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Budeanu's Concept of Reactive and Distortion Power Revisited

Abstract. *Budeanu's concept of reactive and distortion power for systems operating under nonsinusoidal conditions has been subject to criticism throughout the past decades, which finally led to its removal from the latest standards. Recently, some new results are presented that shed a different light on Budeanu's reactive power and its compensation. In this paper, a further analysis is presented to reveal the possibility to endow Budeanu's reactive and distortion powers with physical meaning. For linear and time-invariant systems it is shown that Budeanu's reactive power is representing the mean value of the moving average of the difference between the total magnetic and the electric energy. Budeanu's distortion power, after an appropriate decomposition, represents a measure of the power fluctuations around the active power and Budeanu's reactive power.*

Streszczenie. *Teoria Budeanu dotycząca mocy w obwodach niesinusoidalnych była w ostatnich latach krytykowana i w rezultacie usunikęta z norm. Obecnie wraca się do niektórych elementów tej teorii. W artykule zaprezentowano analizę wskazującą w jakich warunkach teoria Budeanu może być stosowana. Po odpowiedniej dekompozycji moc odkształcona jest miarą fluktuacji mocy wokół wartości średniej mocy czynnej i mocy biernej wg Budeanu.* ***(Teoria mocy biernej i mocy odkształconej według Budeanu – nowe spojrzenie)***

Keywords: Budeanu, reactive power, distortion power, generalized phasors, analytic signals, time-domain, Hilbert transform
Słowa kluczowe: Budeanu, moc w obwodach nieliniowych, transformata Hilberta

Introduction

After the pioneering works of Steinmetz [17] and Iliovici [11], a generalization of the power model for systems operating under periodical nonsinusoidal conditions was proposed by Budeanu [2]. Starting from a harmonic decomposition of the port voltage and current, a straightforward generalization of the active power is given by

(1) $$P_A = \sum_k U_k I_k \cos(\phi_k),$$

with U_k and I_k denoting the rms value of the k-th harmonic component of the voltage and the current, respectively, and ϕ_k denoting the phase difference between the k-th harmonic component of the port voltage and current. Inspired by, e.g., Bunet [1] and Boucherot's theorem [3], Budeanu defined reactive power as

(2) $$Q_B = \sum_k U_k I_k \sin(\phi_k).$$

Budeanu also observed that for nonsinusoidal voltages and currents the quadratic sum of the active and reactive power is not equal to the apparent power S as in the sinusoidal case, and ended up with $S^2 = P_A^2 + Q_B^2 + (\mathrm{REST})^2$. To fill in this gap, a new concept

(3) $$D_B = \sqrt{S^2 - P_A^2 - Q_B^2},$$

called distortion (or deformation) power was proposed.

For decades, Budeanu's power model has enjoyed a lot of support and is set down in many publications and academic textbooks on power phenomena in systems with periodical and distorted waveforms, and for a long time has been part of the IEEE Standard [9]. Nevertheless, from the very beginning it has also been criticized by various opponents. Apart from the fact that it took almost 50 years before the first meters where developed to measure Budeanu's reactive and distortion powers, the objections where mainly concerned with the lack of physical meaning of the distortion power and the summing up of magnitudes of oscillating components of different harmonics [16], see also [1] and the references therein. Budeanu's power model was finally vigorously challenged by Czarnecki [4], attempting to show its uselessness in the context of instrumentation and measurement on the grounds that:

A1. The powers Q_B and D_B do not posses any attributes which might be related to the power phenomena in the system;
A2. The reactive power Q_B does not represent a measure of energy oscillations;
A3. The distortion power D_B does not provide any information about waveform distortion.

In the context of compensation and power factor improvement, it was furthermore concluded in [4] that:

A4. There is no direct relation between Q_B and D_B and the current rms value;
A5. Apparent power cannot be minimized with the help of Budeanu's power model, so that the power factor cannot be increased;
A6. Independent compensation of the powers Q_B and D_B is not possible.

Although these arguments did not instantaneously convince adherents of Budeanu's power model [6], it was finally abandoned from the latest IEEE Standard [10]. However, recently it is shown by Willems [19] that by selecting the so-called *Budeanu reactive current* – if it exists – it is *always* possible to reduce the source current rms value.[1] Indeed, as the Budeanu reactive current is orthogonal to the currents associated with the active and distortion powers, this critically addresses the assertions A5 and A6 (and hence A4) and renders them invalid. In this paper, further new (time-domain) perspectives of Budeanu's power model are derived that also critically address the assertions A1–A3.

Notation

Given two square integrable T-periodic signals $u(t)$ and $i(t)$, we define the inner product as

(4) $$\langle u,i\rangle = \frac{1}{T}\int_0^T u(t)i(t)\mathrm{d}t,$$

and by $\|u\| = \sqrt{\langle u,u\rangle}$ the rms (root-mean-square) value. Time-differentiation is denoted by $u'(t) = \frac{\mathrm{d}}{\mathrm{d}t}u(t)$. Voltages are represented in volts [V] and currents are represented in Ampère [A]. However, these units will be omitted in the text. To simplify the presentation, all voltage and current waveforms are assumed to have zero mean values.

The Budeanu Reactive Current

In this section, the concept of the Budeanu reactive current presented in [19] is reviewed. We start with an illustrative example to show that merely rendering Budeanu's reactive power to zero does not necessarily lead to a decrease of the source current rms value, and hence not to a decrease of the apparent power.

[1] The concept of Budeanu reactive current was already used earlier in the apparent power decomposition proposed in [12], but no reference was made to its merits in compensator design.

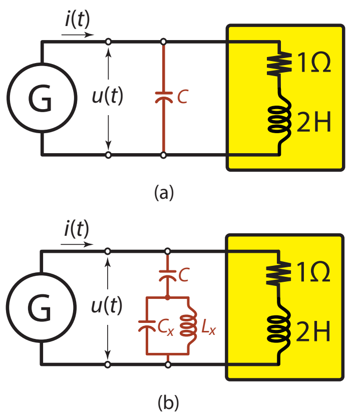

Fig. 1. (a) Uncompensated ($C = 0$ F) and compensation of Q_B using a single capacitor ($C = 0.189$ F); (b) Compensation of Q_B using the Budeanu current ($C = 0.189$ F, $C_x = 0.128$ F and $L_x = 1.805$).

Consider the uncompensated RL circuit shown in Fig. 1(a) driven by a nonsinusoidal voltage

(5) $$u(t) = 10\sqrt{2}\cos(t) + 5\sqrt{2}\cos(5t).$$

The power quantities and the power factor for the uncompensated circuit according to Budeanu's power model are presented in the 2nd column of Table 1.

Table 1. Compensation based on Budeanu's power model.

Quantity	Uncomp.	C-only	$C + L_x \| C_x$	Unit
P_A	20.248	20.248	20.248	W
Q_B	42.475	0	0	VAr
D_B	17.799	53.654	17.799	VA
S	50.309	57.347	26.959	VA
$\lambda = \|P_A\|/S$	0.403	0.353	0.751	—

As shown in the 3rd column, although a shunt capacitor $C = 0.189$ F completely compensates Q_B, the power factor λ is even worse than in the uncompensated case. Hence, the compensation of Budeanu's reactive power in this way may indeed be useless for power factor improvement as the influence of the distortion power makes this compensation far from optimal and gives rise to the assertions A4–A6.

However, as explained in [8] and [19], the main reason why in the above example the compensator current, which renders Budeanu's reactive power to zero, does not reduce the source current – and even increases the distortion power – is that this particular compensator current and the non-active part of the load current are *not* mutually orthogonal. The appropriate choice of the compensator current is selected as follows. Consider a single-phase linear and time-invariant (LTI) system with distorted voltage and current waveforms

(6) $$u(t) = \sum_k U_k\sqrt{2}\cos(k\omega t + \phi_{u_k}),$$

(7) $$i(t) = \sum_k I_k\sqrt{2}\cos(k\omega t + \phi_{i_k}),$$

with $\phi_k = \phi_{u_k} - \phi_{i_k}$, and define the *Budeanu current* [19, 12]:

(8) $$i_B(t) = \frac{Q_B}{\|\hat{u}\|^2}\hat{u}(t),$$

where $\hat{u}(t)$ is obtained by shifting every harmonic 90–degrees backwards, i.e.,[2]

(9) $$\hat{u}(t) = \sum_k U_k\sqrt{2}\sin(k\omega t + \phi_{u_k}).$$

Clearly, $\|u\| = \|\hat{u}\|$ and $\langle u, \hat{u}\rangle = 0$, i.e., $u(t)$ and $\hat{u}(t)$ are mutual orthogonal, so that (8) is mutually orthogonal to Fryze's active current [7]

(10) $$i_A(t) = \frac{P_A}{\|u\|^2}u(t),$$

as well as to the residual current $i_D(t) = i(t) - i_A(t) - i_B(t)$, which represents the distortion component. Hence, we have that

$$\|i\|^2 = \|i_A\|^2 + \|i_B\|^2 + \|i_D\|^2.$$

Consequently, if a compensator supplies the Budeanu reactive current to the load, the Budeanu reactive power seen by the source will be zero and the distortion power remains unaltered. As a result, the apparent power decreases. This shows that by choosing the appropriate compensation current the power factor increases and that the Budeanu reactive power concept, in general, *does* lead to a compensation scheme that reduces the line losses, except for systems in which $Q_B = 0$ already before compensation.

Compensation results for the RL circuit of Fig. 1 supplied with (5) and based on compensation of the Budeanu current (8) are shown in the 4th column of Table 1. The power factor is now improved, while the distortion power remains the same as expected.

It should be emphasized that, in general, the compensator supplying the Budeanu current cannot be realized by a single lossless shunt element. In fact, for the given example, it is composed of the same capacitor $C = 0.189$ F as before, but in series with a parallel connection of a capacitor $C_x = 0.128$ F and an inductor $L_x = 1.805$ H; see Fig. 1(b).

Tellegen's Theorem

Instrumental for our analysis in the remainder of the paper is Tellegen's theorem [14], which states that at each instant the rate of energy (i.e., the power) entering the load through its port gets distributed among the elements of the load network, so that none is lost. For a single-phase load network consisting of branches b (edges) containing resistors, inductors, and capacitors, the instantaneous power at the port is

(11) $$p(t) = \sum_{\text{res}} R_b i_b^2(t) + \sum_{\text{ind}} L_b i_b'(t) i_b(t) + \sum_{\text{cap}} u_b(t) C_b u_b'(t),$$

with $p(t) = u(t)i(t)$, or, equivalently, as

(12) $$w'(t) = p(t) - p_d(t),$$

where $w(t) = w_m(t) + w_e(t)$ is the total instant magnetic and electric energy, with

(13) $$w_m(t) = \frac{1}{2}\sum_{\text{ind}} L_b i_b^2(t), \quad w_e(t) = \frac{1}{2}\sum_{\text{cap}} C_b u_b^2(t),$$

and

(14) $$p_d(t) = \sum_{\text{res}} R_b i_b^2(t)$$

the total instantaneous dissipated power.

[2] The 90–degrees backward shift operation will be formalized later on using the notion of analytic signals and the Hilbert transform.

The Classical Sinusoidal Power Model

Before we address the assertions A1–A3, and to place the remaining developments into perspective, let us briefly recall the well-known classical scenario of a single-phase sinusoidal source transmitting energy to a LTI load. Let the voltage at the load terminals be given by

$$u(t) = U\sqrt{2}\cos(\omega t + \phi_u), \tag{15}$$

then the associated current reads

$$i(t) = I\sqrt{2}\cos(\omega t + \phi_i), \tag{16}$$

so that the right-hand side of (11) can be written as

$$p(t) = P\left[1 + \cos(2\omega t + 2\phi_u)\right] + Q\sin(2\omega t + 2\phi_u),$$

where P and Q represent the active power and the reactive power defined by

$$P = UI\cos(\phi), \quad Q = UI\sin(\phi),$$

respectively, with $\phi = \phi_u - \phi_i$ the phase difference between the current and the voltage. This defines the power triangle

$$S^2 = P^2 + Q^2. \tag{17}$$

Alternatively, a standard method in electrical engineering is to represent (15)–(16) by their time-harmonic *phasors* [5]

$$\underline{u}(t) = \underline{U}e^{j\omega t}, \quad \underline{i}(t) = \underline{I}e^{j\omega t}, \tag{18}$$

with $j^2 = -1$, $\underline{U} = U\sqrt{2}e^{j\phi_u}$, and $\underline{I} = I\sqrt{2}e^{j\phi_i}$. This enables one to define the *complex power*

$$\underline{S} = \frac{1}{2}\underline{u}(t)\underline{i}^*(t) = UIe^{j\phi} = P + jQ, \tag{19}$$

with its magnitude equal to the apparent power, i.e., $|\underline{S}| = S$.

Furthermore, the complex counterpart of (11) is given by

$$\begin{aligned}\frac{1}{2}\sum_b \underline{u_b}(t)\underline{i_b^*}(t) &= \frac{1}{2}\sum_{\text{res}} R_b\underline{i_b}(t)\underline{i_b^*}(t)\\ &+ \frac{1}{2}\sum_{\text{ind}} L_b\underline{i_b'}(t)\underline{i_b^*}(t) + \frac{1}{2}\sum_{\text{cap}} \underline{u_b}(t)C_b\left(\underline{u_b'}(t)\right)^*,\end{aligned} \tag{20}$$

which, upon substitution of $\underline{u_b}(t) = \underline{U_b}e^{j\omega t}$ and $\underline{i_b}(t) = \underline{I_b}e^{j\omega t}$, yields the well-know expression [5]

$$\underline{S} = \sum_{\text{res}} R_b I_b^2 + j2\omega\left[\frac{1}{2}\sum_{\text{ind}} L_b I_b^2 - \frac{1}{2}\sum_{\text{cap}} C_b U_b^2\right]. \tag{21}$$

Comparing the latter with (19) reveals that $P = P_d$ with

$$P_d = \sum_{\text{res}} R_b I_b^2,$$

whereas, setting

$$W_m = \frac{1}{2}\sum_{\text{ind}} L_b I_b^2, \quad W_e = \frac{1}{2}\sum_{\text{cap}} C_b U_b^2,$$

yields

$$Q = 2\omega\left[W_m - W_e\right], \tag{22}$$

where W_m and W_e are the mean values of $w_m(t)$ and $w_e(t)$ as given in (13).

Analytic Signals and the Hilbert Transform

The underlying mathematical principle behind the transition from the sinusoidal time functions (15)–(16) to their time-harmonic phasor representation (18) is the *analytic signal*, widely used in telecommunication and signal processing [18]. For an *arbitrary* real voltage $u(t)$ with frequency spectrum $\tilde{U}(\omega)$, the analytic signal representation is defined as the *positive-frequency* part of the inverse Fourier transform:

$$\underline{u}(t) = \frac{1}{\pi}\int_0^{\infty} \tilde{U}(\omega)e^{j\omega t}\,\mathrm{d}\omega = u(t) + j\hat{u}(t), \tag{23}$$

in which $\hat{u}(t)$ denotes the Hilbert transform of $u(t)$, given by

$$\hat{u}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{u(\zeta)}{t-\zeta}\,\mathrm{d}\zeta, \tag{24}$$

where the Cauchy principal value of the integral is implied.

From standard complex analysis we know that

$$\underline{u}(t) = U(t)\sqrt{2}e^{j\alpha(t)}, \tag{25}$$

where $U(t) = \frac{1}{2}\sqrt{2}|\underline{u}(t)|$ and $\alpha(t) = \arg\{\underline{u}(t)\}$. Thus, for nonsinusoidal waveforms, the analytic voltage signal (25) defines a *generalized phasor* that rotates at a velocity

$$\omega_\alpha(t) = \alpha'(t) = \frac{\hat{u}'(t)u(t) - u'(t)\hat{u}(t)}{u^2(t) + \hat{u}^2(t)}, \tag{26}$$

and has a time-varying magnitude $|\underline{u}(t)|$. Furthermore, observe that the original voltage signal can be recovered from the real part of (23) or (25), i.e., $u(t) = \mathrm{Re}\{\underline{u}(t)\}$.

The angular velocity (26) is often referred to as the *instantaneous frequency*. However, it is important to emphasize that in spite of both being measured in radians per second, harmonic and instantaneous frequency are *different* concepts, which only coincide in the sinusoidal case. Indeed, for a voltage of the form (15), the time-varying angle reduces to $\alpha(t) = \omega t + \phi_u$, and thus $\omega_\alpha(t) \equiv \omega$. See, e.g., [18] for further information.

In a similar fashion, the analytic signal representation of the current takes the form

$$\underline{i}(t) = I(t)\sqrt{2}e^{j\beta(t)}. \tag{27}$$

with $I(t) = \frac{1}{2}\sqrt{2}|\underline{i}(t)|$ and $\beta(t) = \arg\{\underline{i}(t)\}$. The associated angular velocity equals $\omega_\beta(t) = \beta'(t)$.

A graphical representation of the analytic voltage and current signals is depicted in Fig. 2.

Time-Varying Complex Power

Starting from the analytical port voltage and current, (25) and (27), the time-domain nonsinusoidal equivalent of (19) is defined by the time-varying complex power

$$\underline{S}(t) = U(t)I(t)e^{j\varphi(t)} = P(t) + jQ(t), \tag{28}$$

where $\varphi(t) = \alpha(t) - \beta(t)$ denotes the instant phase difference between the time-varying phasors $\underline{u}(t)$ and $\underline{i}(t)$, and

$$P(t) = \frac{1}{2}\left(u(t)i(t) + \hat{u}(t)\hat{i}(t)\right), \tag{29}$$

$$Q(t) = \frac{1}{2}\left(\hat{u}(t)i(t) - u(t)\hat{i}(t)\right), \tag{30}$$

or, equivalently,

$$P(t) = U(t)I(t)\cos\left(\varphi(t)\right), \quad Q(t) = U(t)I(t)\sin\left(\varphi(t)\right),$$

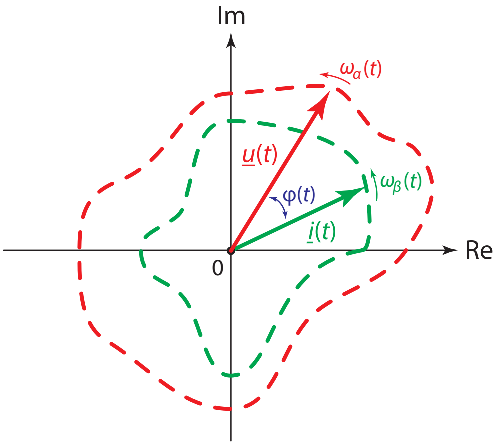

Fig. 2. Analytic signal representation of (periodic) nonsinusoidal voltage and current waveforms. At every time instant, $\varphi(t)$ provides the phase difference between $\underline{u}(t)$ and $\underline{i}(t)$. In the special case that the waveforms are sinusoidal, both the voltage and current phasors have fixed magnitudes and uniformly rotate at a constant angular velocity equal to the harmonic frequency, i.e., $\omega_\alpha(t) = \omega_\beta(t) = \omega$, and $\varphi(t) = \phi$ is constant.

represent the time-varying *real* and *imaginary* power, respectively. Furthermore, the *time-varying apparent power* equals $S(t) = |\underline{S}(t)|$, or, equivalently,

$$S^2(t) = P^2(t) + Q^2(t), \tag{31}$$

which, in turn, naturally suggests the definition of a time-varying power triangle as depicted in Fig. 3.

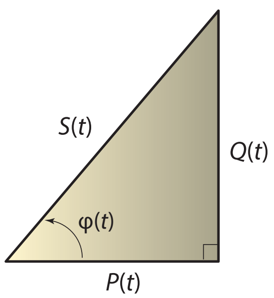

Fig. 3. The time-varying power triangle. For nonsinusoidal waveforms the triangle is generally expanding and contracting at an angular velocity $\omega_\varphi(t) = \varphi'(t)$. In the special case that the waveforms are sinusoidal, the power triangle reduces to (17) with $\varphi(t) = \phi$ and $\omega_\varphi(t) = 0$.

Budeanu's Reactive Power Represents an Average

We are now in position to critically address the assertions A1–A3. For that, let us consider a single-phase LTI system with distorted voltage and current waveforms of the form (6) and (7), respectively.

First of all, since $\langle u, i\rangle = \langle \hat{u}, \hat{i}\rangle$, it is readily observed that the active power under periodic nonsinusoidal conditions (1) is obtained from (29) after averaging over a period, i.e.,

$$P_A \equiv \frac{1}{T}\int_0^T P(t)\mathrm{d}t. \tag{32}$$

It is shown in [13] that Budeanu's reactive power (2) can be expressed in the time-domain using the Hilbert transform (24) as

$$Q_B = \langle \hat{u}, i\rangle = \frac{1}{T}\int_0^T \hat{u}(t)i(t)\mathrm{d}t.$$

Hence, using the fact that $\langle \hat{u}, i\rangle = -\langle u, \hat{i}\rangle$, it is clear that this is equivalent to averaging (30) over a period, i.e.,

$$\boxed{Q_B \equiv \frac{1}{T}\int_0^T Q(t)\mathrm{d}t.} \tag{33}$$

Thus, Q_B represents an *average* quantity! It is equal to the mean value of the imaginary power $Q(t)$ in a fashion similar to the active power P_A, which represents the mean value of the real power $P(t)$. This shows that assertion A1 concerning Q_B is not valid. It is indeed not suitable for (optimal) compensation purposes as properly forcing $Q_B \equiv 0$ by compensating for the Budeanu reactive current (8) only compensates the mean value of the imaginary power. Consequently, there still might be a fluctuating part of the imaginary power present that contributes to the degradation of the power factor.

The Physics Behind Budeanu's Reactive Power

For arbitrary waveforms, substitution of the analytic voltage and current signals (25) and (27) into (20) yields the time-varying counterpart of (21), i.e.,

$$\underline{S}(t) = \sum_{\text{res}} R_b I_b^2(t) + \sum_{\text{ind}} W'_{m_b}(t) + \sum_{\text{cap}} W'_{e_b}(t) + j2\left[\sum_{\text{ind}} \omega_{\beta_b}(t) W_{m_b}(t) - \sum_{\text{cap}} \omega_{\alpha_b}(t) W_{e_b}(t)\right],$$

with $\omega_{\alpha_b}(t) = \alpha'_b(t)$ and $\omega_{\beta_b}(t) = \beta'_b(t)$ the angular velocities associated to the branch voltages and currents, respectively, and

$$W_{m_b}(t) = \frac{1}{2}L_b I_b^2(t), \quad W_{e_b}(t) = \frac{1}{2}C_b U_b^2(t),$$

represent the *time-averaged* magnetic and electric energy associated to each branch of the load network, respectively.

Hence, using (33), we have that

$$\boxed{Q_B \equiv \frac{2}{T}\int_0^T \left[\sum_{\text{ind}} \omega_{\beta_b}(t) W_{m_b}(t) - \sum_{\text{cap}} \omega_{\alpha_b}(t) W_{e_b}(t)\right]\mathrm{d}t,} \tag{34}$$

which clearly shows that Budeanu's reactive power *is* related to (the difference of) magnetic and electric energy storage but only in an average sense. Clearly, in the sinusoidal case (34) reduces to (22). This also renders assertion A2 invalid.

Power Fluctuations Cause Distortion Power

It is correctly observed in [4] that Budeanu's concept of distortion power D_B is not directly related to waveform distortion of the port voltages and currents itself. It may, as the name already suggests, be related to the fluctuations of the real and imaginary power around their mean values (32) and (33), respectively. The purpose of this section is to demonstrate that the norms of these fluctuations can be naturally interpreted as distortion powers, which undermines assertion A1 concerning D_B and at the same time relaxes assertion A3.

Let $D_P(t) = P(t) - P_A$ and $D_Q(t) = Q(t) - Q_B$ represent the power fluctuations around the active and reactive powers P_A and Q_B, respectively. Furthermore, let

$$I_P(t) = I(t)\cos\big(\varphi(t)\big) \text{ and } I_Q(t) = I(t)\sin\big(\varphi(t)\big),$$

then it is easily shown that $\langle I_P, I_Q\rangle = 0$, i.e., the currents $I_P(t)$ and $I_Q(t)$ are mutually orthogonal. Hence, the 'normed' apparent power can be decomposed into two (mutually orthogonal) components:

$$\|U\|^2\|I\|^2 = \|U\|^2\|I_P\|^2 + \|U\|^2\|I_Q\|^2,$$

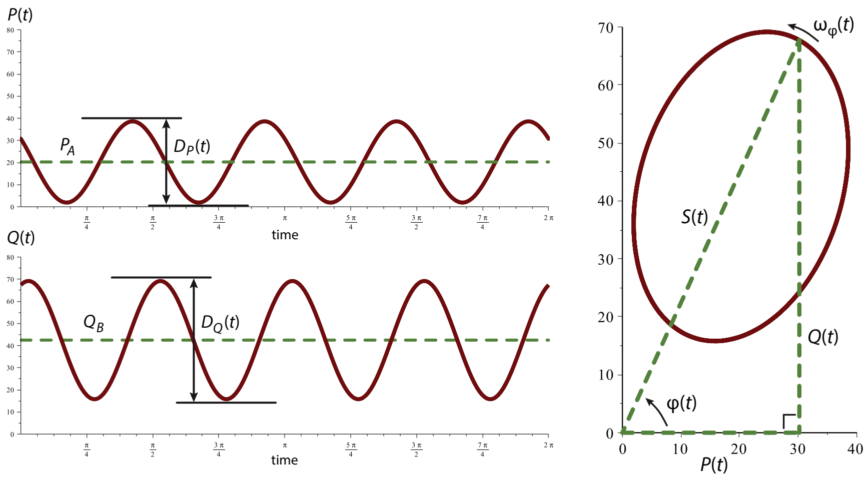

Fig. 4. Fluctuation of the time-varying real and imaginary powers around the active power and Budeanu's reactive power (left), and the associated time-varying power triangle (right).

which, in turn, suggest

$$\|U\|\,\|I_P\| \geq |\langle U, I_P\rangle| \equiv |P_A|, \quad \|U\|\,\|I_Q\| \geq |\langle U, I_Q\rangle| \equiv |Q_B|.$$

If $\|U\|\,\|I_P\| > |\langle U, I_P\rangle|$, the residual is given by

$$D_{P_U}^2 = \|U\|^2\|I_P\|^2 - \langle U, I_P\rangle^2$$
$$= \frac{1}{2T^2}\int_0^T\int_0^T \big(U(s)I_P(t) - U(t)I_P(s)\big)^2 \mathrm{d}s\mathrm{d}t.$$

Similarly, if $\|U\|\,\|I_Q\| > |\langle U, I_Q\rangle|$, we have

$$D_{Q_U}^2 = \|U\|^2\|I_Q\|^2 - \langle U, I_Q\rangle^2$$
$$= \frac{1}{2T^2}\int_0^T\int_0^T \big(U(s)I_Q(t) - U(t)I_Q(s)\big)^2 \mathrm{d}s\mathrm{d}t.$$

This naturally suggest the decomposition of distortion power into two (mutually orthogonal) components:

$$D_B^2 = D_{P_U}^2 + D_{Q_U}^2, \tag{35}$$

where D_{P_U} and D_{Q_U} can be considered as a measure of the fluctuation (distortion) around the active power and Budeanu's reactive power, respectively, relative to the voltage magnitude. Hence, we have

$$\|U\|^2\|I\|^2 = P_A^2 + Q_B^2 + D_B^2 = P_A^2 + D_{P_U}^2 + Q_B^2 + D_{Q_U}^2. \tag{36}$$

In the sinusoidal case, $D_{P_U} = D_{Q_U} = 0$, and (36) reduces to the standard (static) power triangle.

On the other hand, an equally valid starting point would be by selecting instead of $I_P(t)$ and $I_Q(t)$, the voltages

$$U_P(t) = U(t)\cos\big(\varphi(t)\big) \text{ and } U_Q(t) = U(t)\sin\big(\varphi(t)\big).$$

This suggest to decompose the 'normed' apparent power as

$$\|U\|^2\|I\|^2 = \|U_P\|^2\|I\|^2 + \|U_Q\|^2\|I\|^2,$$

and, in a similar fashion as before, gives rise to the distortion powers, D_{P_I} and D_{Q_I}, relative to the current magnitude, and satisfying

$$D_B^2 = D_{P_I}^2 + D_{Q_I}^2. \tag{37}$$

Note that, in general, $D_{P_U} \neq D_{P_I}$ and $D_{Q_U} \neq D_{Q_I}$.

The possible difference between the two decompositions will be exemplified in the following subsection.

Illustrative Example

Consider again the uncompensated ($C = 0$) RL circuit of Fig. 1(a) supplied by the nonsinusoidal voltage (5). The complex power reads

$$\underline{S}(t) = RI^2(t) + LI'(t)I(t) + j\omega_\beta(t)LI^2(t).$$

The waveforms for $P(t)$ and $Q(t)$ are depicted in Fig. 4. Their mean values are given by the active power

$$P_A = \frac{1}{T}\int_0^T \left[RI^2(t) + W_m'(t)\right]\mathrm{d}t = 20.248 \text{ W},$$

and Budeanu's reactive power

$$Q_B = \frac{1}{T}\int_0^T 2\omega_\beta(t)W_m(t)\mathrm{d}t = 42.475 \text{ VAr},$$

with $W_m(t) = \frac{1}{2}LI^2(t)$ the time-averaged magnetic energy.

The power fluctuations $D_P(t)$ and $D_Q(t)$ are also indicated in Fig. 4, together with the associated time-varying power triangle, which is expanding and contracting with the angular velocity $\omega_\varphi(t) = \varphi'(t)$. Since for this particular example the same current is flowing through both the resistor and the inductor, the 'normed' apparent power can be written as

$$\|U\|^2\|I\|^2 = \|RI + LI'\|^2\|I\|^2 + \|\omega_\beta LI\|^2\|I\|^2.$$

It seems therefore most natural to consider the distortion power relative to the port current magnitudes. Indeed, the fluctuation around the active power $P_A = R\|I\|^2$ is caused by the rate of change of $W_m(t)$, i.e., $W_m'(t) = LI'(t)I(t)$. This rate is due to the variation of the current magnitudes and must come from real power, thus causing the fluctuation of $D_P(t)$ for which the distortion power $D_{P_I} = \|U_P\|\,\|I\|$ applies, with $\|U_P\| = \|LI'\|$. The distortion power associated with the fluctuation $D_Q(t)$ equals $D_{Q_I} = \|U_Q\|\,\|I\|$, with $\|U_Q\| = \|\omega_\beta LI\|$. The values of the distortion power, including the alternative decomposition relative to the voltage magnitudes, are reported in Table 2.

Table 2. Distortion power decomposition.

D_B	D_{P_U}	D_{Q_U}	D_{P_I}	D_{Q_I}
17.799	13.245	11.891	12.664	12.508

Concluding Remarks

Starting from the work of Willems [19], further new perspectives are presented that reveal the physical meaning of Budeanu's concept of reactive and distortion power. For linear and time-invariant load networks it is demonstrated, using the Hilbert transform and the associated analytic signal representation, that Budeanu's reactive power can be related to energy oscillations, but only in an average sense. Furthermore, the distortion power is decomposed into a part representing a measure of the fluctuation of power around the active power and a part that represents the fluctuation of power around Budeanu's reactive power. This strongly relaxes the assertions A1–A3 of [4] concerning Budeanu's concept of reactive and distortion power.

The analysis presented in [19] critically addressess the assertions A4–A6 concerning compensation of Budeanu's reactive power in [4], and show that the only appropriate way to render Budeanu's reactive power to zero is by compensating the associated Budeanu current. In connection to [19], it is shown in the present paper that despite the fact that compensation based on the Budeanu current – if it exists – always leads to an improvement of the power factor without altering the distortion power, it may not lead to optimal results as power fluctuations around the average powers may still exist and their compensation using passive filters (so far) seems to be not trivial from a time-domain perspective. On the other hand, based on the approach of [15], the power fluctuations can be compensated using an active filter.

In conclusion, Budeanu's power model has its merits especially in the context of instrumentation and measurements. Budeanu's reactive power indicates if the load exhibits a dominantly inductive character ($Q_B > 0$) or a dominantly capacitive character ($Q_B < 0$). The situation that $Q_B = 0$ simply indicates that the load is neutral from a net perspective. The distortion power provides a measure of the power fluctuations around the active power and Budeanu's reactive power.

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REFERENCES

[1] G. Benysek and M. Pasko (editors). *Power Theories for Improved Power Quality*. Springer-Verlag London, 2012.
[2] C. I. Budeanu. *Puissances réactives et fictives*. Inst. Romain de l'Energie, Bucharest, Romania, 1927.
[3] G. Chateigner, M. Boes, J. Chopin, and D. Verkindère. Puissances, facteur de puissance et théorème de Boucherot. *Technolgie*, 158, Novembre-Décembre 2008.
[4] L. S. Czarnecki. What is wrong with the Budeanu concept of reactive and distortion power and why it should be abandoned. *IEEE Trans. Instr. Meas.*, 36(3):834–837, 1987.
[5] C. A. Desoer and E. S. Kuh. *Basic Circuit Theory*. McGraw-Hill Book company, 1969.
[6] A. E. Emanuel. *Power Definitions and the Physical Mechanism of Power Flow*. Wiley-IEEE Press, 2010.
[7] S. Fryze. Wirk-, blind, und scheinleistung in elektrischen stromkreisen mit nichtsinusoidalem verlauf von strom und spannung (part i, ii, and iii). *Elektr. Z.*, 53:596–599, 625–627, 700–702, 1932.
[8] M. T. Hartman. Orthogonality of functions describing power states in electrical circuits in Budeanu's concept. *In proc. International School on Nonsinusoidal Currents and Compensation (ISNCC)*, pages 127–131, 2010.
[9] IEEE. Standard 1459-2000. *IEEE Power and Energy Society*.
[10] IEEE. Standard 1459-2010. *IEEE Power and Energy Society*.
[11] M. Iliovici. Définition et mesure de la puissance et de l'energie réactives. *Bull. Soc. Franc. Electr.*, (5):931–954, 1925.
[12] H. Lev-Ari and A. M. Stanković. A decomposition of apparent power in polyphase unbalanced networks in nonsinusoidal operation. *IEEE Trans. Power Sys.*, 21(1):438–440, 2006.
[13] Z. Nowomiejski. Generalized theory of electric power. *Archiv für Electrotechnik*, 63:177–182, 1981.
[14] P. Penfield Jr., R. Spence, and S. Duinker. *Tellegen's Theorem and Electrical Networks*. Research Monograph No. 58, MIT Press, 1970.
[15] M. Saitou and T. Shimizu. Generalized theory of instantaneous active and reactive powers in single-phase circuits based on Hilbert transform. *in proc. 33rd Power Elec. Spec. Conf. (PESC)*, 3:1419–1424, 2002.
[16] W. Shepherd and P. Zand. *Energy Flow and Power Factor in Nonsinusoidal Circuits*. Cambridge University Press, 1979.
[17] C. P. Steinmetz. *Theory and Calculation of Alternating Current Phenomena (3rd Edition)*. Electrical World and Engineer, New York, 1900.
[18] D. Vakman. *Signals, Oscillations, and Waves. A Modern Approach*. Artech House Inc., 1998.
[19] J. L. Willems. Budeanu's reactive power and related concepts revisited. *IEEE Trans. Instr. Meas.*, 60(4):1182–1186, 2011.

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