mathcal{F}_i^k} \mathcal{A}\left(U_j^{m,\ell}\right), \text{where } \ell \in \{k, k+1\}$$

Proof: Let us first examine the case $\ell = k$. Considering an arbitrary node $j \in \mathcal{F}_i^k$, it holds that

$$\begin{aligned}\mathcal{A}(C_j) &= \mathcal{A}\left(V_j^{r,k}\right) + \sum_{m \in \mathcal{N}_j^k} \mathcal{A}\left(U_j^{m,k}\right) + \mathcal{A}\left(U_j^{\Omega,k}\right) = \\ &= \mathcal{A}\left(V_j^{r,k}\right) + \sum_{m \in \mathcal{N}_j^k \cap \mathcal{F}_i^k} \mathcal{A}\left(U_j^{m,k}\right) + \sum_{m \in \mathcal{N}_j^k \setminus \mathcal{F}_i^k} \mathcal{A}\left(U_j^{m,k}\right) + \mathcal{A}\left(U_j^{\Omega,k}\right)\end{aligned}$$

Furthermore, $\mathcal{A}(C_j)$ can be written as

$$\begin{aligned}\mathcal{A}(C_j) &= \mathcal{A}\left(V_j^{r,k} \mid_{\mathcal{F}_i^k}\right) + \sum_{m \in \mathcal{N}_j^k \cap \mathcal{F}_i^k} \mathcal{A}\left(U_j^{m,k} \mid_{\mathcal{F}_i^k}\right) + \mathcal{A}\left(U_j^{\Omega,k} \mid_{\mathcal{F}_i^k}\right) \\ &= \mathcal{A}\left(V_j^{r,k} \mid_{\mathcal{F}_i^k}\right) + \sum_{m \in \mathcal{N}_j^k \cap \mathcal{F}_i^k} \mathcal{A}\left(U_j^{m,k}\right) + \mathcal{A}\left(U_j^{\Omega,k} \mid_{\mathcal{F}_i^k}\right)\end{aligned}$$

since nodes $m \in \mathcal{N}_i^k \cap \mathcal{F}_i^k$ are known to i (which performs the evauation)

Coordination Scheme

Combining the above expressions results in:

$$\mathcal{A}\left(V_j^{r,k}\right)=\mathcal{A}\left(V_j^{r,k}\mid_{\mathcal{F}_i^k}\right)-\sum_{m\in\mathcal{N}_j^k\setminus\mathcal{F}_i^k}\mathcal{A}\left(U_j^{m,k}\right)+\left(\mathcal{A}\left(U_j^{\Omega,k}\mid_{\mathcal{F}_i^k}\right)-\mathcal{A}\left(U_j^{\Omega,k}\right)\right)$$

and $U_j^{\Omega,k}=C_j\setminus\Omega$ ~ only on sensing region and pattern ↪

$\mathcal{A}\left(U_j^{\Omega,k}\mid_{\mathcal{F}_i^k}\right)=\mathcal{A}\left(U_j^{\Omega,k}\right)$ which proves the lemma

Coordination Scheme

- <u>Theorem 1</u>: If the node–to–move i has spatial information of the nodes in $\mathcal{F}_i^k$, then evaluation of the network's coverage–increase by ignoring the rest nodes in the network is the same as if the latter had been evaluated supposing existence of all the network's nodes ↪

$$\mathcal{H}^{k+1}\,|_{\mathcal{F}_i^k} - \mathcal{H}^k\,|_{\mathcal{F}_i^k} = \mathcal{H}^{k+1} - \mathcal{H}^k$$

Proof: Considering final equation of previous lecture ↪

Ø from Lemma 1

$$\mathcal{H}^{k+1} - \mathcal{H}^k = \sum_{j\in I_n}\left[\mathcal{A}\left(V_j^{r,k+1}\right) - \mathcal{A}\left(V_j^{r,k}\right)\right] =$$

$$= \sum_{j\in\mathcal{F}_i^k}\left[\mathcal{A}\left(V_j^{r,k+1}\right) - \mathcal{A}\left(V_j^{r,k}\right)\right] + \sum_{j\in I_n\setminus\mathcal{F}_i^k}\left[\mathcal{A}\left(V_j^{r,k+1}\right) - \mathcal{A}\left(V_j^{r,k}\right)\right]$$

Coordination Scheme

Substitute $\mathcal{A}\left(V_j^{r,k}\right), \mathcal{A}\left(V_j^{r,k+1}\right)$ from Lemma 2:

$$\mathcal{H}^{k+1} - \mathcal{H}^k = \sum_{j \in \mathcal{F}_i^k} \left[\mathcal{A}\left(V_j^{r,k+1}|_{\mathcal{F}_i^k}\right) - \mathcal{A}\left(V_j^{r,k}|_{\mathcal{F}_i^k}\right)\right] - \sum_{j \in \mathcal{F}_i^k} \left\{\sum_{m \in \mathcal{N}_j^k \setminus \mathcal{F}_i^k} \left[\mathcal{A}\left(U_j^{m,k+1}\right) - \mathcal{A}\left(U_j^{m,k}\right)\right]\right\}$$

$\emptyset$, because it consists of the alteration of in the unexploited regions of the nodes in $\mathcal{F}_i^k$ that lay in the Voronoi cells of nodes that belong in $\mathcal{N}_j^k \setminus \mathcal{F}_i^k$

Finally, since node i does not have info of existence of nodes $I_n \setminus \mathcal{F}_i^k$,

$$\mathcal{H}^{k+1} - \mathcal{H}^k = \sum_{j \in \mathcal{F}_i^k} \left[\mathcal{A}\left(V_j^{r,k+1}|_{\mathcal{F}_i^k}\right) - \mathcal{A}\left(V_j^{r,k}|_{\mathcal{F}_i^k}\right)\right] + \sum_{j \in I_n \setminus \mathcal{F}_i^k} \left[\mathcal{A}\left(V_j^{r,k+1}|_{\mathcal{F}_i^k}\right) - \mathcal{A}\left(V_j^{r,k}|_{\mathcal{F}_i^k}\right)\right] = \mathcal{H}^{k+1}|_{\mathcal{F}_i^k} - \mathcal{H}^k|_{\mathcal{F}_i^k}$$

Coordination Scheme

- At this point assume circular sensing area of

$$W_i^k = \left\{ x \in \mathbb{R}^2 : \; \|x - x_i^k\| \leq \alpha d\left(x_i^k, V_i^k\right) \right\}$$

Where $0 < \alpha \ll 1$ and $d(x, M) := \inf\{\|x - y\| : \; y \in M\}$

- The optimal position $x_i^{\star k+1}$ is achieved through numerical optimization of:

$$\text{find } \; x_i^{k+1} \in W_i^k :$$

$$\text{maximize } \; \left\{ \mathcal{H}^{k+1} \mid_{\mathcal{F}_i^k} - \mathcal{H}^k \mid_{\mathcal{F}_i^k} \right\}$$

$$\text{subject to: } \; \mathcal{H}^{k+1} \mid_{\mathcal{F}_i^k} > \mathcal{H}^k \mid_{\mathcal{F}_i^k}$$

- Then the control action can be selected as:

$$u_i^k = x_i^{\star\, k+1} - x_i^k, \; \|u_i\| \leq \alpha$$

Issues on spatial information exchange

- Let $\mathcal{R}_i^k(\mathrm{I})$ and $\mathcal{R}_i^k(\mathrm{I})_{wcs}$ min communication radius of i at step k in order to exchange info from a centralized and decentralized point of view respectively
- The minimum range for centralized is:

$$R_i^k\left(\mathcal{N}_i^k\right) = 2\max\left\{d\left(x_i^k, \Delta_{ij}^k\right) : \; j \in \mathcal{N}_i^k\right\} = \max\left\{\left\|x_i^k, x_j^k\right\| : \; j \in \mathcal{N}_i^k\right\}$$

This numerical procedure ends when:

$$R_i^k\left(\mathcal{N}_i^k\right)_{\mathrm{wcs}} = 2\max\left\{\left\|x_i^k - v_{i,j}^k\right\| : j \in I_{\mathrm{N}\left(V_i^k\right)}\right\}$$

- Thus, the proposed algorithm is formed as follows:

Issues on spatial information exchange

$\Diamond$ *Goal:* Identify current Delaunay neighbors and Voronoi cell

$R_i^k \leftarrow 0,\ S_i^k \leftarrow \emptyset$

$\hat{V}_i^k \leftarrow \Omega,\ \hat{\mathbb{N}}_i^k \leftarrow \emptyset$

while $R_i^k \leq 2\max\left\{\left\|x_i^k - \hat{v}_{i,j}^k\right\| : j \in I_{\mathrm{N}(\hat{V}_i^k)}\right\}$ **and** $S_i^k \subset \Omega$ **do**

  increase R_i^k

  update S_i^k

  if node j detected **then**

    $\hat{\mathbb{N}}_i^k \leftarrow \hat{\mathbb{N}}_i^k \cup j$

    update $\hat{V}_i^k$

  end if

end while

$V_i^k \leftarrow \hat{V}_i^k$

isolate $\mathbb{N}_i^k$ from the set $\hat{\mathbb{N}}_i^k$

Issues on spatial information exchange

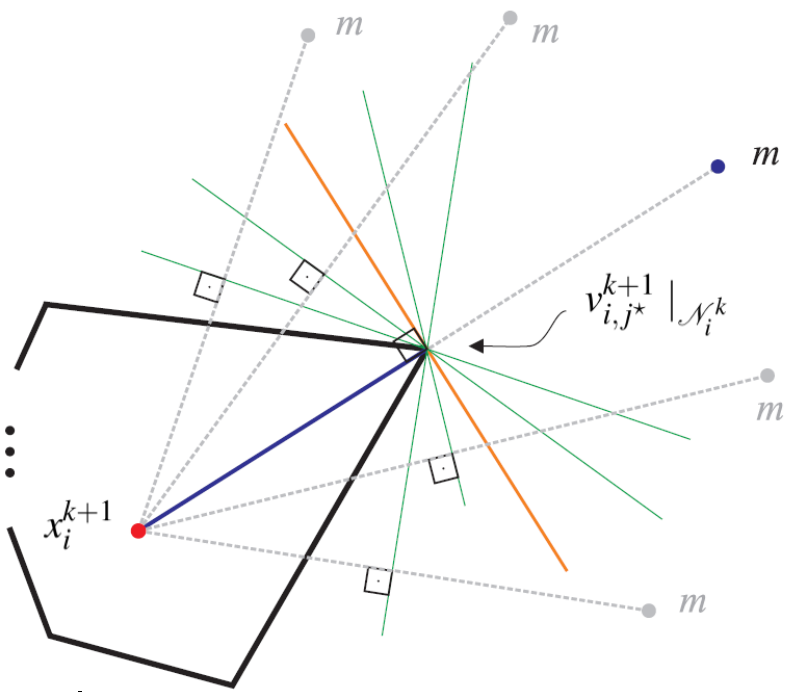
- New target: Ensure connectivity of i with $\bigcup_{x_i^{k+1} \in W_i^k} \mathcal{N}_i^{k+1}$, from a decentralized point of view
- $\hookrightarrow$ worst case scenario:

  $m \in I_n$ enters $\mathcal{N}_i^{k+1}$ at x_i^{k+1}.If $h_{ij}$the equally dividing line

$$h_{ij} = \left\{x \in \mathbb{R}^2 : \ \|x - x_i\| = \left\|x - x_j\right\|\right\}, \ \ i, j \in I_n, \ i \neq j$$

- Moreover, let $v_{i,j^\star}^{k+1} \mid_{\mathcal{N}_i^k}$ be the farthest vertex of $V_i^{k+1} \mid_{\mathcal{N}_i^k}$, where: $j^\star = \arg\max \left\{ \left\| x_i^{k+1} - v_{i,j}^{k+1} \mid_{\mathcal{N}_i^k} \right\| : \ j \in I_{\mathrm{N}\left(V_i^{k+1}\mid_{\mathcal{N}_i^k}\right)} \right\}$
- The worst case scenario for the position of node m is when it lays along the line that connects x_i^{k+1} and $v_{i,j^\star}^{k+1} \mid_{\mathcal{N}_i^k}$

Issues on spatial information exchange



Εικόνα 3: Worst case scenario for the existence of a node *m* that is to enter $\mathcal{N}_i^{k+1}$

Πηγή: Y. Stergiopoulos, "Cooperative Control of Networked Robotic Systems", *Ph.D. Dissertation,* September 2014, URL: http://hdl.handle.net/10889/8238

- This worst-case for m is expressed as:

$$x_m^{k+1} = x_i^{k+1} + 2\left(v_{i,j^\star}^{k+1}|_{\mathcal{N}_i^k} - x_i^{k+1}\right)$$

- Then, the adequate communication range for x is:

$$R_i^k\left(\mathcal{N}_i^{k+1}\right)_{\text{wcs}} = \left\|x_i^k - \left(x_i^{k+1} + 2\left(v_{i,j^\star}^{k+1}|_{\mathcal{N}_i^k} - x_i^{k+1}\right)\right)\right\| = \left\|\left(x_i^k + x_i^{k+1}\right) - 2v_{i,j^\star}^{k+1}|_{\mathcal{N}_i^k}\right\|$$

Issues on spatial information exchange

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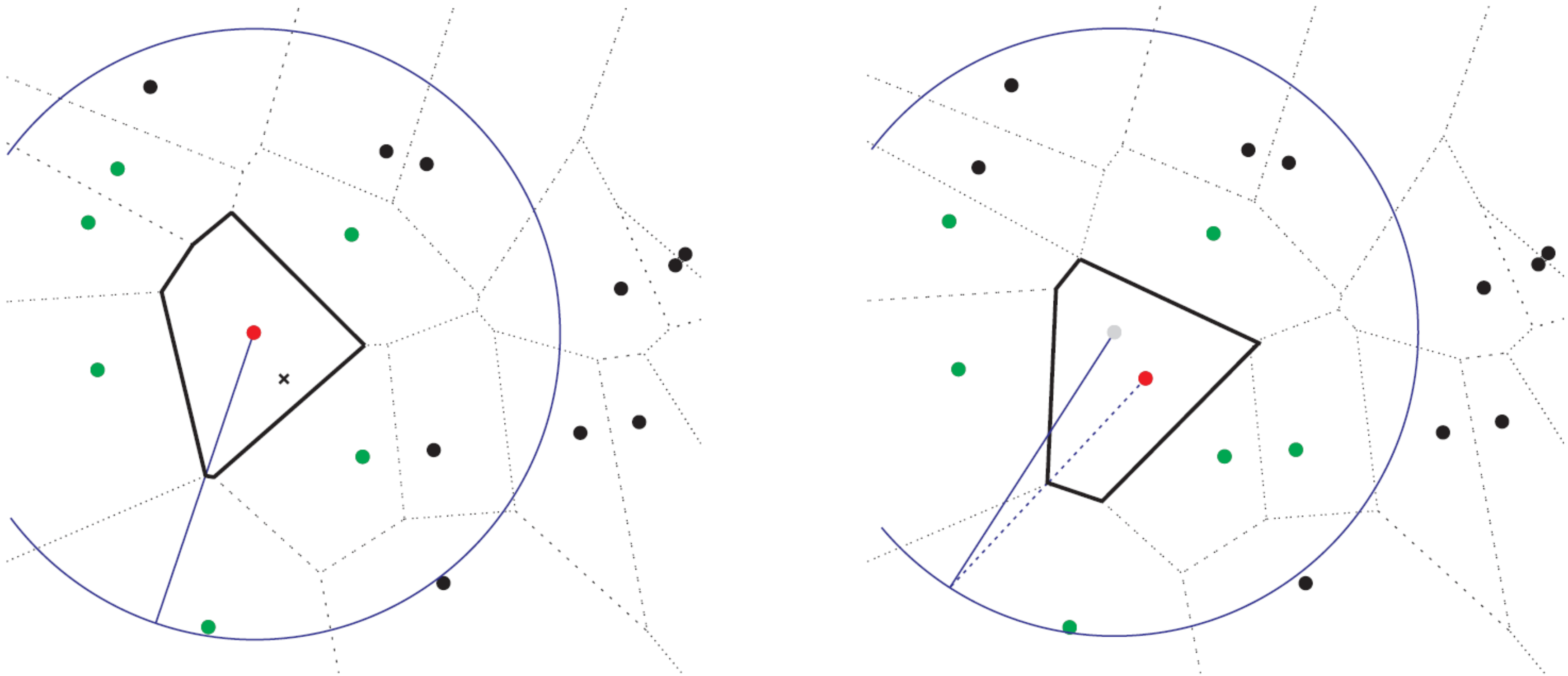
*Blue circle is the min communication radius



Εικόνα 4: Communication radius of the node–to–move (red color) required in order to guarantee connectivity with $\mathcal{N}_i^k$ (left) and $\mathcal{N}_i^{k+1}$ (right – worst case scenario)

Issues on spatial information exchange

- Final conclusion(Corollary):

The communication radius of node i at step k in order to guarantee connectivity with both current and all possible future Delaunay neighbors, should be at least

$$R_i^k\left(\mathcal{F}_i^k\right)_{\mathrm{wcs}} = \max\left\{ R_i^k\left(\mathcal{N}_i^k\right)_{\mathrm{wcs}},\ R_i^k\left(\bigcup_{x_i^{k+1}\in W_i^k} \mathcal{N}_i^{k+1}\right)_{\mathrm{wcs}} \right\}$$

- The derived algorithm is formed as follows:

Issues on spatial information exchange

$\Diamond$ *Goal:* Identify nodes whose Voronoi cells are possibly affected

$R_i^k \left(\bigcup_{x_i^{k+1} \in W_i^k} \mathcal{N}_i^{k+1} \right)_{\text{wcs}} \leftarrow 0$

identify $\mathcal{N}_i^k$ and V_i^k via Algorithm 1

perform gridding on W_i^k

for each $x_i^{k+1} \in W_i^k$ **do**

    evaluate $V_i^{k+1} \mid_{\mathcal{N}_i^k}$

    evaluate $v_{i,j^*}^{k+1} \mid_{\mathcal{N}_i^k}$

    evaluate $R_i^k \left(\mathcal{N}_i^{k+1} \right)_{\text{wcs}}$

    $R_i^k \left(\bigcup_{x_i^{k+1} \in W_i^k} \mathcal{N}_i^{k+1} \right)_{\text{wcs}} \leftarrow \max \left\{ R_i^k \left(\bigcup_{x_i^{k+1} \in W_i^k} \mathcal{N}_i^{k+1} \right)_{\text{wcs}}, R_i^k \left(\mathcal{N}_i^{k+1} \right)_{\text{wcs}} \right\}$

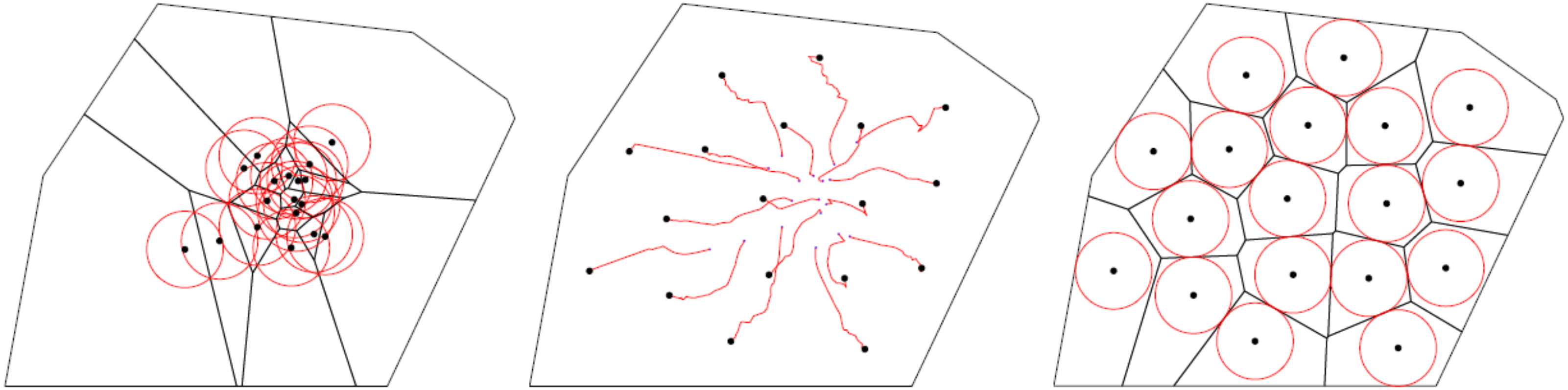
end for

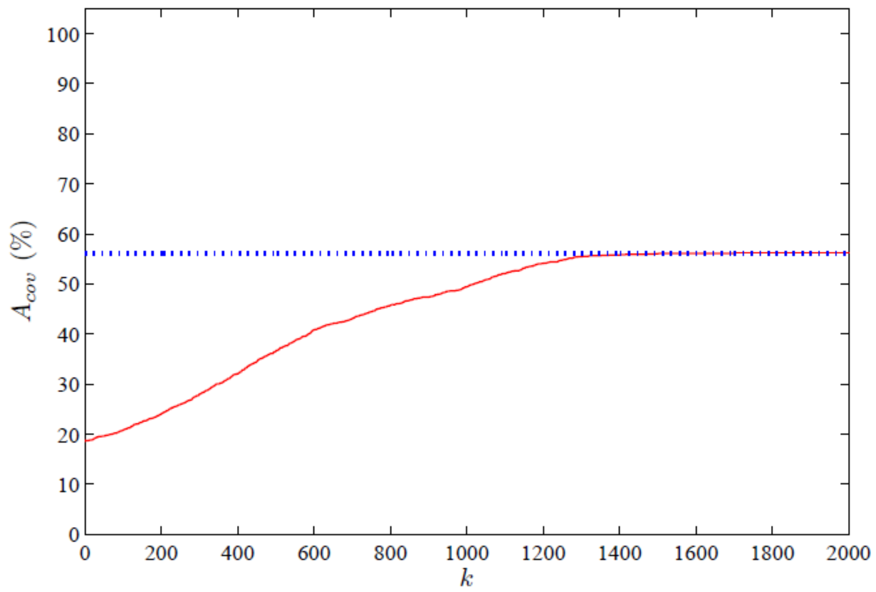
$R_i^k \left(\mathcal{F}_i^k \right)_{\text{wcs}} \leftarrow \max \left\{ R_i^k \left(\mathcal{N}_i^k \right)_{\text{wcs}}, R_i^k \left(\bigcup_{x_i^{k+1} \in W_i^k} \mathcal{N}_i^{k+1} \right)_{\text{wcs}} \right\}$

update S_i^k

identify $\mathcal{F}_i^k$

Issues on spatial information exchange





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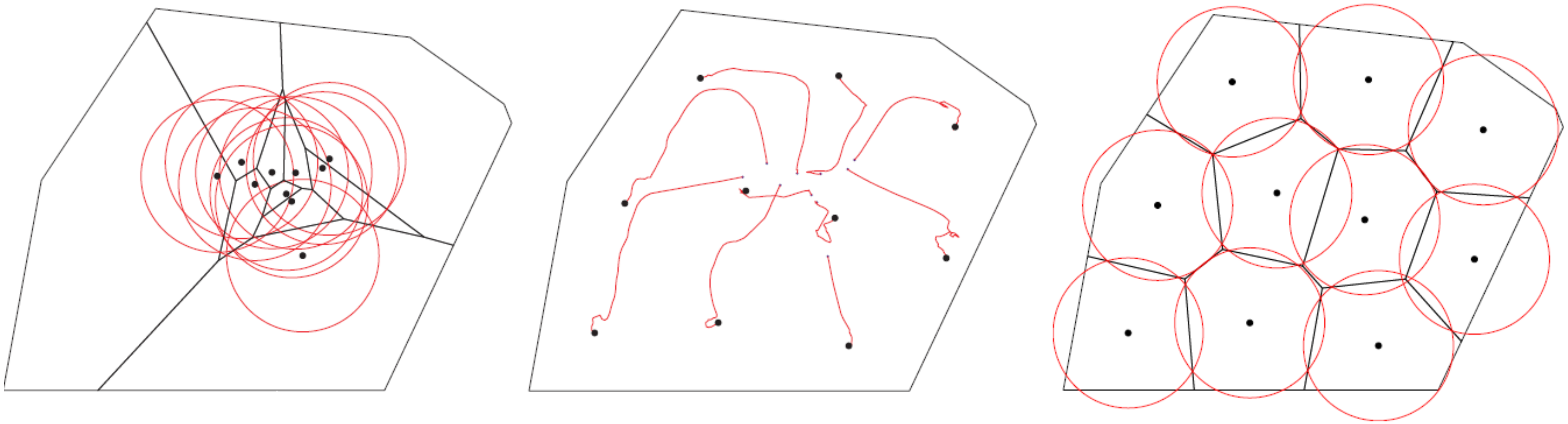
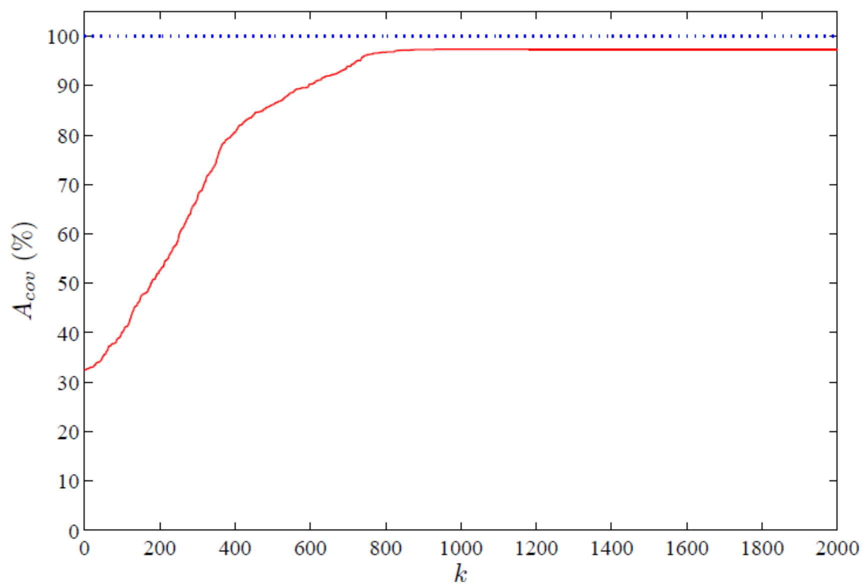
Εικόνα 5: Sparse–network case study: [Left] Initial network configuration. [Middle] Network evolution through time. [Right] Final network optimum state. [Bottom] Percentage of covered area w.r.t. time

Issues on spatial information exchange





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Εικόνα 6: Congested–network case study: [Left] Initial network configuration. [Middle] Network evolution through time. [Right] Final network optimum state. [Bottom] Percentage of covered area w.r.t. time

More info

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Τέλος Ενότητας

Χρηματοδότηση

- Το παρόν εκπαιδευτικό υλικό έχει αναπτυχθεί στο πλαίσιο του εκπαιδευτικού έργου του διδάσκοντα.
- Το έργο «**Ανοικτά Ακαδημαϊκά Μαθήματα στο Πανεπιστήμιο Πατρών**» έχει χρηματοδοτήσει μόνο την αναδιαμόρφωση του εκπαιδευτικού υλικού.
- Το έργο υλοποιείται στο πλαίσιο του Επιχειρησιακού Προγράμματος «Εκπαίδευση και Δια Βίου Μάθηση» και συγχρηματοδοτείται από την Ευρωπαϊκή Ένωση (Ευρωπαϊκό Κοινωνικό Ταμείο) και από εθνικούς πόρους.

Ευρωπαϊκή Ένωση
Ευρωπαϊκό Κοινωνικό Ταμείο
ΕΠΙΧΕΙΡΗΣΙΑΚΟ ΠΡΟΓΡΑΜΜΑ
ΕΚΠΑΙΔΕΥΣΗ ΚΑΙ ΔΙΑ ΒΙΟΥ ΜΑΘΗΣΗ
επένδυση στην κοινωνία της γνώσης
ΥΠΟΥΡΓΕΙΟ ΠΑΙΔΕΙΑΣ ΚΑΙ ΘΡΗΣΚΕΥΜΑΤΩΝ
ΕΙΔΙΚΗ ΥΠΗΡΕΣΙΑ ΔΙΑΧΕΙΡΙΣΗΣ
ΕΣΠΑ 2007-2013
πρόγραμμα για την ανάπτυξη
ΕΥΡΩΠΑΪΚΟ ΚΟΙΝΩΝΙΚΟ ΤΑΜΕΙΟ
Με τη συγχρηματοδότηση της Ελλάδας και της Ευρωπαϊκής Ένωσης

Σημειώματα

Σημείωμα Ιστορικού Εκδόσεων Έργου

Το παρόν έργο αποτελεί την έκδοση 1.0

Σημείωμα Αναφοράς

- Copyright Πανεπιστήμιο Πατρών, Αντώνιος Τζές, Ευάγγελος Δερματάς, «Ρομποτικά Συστήματα. Area Coverage control techniques». Έκδοση: 1.0. Πάτρα 2015. Διαθέσιμο από τη δικτυακή διεύθυνση: https://eclass.upatras.gr/courses/EE804/index.php

Σημείωμα Αδειοδότησης

Διατήρηση Σημειωμάτων

Οποιαδήποτε αναπαραγωγή ή διασκευή του υλικού θα πρέπει να συμπεριλαμβάνει:

- το Σημείωμα Αναφοράς
- το Σημείωμα Αδειοδότησης
- τη δήλωση Διατήρησης Σημειωμάτων
- το Σημείωμα Χρήσης Έργων Τρίτων (εφόσον υπάρχει)

μαζί με τους συνοδευόμενους υπερσυνδέσμους.

Σημείωμα Χρήσης Έργων Τρίτων (1/1)

Το Έργο αυτό κάνει χρήση των ακόλουθων έργων:

Εικόνες/Σχήματα/Διαγράμματα/Φωτογραφίες

Εικόνα 1: Alteration of the Delaunay neighbors of a node caused by the motion of the latter, Y. Stergiopoulos, "Cooperative Control of Networked Robotic Systems", Ph.D. Dissertation, September 2014, URL: http://hdl.handle.net/10889/8238

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