# Preliminaries, Profiling Tools and Measuring Execution time of Programs

# **Objectives**.

- Profile C/C++ programs using gprof profiler (Linux)
- Profile C/C++ programs using perf tool (Linux)
- Profile C/C++ programs using Valgrind tool (Linux)
- Profile C/C++ programs using Intel Vtune tool (supported by all Operating systems)
- Learn how to compile and run C programs using Linux Terminal
- Familiarize yourselves with pointers in C
- To use accurate timers to measure the execution time of code blocks

## Aim

The main **aim** of this lab session is to learn how to profile C/C++ applications. In this session, we will be using Linux as there are several well-known and well-used free tools such as gprof and Valgrind.

# Section1 – Preliminaries

## How to Compile and run a C/C++ program in Linux.

In this lab session we will be using gedit text editor to write our programs and the Linux Terminal to compile and run our programs. Please note that there are more efficient programming environments to develop software such as Eclipse (it is free), but for small programs, this is not necessary.

You can compile a .c file using the following command:

#### gcc source.c -o exec -other\_optional\_options

gcc is a well-known and used compiler. The executable name is specified just after the '-o' ('o' stands for output). The rest options are optional.

Then, you can run the executable by using the following command:

./exec

## Passing Input arguments – Argc, Argv

So far, all the programs we have written can be run with a single command. For example, if we compile an executable called exec, we can run it from within the same directory with the following command at the GNU/Linux command line:

./exec

However, what if you want to pass information from the command line to the program you are running?

Up until now, the skeletons we have used for our C programs have looked something like this:

#### int main() { }

From now on, our examples may look a bit more like this:

#### int main (int argc, char \*argv[]) { }

As you can see, main now has arguments. The name of the variable argc stands for "argument count"; argc contains the number of arguments passed to the program. The name of the variable argv stands for "argument vector". A vector is a one-dimensional array, and argv is a one-dimensional array of strings. Each string is one of the arguments that was passed to the program.

Compile the input\_arguments.c file using the following command gcc input\_arguments.c -o exec. Then run the program normally, but add some parameters too :

```
./exec arg1 arg2
```

The output will be

Task1. Study input\_arguments.c program. Make sure you understand what this program does

### **Using Pointers**

#### Using Pointers on 1-d arrays

Every variable is stored into a memory location and every memory location has a memory address. A memory address can be accessed by using the ampersand (&) operator. Consider the following example

#include <stdio.h>
int main () {
 int var1=4;
 printf("The memory address of %p contains %d \n", &var1, var1 );

```
return 0;
}
```

The code above prints:

#### The memory address of 0x7ffdd32f4b9c contains 4.

Keep in mind that the memory address that var1 is stored might be different every time you compile the program. To print the memory address of a variable, the '%p' format specifier is used. '%p' is a format specifier which is used if we want to print data of type (void \*) or in simple words address of pointer or any other variable. The output is displayed in hexadecimal value. However, the memory addresses can be printed in integer format too, if we use '%d' instead of '%p' ; in this case, a warning will be shown 'warning: format '%d' expects argument of type (int', but argument 4 has type 'int \*', which can be ignored.

#### Task2. Study 'pointers\_1d\_array\_initialization.c' file.

This file has a routine that initializes a one-dimensional array and three routines that print the array's values, in three different ways, which are explained below. Please note that the array elements are always stored in consecutive memory locations (always).

- 1. printf("\n element %d equals to %d",i, A[i]);
- printf("\n element %d equals to %d",i, \*(A+i) );
- printf("\n element %d equals to %d",i, \*(ptr+i) );

The three bullets above are equivalent. What does \*(A+i) mean? A is an array, thus A is the memory address of the first array element. (A+i) is the memory address of the ith array element. \* refers to contents of memory address. Thus, \*(A+i) means : give me the content (value) of the ith array element. The same holds for the 3<sup>rd</sup> bullet as '*ptr=&A[0]*', which means that the pointer shows to the memory address of A[0].

Compile and run the program using different 'N' values, to make sure that all the three routines print the right values.

In the C Programming Language, the '*#define*' directive allows the definition of macros within your source code. This macro definition allows a constant value to be declared for use throughout your code. Macro definitions are not variables and cannot be changed by your program code like variables.

Keep in mind that in C language, all the routines must be declared before main function.

It is always a good practice to assign a NULL value to a pointer variable in case you do not have an exact address to be assigned. A pointer that is assigned NULL is called a null pointer. This is done by using

#### Using Pointers on 2d arrays

Task3. Extend the program in 'pointers\_1d\_array\_initialization.c' file to 2-d arrays

Make the one-dimensional array, two-dimensional and modify the four routines appropriately. Make sure your program prints the right values. Rename the new file as 'pointers\_2d\_array\_initialization.c'. In C language, the array's elements are always stored row-wise in memory; this means that first the elements of the first row are stored into memory, then the elements of the second row etc.

#### Task4. Print the array's memory addresses

The 'print\_array\_memory\_addresses.c' file prints the memory addresses of the 2d array in the above example (pointers\_2d\_array\_initialization.c). The memory addresses are printed in hex format. This is further explained in task1 above.

How many bytes are allocated for every element? The answer is 4bytes, as the data type is of typr 'int' which is four bytes. As you can observe, the array's elements are stored into consecutive memory locations. Change the data type of the array from 'int' to 'short int' and then into 'long int'. Are the memory addresses different now? Why? Given that 'short int' occupies 2 bytes, each array element needs 2 bytes.

# **Further Reading**

If you want to learn more about C programming you can study the following links:

- 1) Tim Bailey, 2005, An Introduction to the C Programming Language and Software Design, available at: <u>http://www-personal.acfr.usyd.edu.au/tbailey/ctext/ctext.pdf</u>
- 2) C examples, Programiz, available at <u>https://www.programiz.com/c-programming/examples</u>
- 3) C Programming examples with Output, Beginners book, available at https://beginnersbook.com/2015/02/simple-c-programs/
- 4) C Programming Tutorial, from tutorialspoint.com, available at <u>https://www.unf.edu/~wkloster/2220/ppts/cprogramming\_tutorial.pdf</u>

# Section 2 – Profiling Tools

## **Gprof profiler (Linux)**

Gprof profiler is a tool which collects statistics on programs. It works by inserting appropriate code in the beginning and in the end of each function so as to collect information about the execution time. Gprof is not a debugger, so make sure your program is working. Gprof does not work for parallel programs.

Type the following command on terminal to see the manual of gprof

man gprof

#### How to use gprof:

**Step1:** Compile using '-pg' option. This option adds extra code to the generated binary so as gprof can report detailed timing statistics.

**Step2:** Run the program normally

**Step3:** Type 'gprof executable\_name -other\_optional\_options'

**Store the gprof results into a file**: You can store the results into a file by adding the '>' character. The complete command is:

gprof executable\_name > results.txt

**Gprof options:** Gprof profiler supports a large number of options. To find more about them use the man command as above. An helpful option is '-b' that shows only the important information.

Task1: profile the benchmark.c file.

#### Understanding Gprof output:

The gprof output will look like Fig.1. The output is divided into two parts, i.e., Flat profile and Call graph.

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•				Terminal Help				
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	[1]	100.0	0.00	7.09	1/1	main [1]		
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$\sim$			0.01	0.01	1/1	writedata_vec_add [5]		
			0.01	0.00	1/1	vec_add [7] writedata MVM [9]		
•			0.00	0.00	1/1	initialize [11]		
			0.00	0.00	1/1	MVM [10]		
			0.04	3.50	1/1	main [1]		
	[2]	50.0	0.04	3.50		writedata_MMM [2]		
			3.50	0.00	1/1	MMM_default [3]		
			3.50	0.00	1/1	writedata_MMM [2]		
	[3]	49.4	3.50	0.00		MMM_default [3]		
			3.50	0.00	1/1	 main [1]		
	[4]	49.4	3.50	0.00		MMM [4]		
			0.01	0.01	1/1	 main [1]		
	[5]	0.3	0.01	0.01	1	writedata_vec_add [5]		
			0.01	0.00	1/1	vec_add_default [8]		
			0.01	0.00	1/1	writedata_MVM [9]		
	[6]	0.1	0.01	0.00		MVM_default [6]		
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			0.00	0.00	1/1	Macin [1]		

Fig.1 Gprof Output for benchmark.c program

#### Flat Profile:

- The first column (%) shows the execution time percentage of each function.
- The second column (cumulative) shows the sum of the execution time in seconds by this function and those listed above it.
- The third column (self) shows the number of seconds accounted for by this function alone.
- The fourth column (calls) shows the number of times this function was invoked, if this function is profiled, else blank.
- The fifth column shows the average number of milliseconds spent in this ms/call function per call, if this function is profiled, else blank.
- The sixth column shows the average number of milliseconds spent in this ms/call function and its descendents per call, if this function is profiled, else blank.
- The name column shows the name of the function.

**Call Graph**: The call graph describes the call tree of the program. Each entry in this table consists of several lines. The line with the index number at the left hand margin lists the current function. The lines above it list the functions that called this function, and the lines below it list the functions this one called.

- % time This is the percentage of the `total' time that was spent in this function and its children.
- self This is the total amount of time spent in this function.

- children This is the total amount of time propagated into this function by its children.
- called This is the number of times the function was called. If the function called itself recursively, the number only includes non-recursive calls, and is followed by a `+' and the number of recursive calls.

## Further Reading for gprof

1. GPROF Tutorial – How to use Linux GNU GCC Profiling Tool, available at <u>https://www.thegeekstuff.com/2012/08/gprof-tutorial/</u>

## Valgrind Tool (Linux)

The Valgrind tool suite provides a number of debugging and profiling tools that help you make your programs faster and more correct. A detailed desciption is found in Valgrind's webpage <a href="https://www.valgrind.org/docs/manual/quick-start.html#quick-start.intro">https://www.valgrind.org/docs/manual/quick-start.html#quick-start.intro</a> . You can download and install Valgring by just typing opening a terminal and typing 'sudo apt-get install valgrind'. Alternatively, you can download Valgrind from <a href="https://www.valgrind.org/downloads/?src=www.discoversdk.com">https://www.valgrind.org/downloads/?src=www.discoversdk.com</a> ; you can install it by following the steps: a) extract the compressed file downloaded, b) use terminal and go to the valgrind directory and type: './configure', then type 'make', then type 'make install'.

In this module, we will be using Cachegrind tool of Valgrind. Cachegrind simulates how your program interacts with a machine's cache hierarchy. It simulates a machine with independent first-level instruction and data caches (I1 and D1), backed by a unified second-level cache (L2). Note that modern PCs, normally have three levels of cache and in this case Cachegrind simulates the first-level and last-level of caches only. Therefore, Cachegrind always refers to the I1, D1 and LL (last-level) caches. For more information read this <u>https://valgrind.org/docs/manual/cg-manual.html</u>.

#### How to use Cachegrind:

**Step1**: compile using '-g' option

Step2: use the following command 'valgrind --tool=cachegrind ./executable'

Task1: use valgrind to simulate how the benchmark.c file interacts with cache memories.

The output will look like this:

```
==11586== | refs: 1,041,351,336
==11586== 11 misses:
                       1.108
==11586== LLi misses:
                       1,101
                        0.00%
==11586== I1 miss rate:
==11586== LLi miss rate:
                        0.00%
==11586==
==11586== D refs: 387,398,347 (244,246,437 rd + 143,151,910 wr)
==11586== D1 misses: 411,589 ( 160,745 rd + 250,844 wr)
==11586== LLd misses:
                       405,386 ( 154,589 rd + 250,797 wr)
==11586== D1 miss rate:
                        0.1% (
                                  0.1% +
                                             0.2%)
==11586== LLd miss rate:
                         0.1% ( 0.1% +
                                              0.2%)
```

==11586==			
==11586== LL refs:	412,697 ( 1	L61,853 rd +	250,844 wr)
==11586== LL misses:	406,487 (	155,690 rd	+ 250,797 wr)
==11586== LL miss rate:	0.0% (	0.0% +	0.2%)

- Irefs stands for Instruction references
- 11 misses stands for L1 instruction cache misses
- LLi misses stands for Last level cache instruction misses
- Drefs stands for data references
- D1 misses stands for L1 data cache misses
- *rd* stands for reads while *wr* stands for writes

**Detailed Profiling**: Every time we run Valgrind, a file is generated and stored into the working directory. This file name is 'cachegrind.out.12065', the last digits differ from simulation to simulation. To see a more detailed analysis use the following command:

cg\_annotate --auto=yes cachegrind.out.12065

For more annotate options see <u>https://www.cs.cmu.edu/afs/cs.cmu.edu/project/cmt-</u> <u>40/Nice/RuleRefinement/bin/valgrind-3.2.0/docs/html/cg-manual.html</u>.

## Further Reading for Valgrind:

1. The Valgrind Quick Start Guide, available at <u>https://www.valgrind.org/docs/manual/quick-start.html#quick-start.intro</u>

2. Cachegrind: a cache and branch-prediction, available at <u>https://valgrind.org/docs/manual/cg-</u>manual.html

## Perf Tool (Linux) - optional

The perf tool offers a rich set of commands to collect and analyze performance and trace data. A detailed description of perf can be found in <a href="https://perf.wiki.kernel.org/index.php/Tutorial">https://perf.wiki.kernel.org/index.php/Tutorial</a> . To take full advantage of Perf tool, native Linux is needed (not virtual machine)

How to use Perf: Perf includes many features. Some of them are listed below:

- perf stat ./executable : It shows CPU Performance statistics
- perf stat -d ./executable : It shows more detailed CPU Performance statistics
- *perf stat -d sleep 1 ./executable* : it shows detailed CPU Performance statistics, but by running the program only for 1 second
- *perf stat -d -C 0,2 ./executable* : it shows detailed CPU Performance statistics for the CPU core 0 and CPU core2.
- *perf stat -a ./p :* It shows CPU Performance statistics by using all the CPU cores

- *perf stat -d -e cycles ./executable* : shows detailed CPU Performance statistics including the number of CPU cycles by accessing the appopriate hardware counter.
- *perf stat -r 3 ./executable* : 'r' stand for repeat. -r 3, runs the program 3 times. It is possible to use perf stat to run the same program multiple times and get for each count, the standard deviation from the mean
- *perf list* : Lists all the options with regards to the hardware counters and events. The '-e' option allows us to monitor the hardware counters. You cannot display the values of the hardware counters by using a virtual machine.
- perf stat -d -C 0,1,2,3 -e cycles, instructions, ref-cycles, cpu-clock, cache-misses, cache-references, L1dcache-load-misses, L1-dcache-loads, L1-dcache-stores, L1-icache-load-misses, LLC-loadmisses, LLC-loads, LLC-store-misses, LLC-stores, cache-misses, cache-references, mem-loads, memstores ./executable : shows detailed statistics of the aforementioned hardware counters. Do not use space between two options
- perf stat -M Summary ./executable : in 'perf list' there are some sections such as Pipeline and Summary. To print those use -M option

#### Perf record and report:

- perf record -e cycles, instructions, ref-cycles, cpu-clock, cache-misses, cache-references, L1-dcache-load-misses, L1-dcache-loads, L1-dcache-stores, L1-icache-load-misses, LLC-load-misses, LLC-loads, LLC-store-misses, LLC-stores, cache-misses, cache-references, mem-loads, mem-stores
   ./executable : perf record collects samples which can then analyzed, possibly on another machine, using the perf report and perf annotate commands.
- perf report : By using the previous command (perf record), the samples collected are saved into a binary file called, by default, perf.data. The perf report command reads this file and generates a concise execution profile.

Note: you might need to open the following file and amend its value with 0, /proc/sys/kernel/perf\_event\_paranoid.

Task1: Use perf tool to profile the program in the benchmark.c file.

Keep in mind that the hardware counters are used by other processes too, not just by the program we are running.

## Further Reading for Perf

Tutorial, Linux kernel profiling with perf, available at <u>https://perf.wiki.kernel.org/index.php/Tutorial</u>
 perf Examples, available at <u>http://www.brendangregg.com/perf.html</u>

## Intel Vtune (Windows, Linux, Mac) - optional

Intel Vtune is perhaps the most powerful and user-friendly tool to optimize application performance. It can be used for Intel processors only. Note that it supports a graphical interface. You can download

Vtune here <u>https://software.intel.com/content/www/us/en/develop/tools/oneapi/base-toolkit/download.html</u>. A user guide is found in [1].

## Further Reading for Vtune

1. Get started with Intel Vtune, available at <u>https://software.intel.com/content/www/us/en/develop/documentation/get-started-with-</u><u>vtune/top/windows-os.html</u>

# Section 3 - Using Accurate Timers to Measure Execution Time

Below different ways of measuring the execution time of blocks of code are provided. Note that the aim of this section is to learn how to use the timers; not further investigate their implementation details.

#### Accurate Timer on Linux

The *clock\_gettime* system call allows us to measure the execution time of programs. This function uses the following struct.

```
struct timespec {
time_t tv_sec; /* seconds */
long tv_nsec; /* nanoseconds */
};
```

The *clock\_gettime* system call is used as follows:

#include <time.h>

int clock\_gettime(clockid\_t clk\_id, struct timespec \*tp);

The second parameter is the time structure that will be filled in by the system call to contain the value of the clock. The first parameter, clock ID, allows you to specify the clock you are interested in using; these can be found in this link <u>https://linux.die.net/man/3/clock\_gettime</u>.

**Task1**: Study the '*timers.c*' file and make sure you understand what it does. This example uses clock\_gettime using two different options, CLOCK\_MONOTONIC and CLOCK\_PROCESS\_CPUTIME\_ID.

For measuring elapsed time, CLOCK\_MONOTONIC is recommended. CLOCK\_MONOTONIC represents the absolute elapsed wall-clock time since some arbitrary, fixed point in the past.

The CLOCK\_PROCESS\_CPU\_TIME\_ID clock measures *only* the CPU time consumed by the process. If the kernel puts the process to sleep, the time it spends waiting is not counted. If a process has multiple threads, CLOCK\_THREAD\_CPUTIME\_ID is similar but measures only the CPU time spent on the thread that is making the request.

This example marks the start time by getting the value of CLOCK\_MONOTONIC. The process then sleeps for a second and marks the stop time by getting the value of CLOCK\_MONOTONIC a second time. It then does the same thing again but uses CLOCK\_PROCESS\_CPUTIME\_ID. Since the CPU is not used for the time that the process is sleeping, the results are substantially shorter.

#### Accurate Timer on Visual Studio

The most accurate timer is the '*high\_resolution\_clock*' which is supported in C++ only. Keep in mind that you can write C code inside a C++ file/template. You can use it as follows

```
#include <chrono>
int main() {
auto start = std::chrono::high_resolution_clock::now();
    routine();
auto finish = std::chrono::high_resolution_clock::now();
std::chrono::duration<double> elapsed = finish - start;
std::cout << "Elapsed time: " << elapsed.count() << " s\n";
...
}</pre>
```

#### Accurate Timer on both Linux and Visual Studio supported by OpenMP

In weeks 5-7 we will be using OpenMP framework which supports *omp\_get\_wtime()* routine. This works for both Linux and Visual Studio. In Linux you must compile using '-fopenmp'. You can use it as follows.

```
#include <omp.h>
int main(){
    double start,end;
```

start=omp\_get\_wtime();
 routine();
end=omp\_get\_wtime();

printf("Elapsed Time in seconds is %f",end-start);

}

...

## Section 4 - The Roofline Model

The roofline model [3] provides an easy way to get performance bounds for compute-bound and memory-bound loop kernels. It allows us to know how far the achieved performance is from the optimum. It is based on the concept of computational intensity, sometimes also called arithmetic or operational Intensity. The arithmetic intensity is given by the following formula: *'FP.arithmetical.instructions / number.of.bytes.loaded.stored'*. This model has several limitations [3], e.g., does not consider all features of modern processors and ignores integer computations.

**Roofline Model for MMM algorithm**: MMM has N<sup>3</sup> iterations and each iteration contains 4 Floating Point (FP) L/S operations and 2 FP arithmetical operations. So, the arithmetical intensity of MMM (the ratio between FP arithmetical operations and number of bytes loaded/stored), when the arrays are of type 'float', is 2/(4\*4bytes)=1/8.

MMM loop kernel contains integer arithmetical instructions too (we could generate the assembly and check them out by typing *gcc source.c -S assembly.s -O2*). The integer arithmetical instructions are responsible for a) computing the L/S memory addresses (e.g., ...=A[i][j] will be broken down to multiple assembly instructions), b) controlling the iterators (increment i, compare i to N, branch back); however, these integer operations take 1 CPU cycle each and they are performed in parallel with the FP ones. So, for this loop kernel we could assume that performance is not affected by the integer operations. Furthermore, the roofline model does not consider integer operations, and this is a serious limitation of this model.



Fig.4. The Roofline Model [3]

Algorithms that have a low arithmetical intensity are **memory-bound**, while algorithms that have a high arithmetical intensity are **compute-bound**. Memory-bound means that their performance is bounded on the memory latency and bandwidth values; in simple words, no matter how high the CPU frequency is, or no matter how many cores the CPU supports, performance depends on how fast the data are loaded/stored from/to memory.

The peak FP performance is the maximum we can get, and it refers to compute-bound loop kernels with a perfect balance between simple and complex FP operations. The latter is an advanced topic and it is not further explained here; for those who want to go deeper and learn more about it, ask the module leader. The peak FP performance is CPU dependent.

Tip. each instruction has a latency and a throughput value, where the latter one is always larger or equal to the first; thus, to achieve the optimum performance of a bunch of instructions, e.g., multiply instructions, multiple multiply instructions must be feed the instruction pipeline one after another.

The peak memory bandwidth depends on the DDR memory and memory controller hardware characteristics. Furthermore, if the data fit and remain in the cache, the peak memory bandwidth is the cache bandwidth. As a reminder, a DDR memory access takes about 100-200 CPU cycles, an L3 memory accesses about 40-70 CPU cycles, an L2 memory access 6-14 CPU cycles, while an L1 memory access takes about 1-4 CPU cycles.

So, if the peak memory bandwidth is 21GBytes/sec, then the maximum MMM performance will be  $21GB/sec \times (1/8)=2.65gigaflops$ . If the arrays fit in the precious cache memories, then the memory bandwidth is higher and thus performance is increased.

How can we improve the performance of memory-bound algorithms? The main strategies are as follows. Reducing the number of memory accesses through the whole memory hierarchy; this relates to reducing the number of cache-misses too. Another strategy is to use software prefetching. The above can be achieved by using code optimizations such as loop tiling, register blocking, array copying, loop merge/distribution etc. We will study those next week.

## **Further Reading:**

1. The Valgrind Quick Start Guide, available at https://www.valgrind.org/docs/manual/quickstart.html#quick-start.intro

2. Cachegrind: a cache and branch-prediction, available at https://valgrind.org/docs/manual/cgmanual.html

3. Samuel Williams, Andrew Waterman, and David Patterson. 2009. Roofline: an insightful visual performance model for multicore architectures. Commun. ACM 52, 4 (April 2009), 65-76. DOI=10.1145/1498765.1498785, available at

https://people.eecs.berkeley.edu/~kubitron/cs252/handouts/papers/RooflineVyNoYellow.pdf