A Deterministic Simulation Model for Sojourn Time in Urban Cells with rectangular geometry

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Abstract

A deterministic simulation model for the calculation of sojourn time statistics is proposed. The model is designed for urban cells taking into account city movement in street pattern, traffic lights and crossings. Sojourn time and remaining sojourn time are assumed to be random variables that follow a gamma distribution. The parameters of gamma are found for several possible parameter values using Monte Carlo simulation. Then, the values are fitted to the empirical model using genetic algorithms with grammatical evolution. The model can be used to calculate sojourn time statistical parameters for any square shaped or rectangular shaped cell.

Key words: Sojourn Time, Gamma distribution, Grammatical Evolution.

Introduction

Sojourn Time is defined as the time spent by a vehicle inside a cell. It was firstly defined for a handover call, in which the vehicle starts its journey at the edge of the cell. For a new call, where the vehicle starts from a random position inside the cell, the dwell time is called remaining sojourn time. These two random variables, obviously depend on characteristics of the cell area. In case of an urban cell, the important characteristics are the road pattern, the traffic lights, the crossings and the city traffic.

Sojourn time statistics have been investigated in the past due to its usability in teletraffic analysis and cell planning. In [1], a simple analytical model is presented where movement in a straigth line is considered. The results in [1] show an exponential-like behaviour. In [2], a more realistic approach is chosen where the vehicles move in a series of vectors and perform random stops in between. In this case, the result is a statistical model that describes a specific cell environment. An extention to [2] is found in [3], where the same statistical model is enhanced with more complex stochastic models to better describe velocity, street length and probabilities of turning. An analytical approach of the statistical model of [2] and [3] is found in [4]. However, applying these models in a real-life cell remains challenging. Other previous works, relative to the present, can be found in [5], [6] and [7].

The proposed model is a statistical model which can be seen as an expansion of [8]. Sojourn time is assumed to follow a gamma distribution as in [2], [3] [4] and [8]. The model connects an intuitive parameter set with the gamma distribution parameters. A map can be used to extract the model parameters for any cell that fulfills certain requirements, and then use them as inputs to the model. Measurements carried through in a real cell show a satisfactory model performance.

The mobility model is described in section 1 and the empirical formulas are given in section 2. A real cell study is found in section 3 where the model is compared to measurements and finally, the paper is concluded in section 4.

1 The Proposed Deterministic Simulation Model

In this section, the proposed model is described. The design process is analyzed and the necessary assumptions are explained.

1.1 Model design

The model process is described in figure 1. A intuitive parameter set is created and some assumptions are made. For every case of parameter set a Monte Carlo simulation is performed and the results are fitted to the gamma distribution. The gamma parammeters a and b are then extracted with a median regression method. After the simulation loop, $h_1$and h_2 data are available and they are used as inputs to a graph fitting procedure by means of a genetic algorithm with function evolution. The final model is an expression of a and b in terms of the parameter set.

The parameter set constitutes of the probability of traffic, p_{tr}, the average block area, A_b and the rectangle side ratio r_b. The first parameter is an indication of traffic lights or stop signs and it is calculated as in 1:

$$p_{tr} = \frac{N_{tr} + 0.5\,(N - N_{tr})}{N} \tag{1}$$

Where N_{tr} is the number of crossings with traffic lights and $N \geq N_{tr}$ is the number of total crossings in the cell. It is noted that using equation 1, the resulting model describes a cell where the movement is uniformly distributed among all roads. In section 3, a variation where major roads are given higher probability of usage is presented.

The average area of block, A_b, can be used to extract the length of the roads. In case of square blocks:

$$d = \sqrt{A_b} + R_W \tag{2}$$

Where d is the acme and R_W is the road width. In case of a cell with blocks of rectangular geometry and side ratio $r_b \geq 1$:

$$d_1 = \sqrt{r_b \cdot A_b} + R_W \tag{3}$$

$$d_2 = \sqrt{\frac{A_b}{r_b}} + R_W \tag{4}$$

Where d_1 is the long side and d_2 the short side of the rectangular block. The extraction of r_b is relatively easy when all the blocks in the cell are identical. In case of non identical blocks, an average value must be used resulting in a possible error.

1.2 Deterministic laws of mobility

The random walk used in the model is showcased in figure 2. The position vector $\bar{r}$ for step i will be:

$$\bar{r}_i = \bar{r}_0 + \sum_{k=1}^{i} \bar{m}_k \tag{5}$$

Where, $\bar{r}_0$ is the starting position, and $|\bar{m}_k|$ is the distance that a vehicle makes in step k. $|\bar{m}_k|$ is calculated by equations 2, 3 and 4. In every crossing the vehicle makes a turn $\varphi_i = \arccos\left(\frac{\bar{m}_{i-1}\bar{m}_i}{|\bar{m}_{i-1}||\bar{m}_i|}\right)$ which is a discrete random variable with $\left[0, \frac{\pi}{2}, -\frac{\pi}{2}\right]$ possible values. Assuming u_i to be an always positive

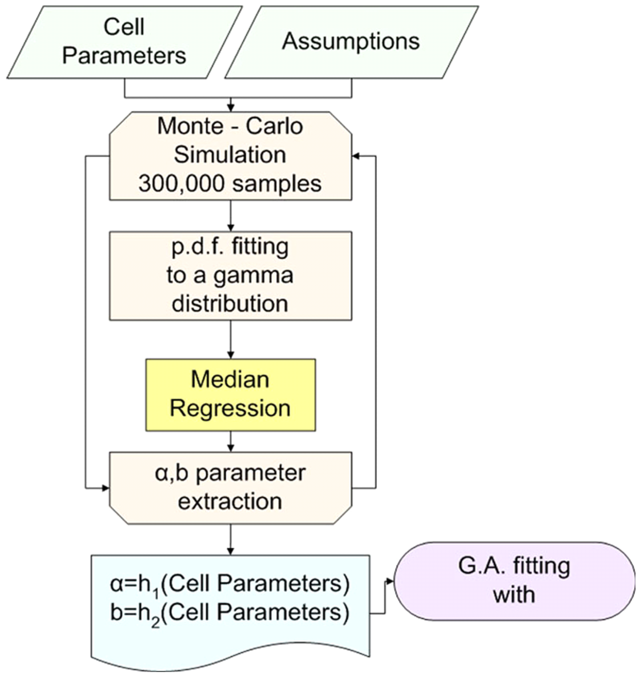

Fig. 1. Logical diagram of the model design procedure.

gaussian random variable that describes the average velocity of the vehicle in every step, the sojourn time for a random walk with n steps will be:

$$st = \sum_{i=1}^{n-1} \left(\frac{|\bar{m}_i|}{u_i} + \tau \cdot tr_i \right) - \lambda \cdot \left(\frac{|\bar{m}_n|}{u_n} + \tau \cdot tr_n \right) \tag{6}$$

Where tr_i is a binomial random variable with $p = p_{tr}$, τ is an average delay in a traffic light or a stop sign and λ is the percentage of the last step which is outside the cell area.

$$\lambda = \sqrt{1 + \frac{d_n^2}{|\bar{r}_n|} - \frac{d_n^2}{|\bar{r}_{n-1}|}} - 1 \tag{7}$$

Where d_n, is d_1 or d_2 depending on the orientation of the last step.

1.3 Assumptions

1.3.1 Velocity

Measurements in [3] have shown that average velocity in cities follows a semi-gaussian semi-rician distribution. In [8], however, it is shown that this behaviour is the result of the distortion of a gaussian velocity caused by the

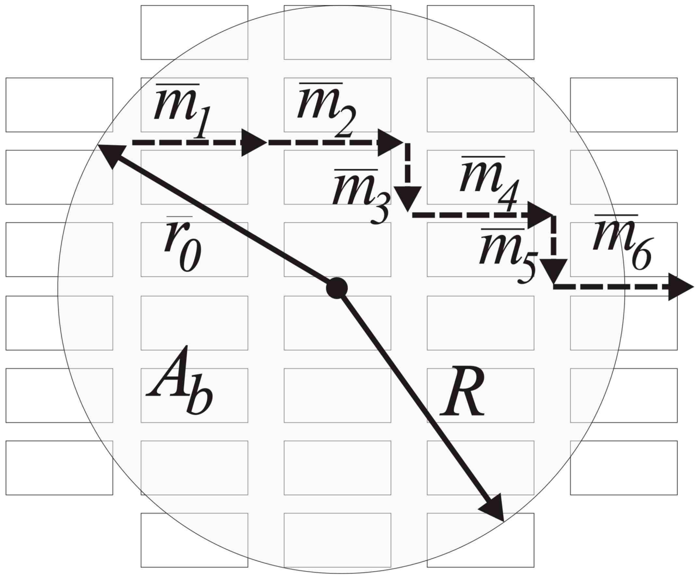

Fig. 2. A Random Walk example.

pause times. This means that, modelling the velocity as gaussian and including probabilistic pause times at crossings is equivalent to the modelling of [3].

Moreover, in the proposed model, average velocity in every step is modeled as half gaussian with fixed mean and deviation ($\bar{u}_i = 40kmh$, $\sigma_{u_i} = 10kmh$). This implies that velocity is not a main characteristic of sojourn time as opposed to other previous models. The justification of this lies in the reasoning that the behaviour of the driver is imposed by the traffic lights and the stop signs. The effect of traffic jams is not taken into account in this context. As a result, velocity considered a half gaussian random variable with fixed parameters which is distorted by the delays in the crossings.

1.3.2 Crossing Delay

The crossing delay is τ with probability p_{tr} and 0 with probability $1 - p_{tr}$ as described by the binomial random variable tr_i. This is a simple approach of modeling the delay in a traffic light or a stop sign. Delay is 0 in case of a priority road crossing. The value $\tau = 10\,\mathrm{sec}$ is typically used.

1.3.3 Turning in the crossings

The modeling of turns is very important for the model to yield realistic behaviour. In bibliography ([3] and [8]), it is proposed that the probable cases listed in descending order are: 0, $-\frac{\pi}{2}$ (right) and $\frac{\pi}{2}$ (left). Making a π turn is considered highly improbable (< 0.01). Moreover, conditional probabilities of turning are used to make the model more realistic. Turning left or right becomes very improbable after already having turned two times in that direction more than in the opposite. This makes sure that a driver with a chosen destination is simulated much more times than one that roams around the cell in random directions. If conditional probabilities are neglected, the resulting model yields larger values on the average. The values used in our model are: $p_0 = 0.7$, $p_{\frac{\pi}{2}} = 0.135$, $p_{-\frac{\pi}{2}} = 0.165$, $p'_0 = 0.9$, $p'_{\frac{\pi}{2}} = 0.4$, $p'_{-\frac{\pi}{2}} = 0.6$, where ' indicates the conditional probabilities.

1.3.4 Block shape

The block shape can affect the result of sojourn time. Specifically, three cases are investigated. The square pattern, the rectangular pattern with the priority roads positioned in the long side of the rectangle and the rectangular pattern with the priority roads positioned in the short side. The side irregularity is given by the side ratio r_b. Assuming that d_1 is in the direction of the priority roads, it is evident that $r_b = \frac{d_2}{d_1} = 1$, $r_b < 1$ and $r_b > 1$, for the three cases respectively.

1.4 Genetic Algorithm with Grammatical Evolution

The results of the Monte Carlo simulations are fitted to a function using a genetic algorithm with grammatical evolution. Grammatical Evolution is an evolutionary process that can create programs in an arbitrary language, [9]. The production is performed using a mapping process governed by a grammar expressed in Backus Naur Form. The problem of data fitting can be formulated as: Given M points and associated values (x_i, y_i), $i \in [1, M]$, with $x_i \in R^n$, estimate a function $f : R^n \to R$ that minimizes the least squares error.

$$E_T = \sum_{i=1}^{M} \left(f\left(x_i\right) - y_i\right)^2 \tag{8}$$

The specific method takes as input the points (x_i, y_i) and creates a functional form that minimizes the quantity in equation 8 through the procedure of Grammatical Evolution.

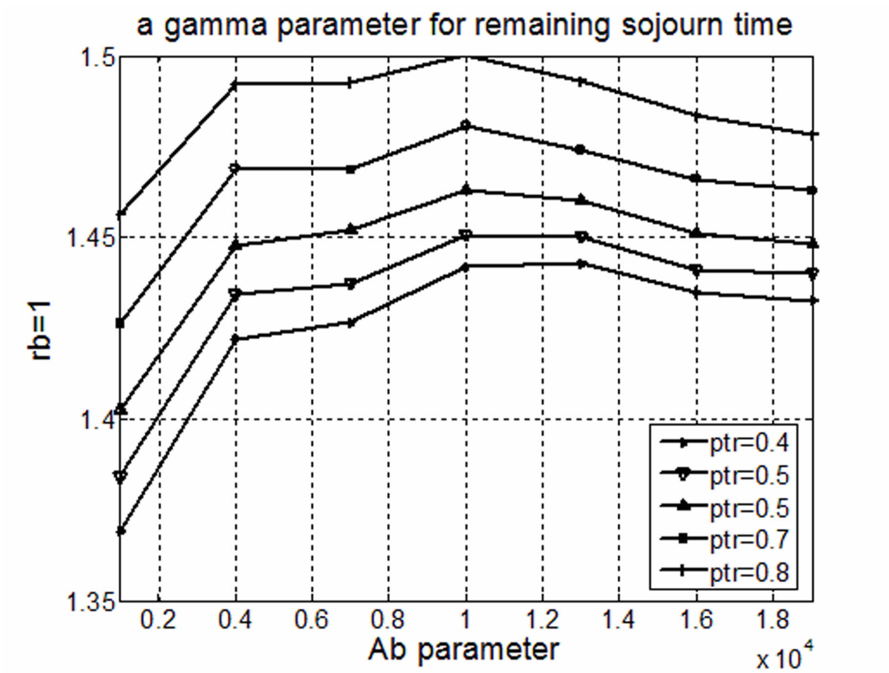

Fig. 3. The function $a_n = F_1\left(A_b, p_{tr}, r_b\right)$ for $r_b = 1$.

2 Results

The results are presented in figures 3-10. Only cases where $r_b = 1$ and $r_b = 2$ are presented. Parameter b has proven to be more stable than a. For the chosen range of parameters values, there is a clear dependance of a_n, b_n, a_h and b_h, on the parameter set.

Using the model of [9], the fit yields the following results:

$$F_1\left(A_b, p_{tr}, r_b\right) = 1.468134^{\cos\left[\frac{\ln(p_{tr})}{\log\left(\frac{A_b}{69.89(p_{tr}+\sin(p_{tr}))^{1.99}}\right)}\right]} \tag{9}$$

$$F_2\left(A_b, p_{tr}, r_b\right) = 41.1 + \tan\left\{A_b + \left|\cos\left[A_b - \frac{A_b^{p_{tr}}}{16001 p_{tr} + \frac{e^{7.4}}{\sin(16.9 p_{tr})}}\right]\right|\right\} \tag{10}$$

$$F_3\left(A_b, p_{tr}, r_b\right) = \ln\left\{\ln\left(A_b p_{tr}\right) \cdot \left[A_b p_{tr} - 811.12\left|p_{tr}\right| - 1.41 + \frac{A_b^{p_{tr}}}{2r_b - p_{tr}}\right]\right\} \tag{11}$$

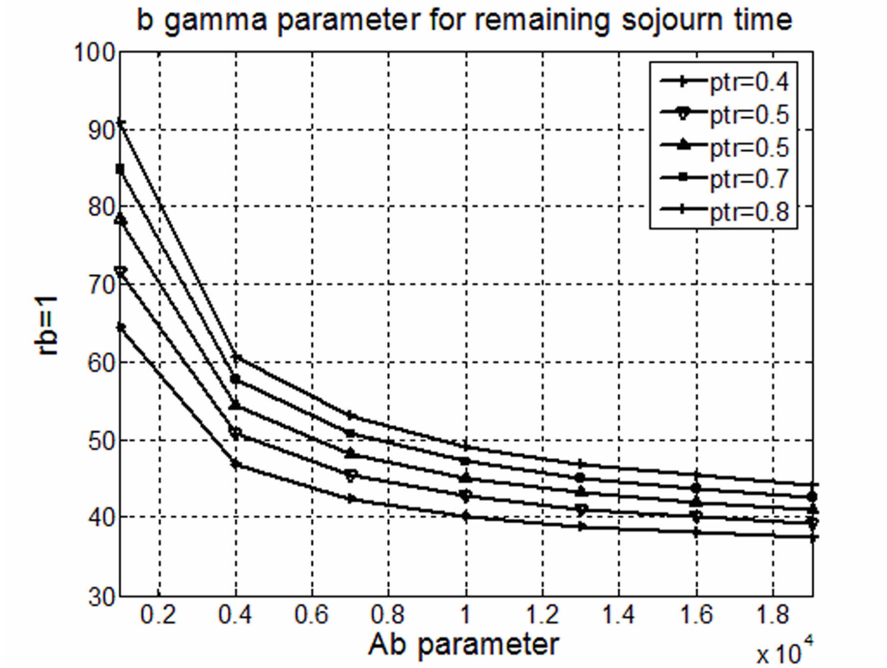

Fig. 4. The function $b_n = F_2\left(A_b, p_{tr}, r_b\right)$ for $r_b = 1$.

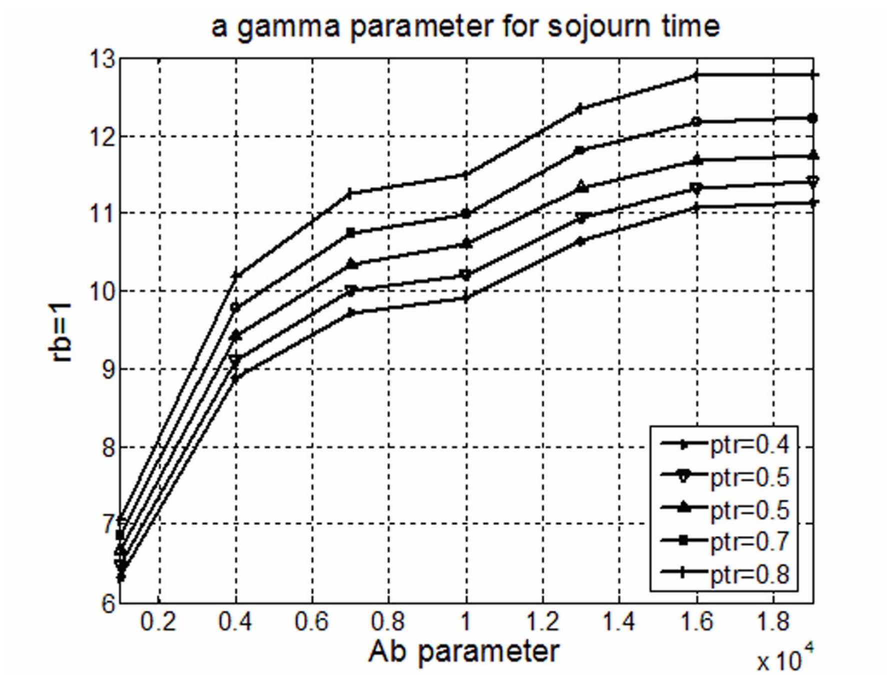

Fig. 5. The function $a_h = F_3\left(A_b, p_{tr}, r_b\right)$ for $r_b = 1$.

$$F_4\left(A_b, p_{tr}, r_b\right) = \frac{10.4 - \tan\left(A_b\right)\left|3.8^{p_{tr}}\right|}{\ln\left\{\left|\sin\left[A_b + \frac{\ln(r_b)}{7.257e^{r_b}}\right] + \sin\left[\frac{p_{tr}^{\ln(p_{tr})}}{A_b \sin(p_{tr})}\right]\right|\right\}} \tag{12}$$

The mean least square error of fitting was found to be: $e_1 = 0.0024$, $e_2 = 8.46$,

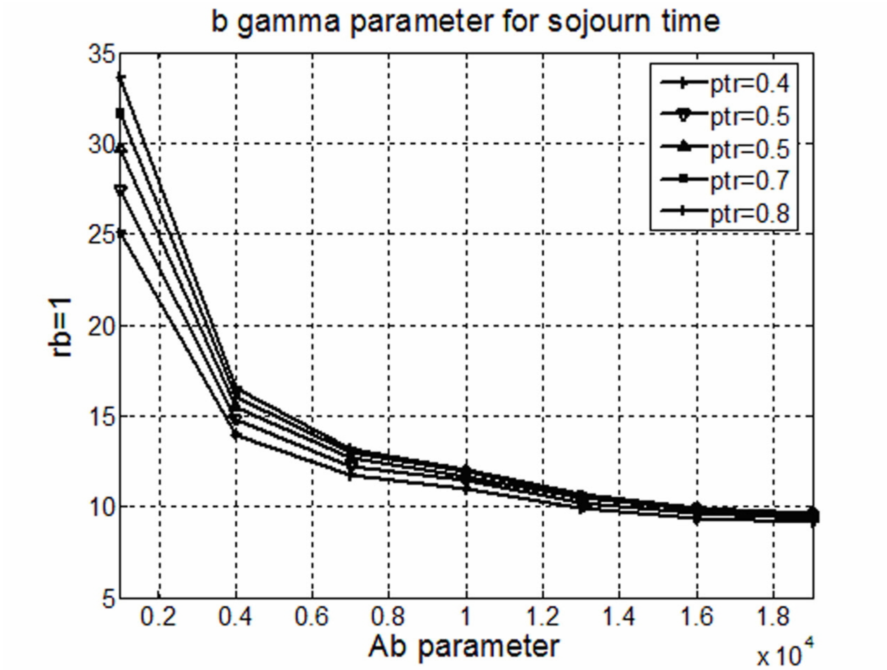

Fig. 6. The function $b_n = F_2\left(A_b, p_{tr}, r_b\right)$ for $r_b = 1$.

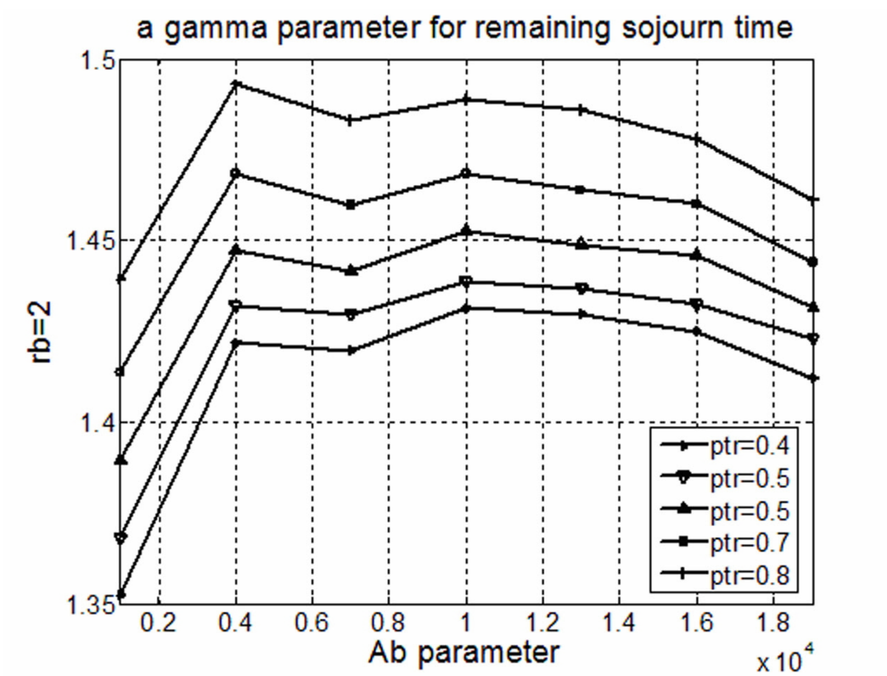

Fig. 7. The function $a_n = F_1\left(A_b, p_{tr}, r_b\right)$ for $r_b = 2$.

$e_3 = 0.165$ and $e_4 = 0.322$. 2000 generations were used in the fit process.

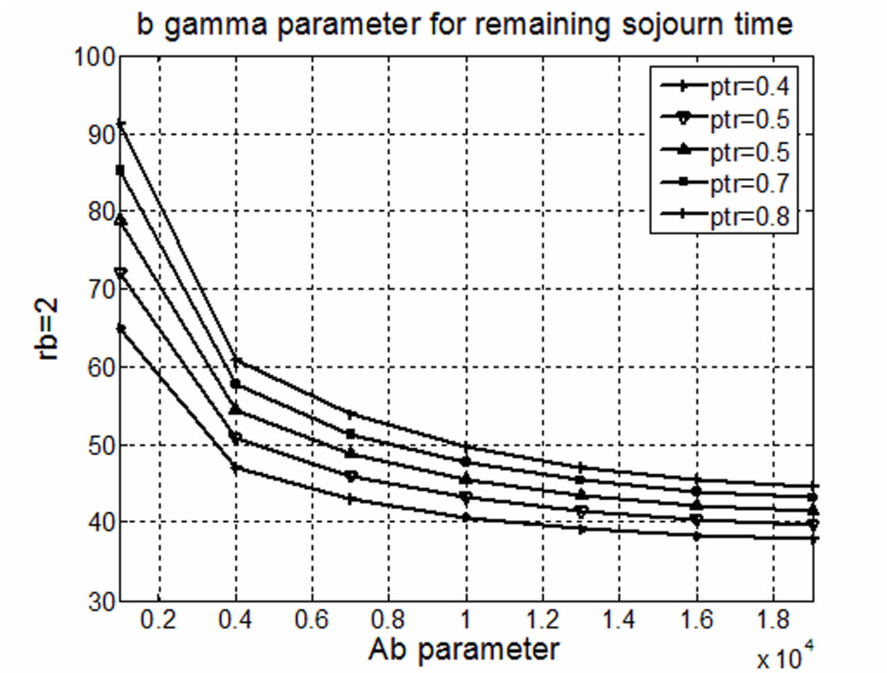

Fig. 8. The function $b_n = F_2\left(A_b, p_{tr}, r_b\right)$ for $r_b = 2$.

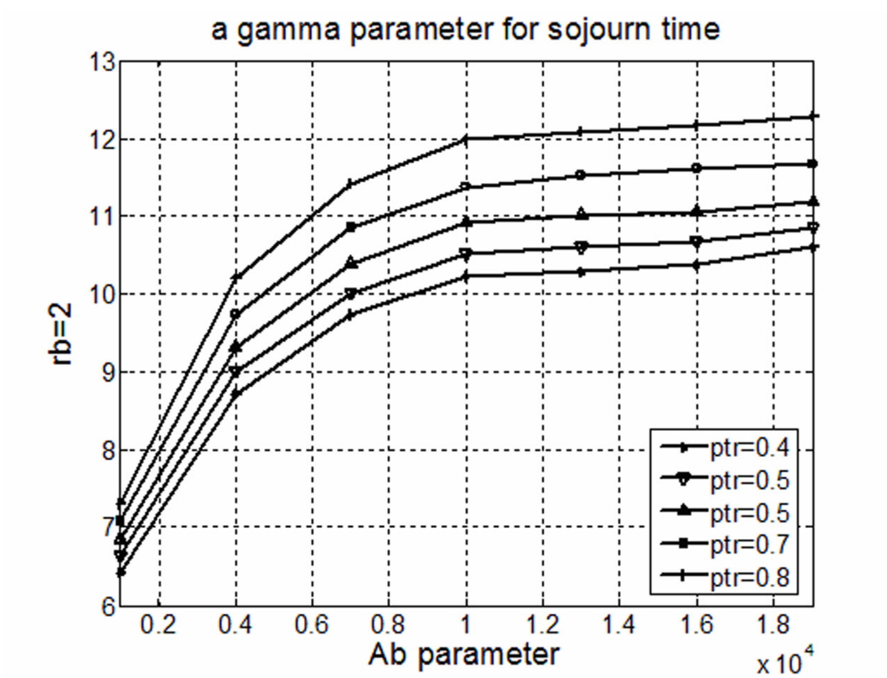

Fig. 9. The function $a_h = F_3\left(A_b, p_{tr}, r_b\right)$ for $r_b = 2$.

3 Case Study in a real cell

A case study is presented for the cell of figure 11. Using the map, the values $A_b = 9070m^2$ and $r_b = 1.8$ are easily extracted. For p_{tr}, a traffic assumption

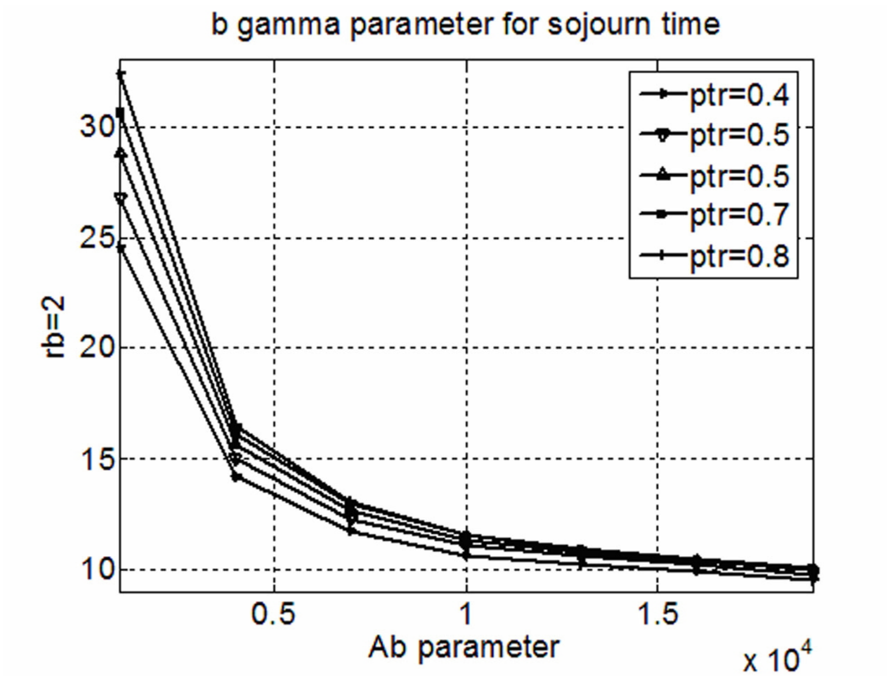

Fig. 10. The function $b_h = F_4\left(A_b, p_{tr}, r_b\right)$ for $r_b = 2$.

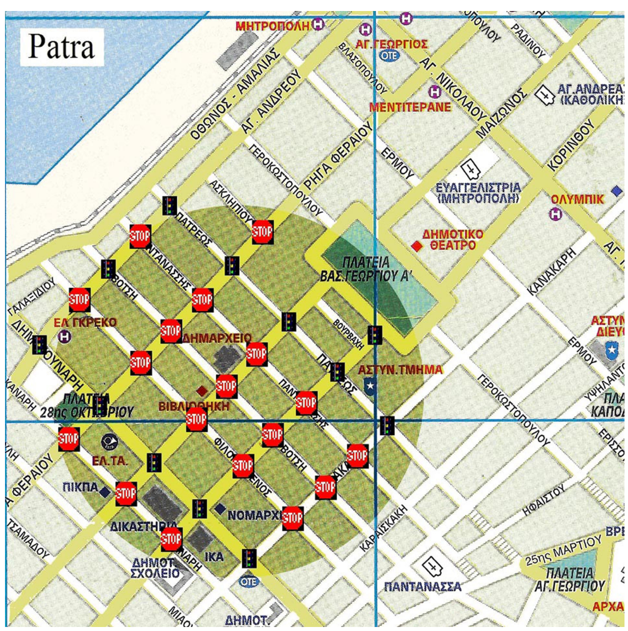

Fig. 11. The cell in Patras under test. Mesearuments took place in this cell.

is made. The 80% of the vehicles are assumed to use the priority roads only and the rest are assume to move randomly in the cell. Therefore, $p_{tr} = 0.8 \cdot p_{tr}^p + 0.2 \cdot p_{tr}^c$, where $p_{tr}^p = 0.571$ is the p_{tr} calculation for priority roads and $p_{tr}^c = 0.7$ for the whole cell. The value $p_{tr} = 0.597$ was found.

Using the model, $a_n = 1.447$, $b_n = 46$, $a_h = 10.5$, and $b_h = 14$ are calculated. The results are compared to measurements taken in the same area in figure 12.

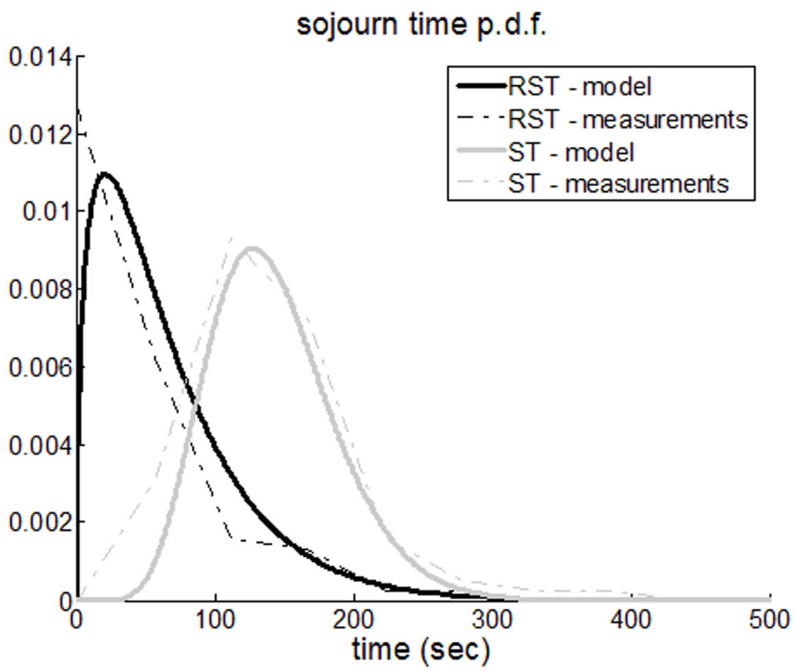

Fig. 12. Comparison of the proposed model and measurements for the cell of figure 11. The values used for the model are: $a_n = 1.447$, $b_n = 46$, $a_h = 10.5$ and $b_h = 14$.

The model conformance to measurements is improved, in comparison to [8], due to the use of conditional probabilities of turnng and better fitting method.

4 Conclusion

A deterministic simulation model is proposed. The model can be used along with a map in order to provide sojourn time and remaining sojourn time statistics. The map extraction of the parameter of traffic is described in detail. The model has been tested in a real cell, and the results show a satisfactory measurement conformance. This model can be used for cell planning and handover ratio estimation in any urban cell that has a rectangular block shape.

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