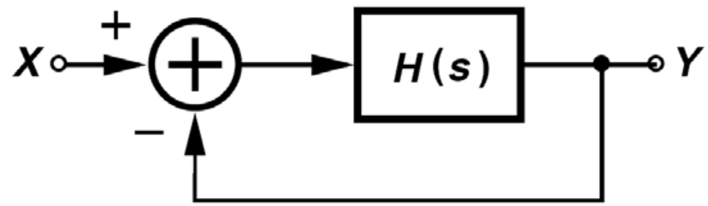
Feedback View of Oscillators

- An oscillator may be viewed as a "badly-designed" negative-feedback amplifier—so badly designed that it has a zero or negative phase margin.

$$\frac{Y}{X}(s) = \frac{H(s)}{1 + H(s)}$$



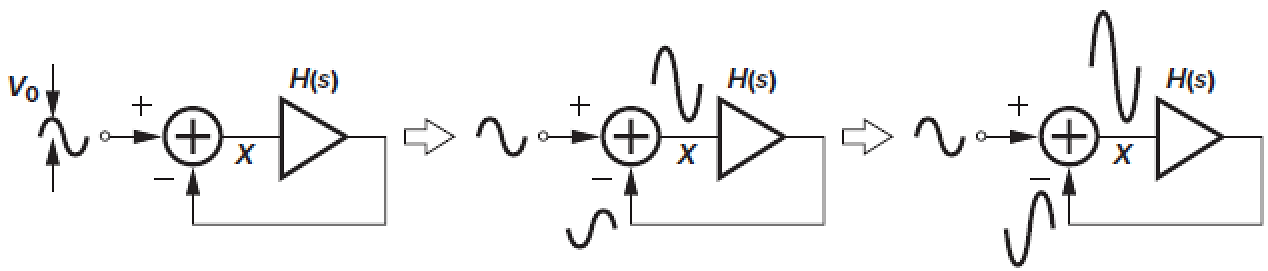


Figure 15.2 Evolution of oscillatory system with time.

In summary, if a negative-feedback circuit has a loop gain that satisfies two conditions:

$$|H(j\omega_0)| \geq 1$$

$$\angle H(j\omega_0) = 180^\circ$$

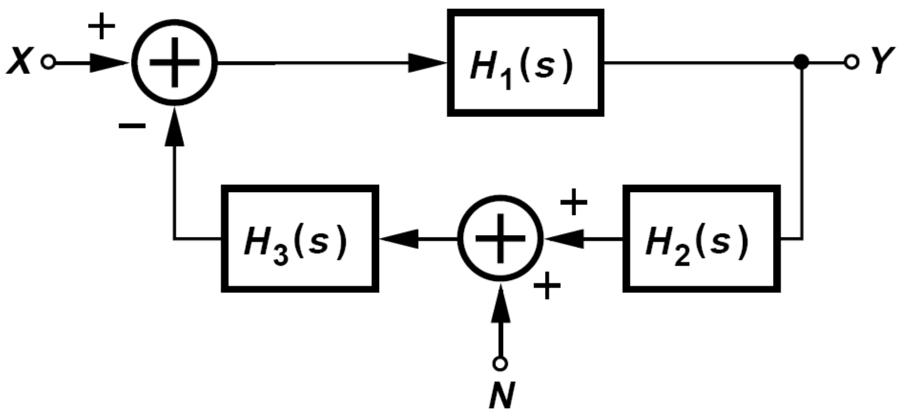
then the circuit may oscillate at ω_0. Called "Barkhausen criteria," these conditions are necessary

Feedback View of Oscillators (II)

For the above system to oscillate, must the noise at ω_1 appear at the input?

No, the noise can be anywhere in the loop. For example, consider the system shown in figure below, where the noise *N* appears in the feedback path. Here,

$$Y(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)H_3(s)} X(s) + \frac{H_1(s)H_3(s)}{1 + H_1(s)H_2(s)H_3(s)} N(s).$$



Thus, if the loop transmission, $H_1H_2H_3$, approaches -1 at ω_1, *N* is also amplified indefinitely.

Y/X in the Vicinity of $\omega = \omega_1$

Derive an expression for Y/X in figure below in the vicinity of $\omega = \omega_1$ if $H(j\omega_1) = -1$.

We can approximate $H(j\omega)$ by the first two terms in its Taylor series:

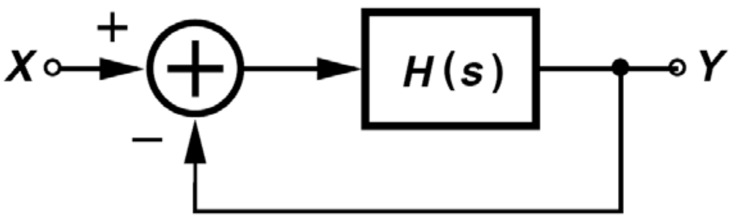
$$H[j(\omega_1 + \Delta\omega)] \approx H(j\omega_1) + \Delta\omega \frac{dH(j\omega_1)}{d\omega}$$

Since $H(j\omega_1) = -1$, we have

$$\frac{Y}{X}[j(\omega_1 + \Delta\omega)] = \frac{H(j\omega_1) + \Delta\omega \frac{dH(j\omega_1)}{d\omega}}{\Delta\omega \frac{dH(j\omega_1)}{d\omega}}$$

$$\approx \frac{H(j\omega_1)}{\Delta\omega \frac{dH(j\omega_1)}{d\omega}}$$

$$\approx \frac{-1}{\Delta\omega \frac{dH(j\omega_1)}{d\omega}}$$

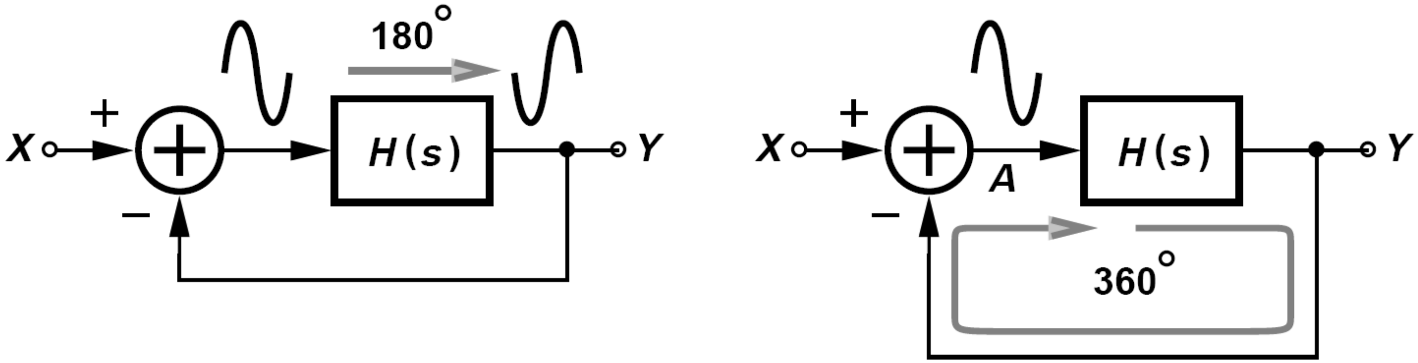


As expected, $Y/X \to \infty$ as $\Delta\omega \to 0$, with a "sharpness" proportional to $dH/d\omega$.

Barkhausen's Criteria

$$|H(s = j\omega_1)| = 1$$

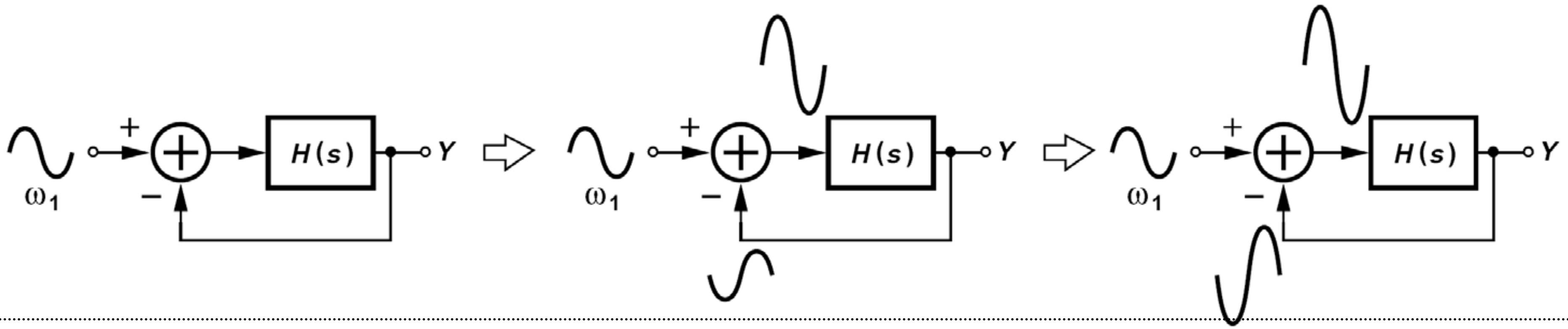
$$\angle H(s = j\omega_1) = 180^\circ$$



- For the circuit to reach steady state, the signal returning to A must exactly coincide with the signal that started at A. We call $\angle H(j\omega_1)$ a "frequency-dependent" phase shift to distinguish it from the 180° phase due to negative feedback.
- Even though the system was originally configured to have negative feedback, *H(s)* is so "sluggish" that it contributes an additional phase shift of 180° at ω_1, thereby creating positive feedback at this frequency.

Significance of $|H(jw_1)| = 1$

- For a noise component at ω_1 to "build up" as it circulates around the loop with positive feedback, the loop gain must be at least unity.
- We call $|H(j\omega_1)| = 1$ the "startup" condition.



- What happens if $|H(j\omega_1)| > 1$ and $\angle H(j\omega_1) = 180^\circ$? The growth shown in figure above still occurs but at a faster rate because the returning waveform is amplified by the loop.
- Note that the closed-loop poles now lie in the right half plane.

$$V_X = V_0 + |H(j\omega_0)|V_0 + |H(j\omega_0)|^2 V_0 + |H(j\omega_0)|^3 V_0 + \cdots$$

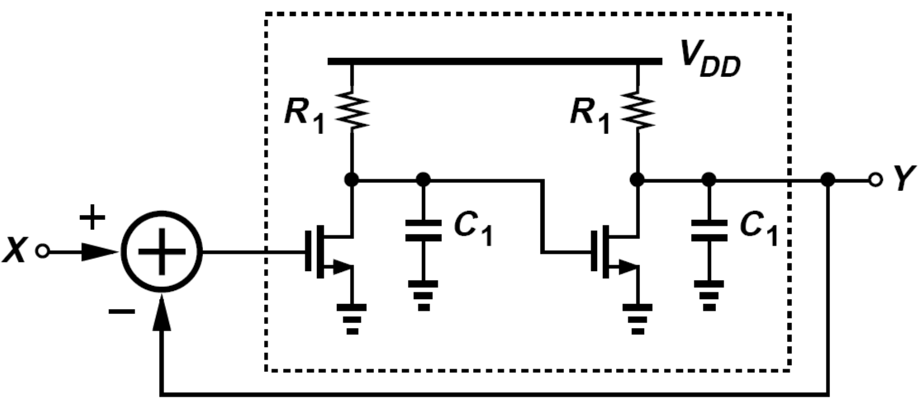
If $|H(j\omega_0)| > 1$, the above summation diverges, whereas if $|H(j\omega_0)| < 1$, then

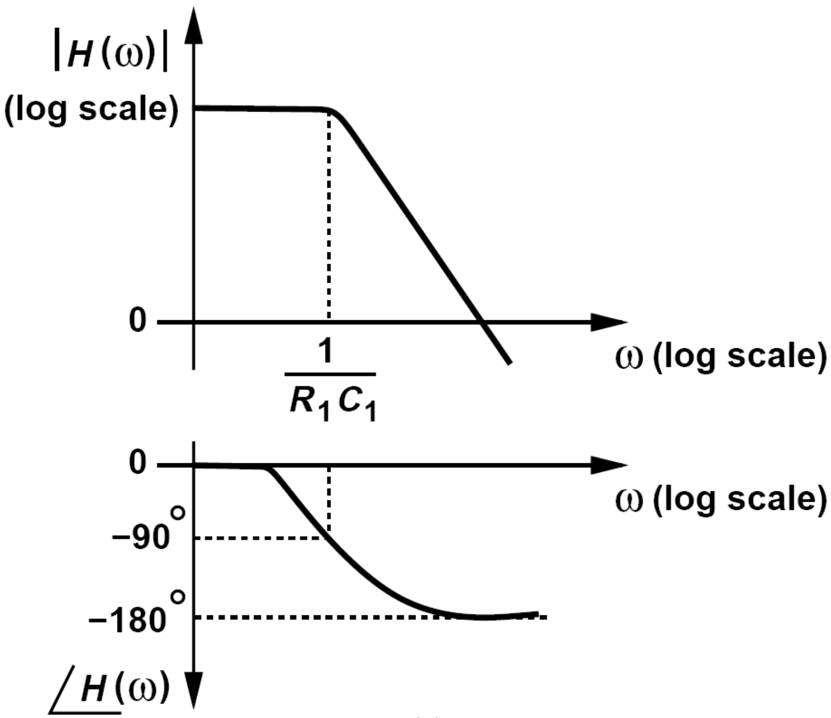
$$V_X = \frac{V_0}{1 - |H(j\omega_0)|} < \infty$$

Can a Two-Pole System Oscillate? (I)

Can a two-pole system oscillate?

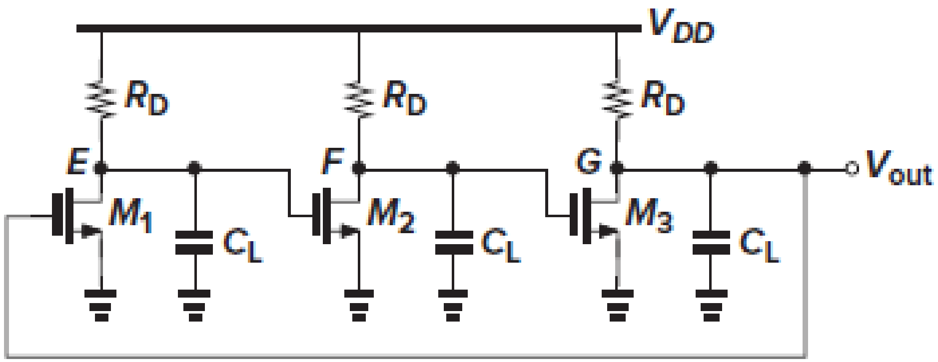
Suppose the system exhibits two coincident real poles at ω_p. Figure below (left) shows an example, where two cascaded common-source stages constitute $H(s)$ and $\omega_p = (R_1C_1)^{-1}$. This system cannot satisfy both of Barkhausen's criteria because the phase shift associated with each stage reaches 90° only at $\omega = \infty$, but $|H(\infty)| = 0$. Figure below (right) plots $|H|$ and $\angle H$ as a function of frequency, revealing no frequency at which both conditions are met. Thus, the circuit cannot oscillate.





Three stage Ring oscillator

$$H(s) = -\frac{A_0^3}{(1+\frac{s}{\omega_0})^3}$$



The circuit oscillates only if the frequency-dependent phase shift equals 180°, i.e., if each stage contributes 60°. The frequency at which this occurs is given by

$$\tan^{-1}\frac{\omega_{osc}}{\omega_0} = 60^\circ \tag{15.7}$$

and hence

$$\omega_{osc} = \sqrt{3}\omega_0 \tag{15.8}$$

The minimum voltage gain per stage must be such that the magnitude of the loop gain at ω_{osc} is equal to unity:

$$\frac{A_0^3}{\left[\sqrt{1+(\frac{\omega_{osc}}{\omega_0})^2}\right]^3} = 1 \tag{15.9}$$

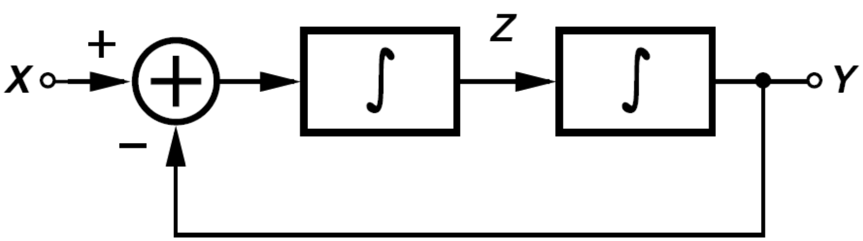
It follows from (15.8) and (15.9) that

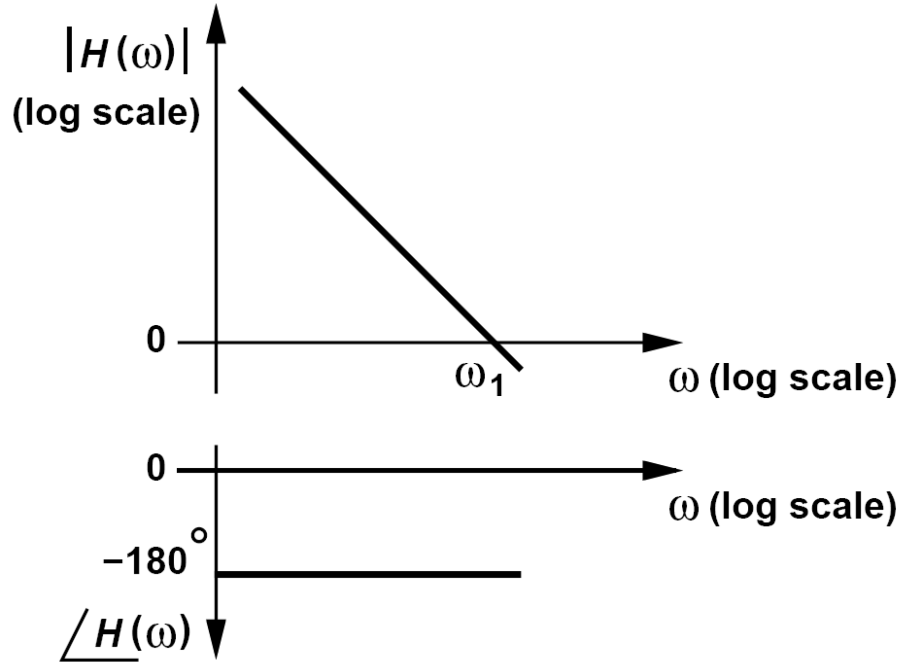
$$A_0 = 2 \tag{15.10}$$

Can a Two-Pole System Oscillate? (Ⅱ)

Can a two-pole system oscillate?

But, what if both poles are located at the origin? Realized as two ideal integrators in a loop, such a circuit does oscillate because each integrator contributes a phase shift of -90° at any nonzero frequency. Shown in figure below (right) are |*H*| and ∠*H* for this system.





Frequency and Amplitude of Oscillation in Previous Example

The feedback loop of figure above is released at t = 0 with initial conditions of z_0 and y_0 at the outputs of the two integrators and x(t) = 0. Determine the frequency and amplitude of oscillation.

Assuming each integrator transfer function is expressed as *K/s*,

$$\frac{Y}{X}(s) = \frac{K^2}{s^2 + K^2}$$

$$\frac{d^2y}{dt^2} + K^2 y = K^2 x(t)$$

Substitute x and y,

$$-A\omega_1^2 \cos(\omega_1 t + \phi_1) + K^2 A \cos(\omega_1 t + \phi_1) = 0$$

$$\omega_1 = K$$

Interestingly, the circuit automatically finds the frequency at which the loop gain K^2/ω^2 drops to unity.

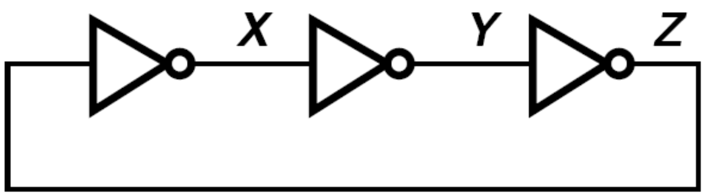
$$y(0) = A\cos\phi_1 = y_0$$

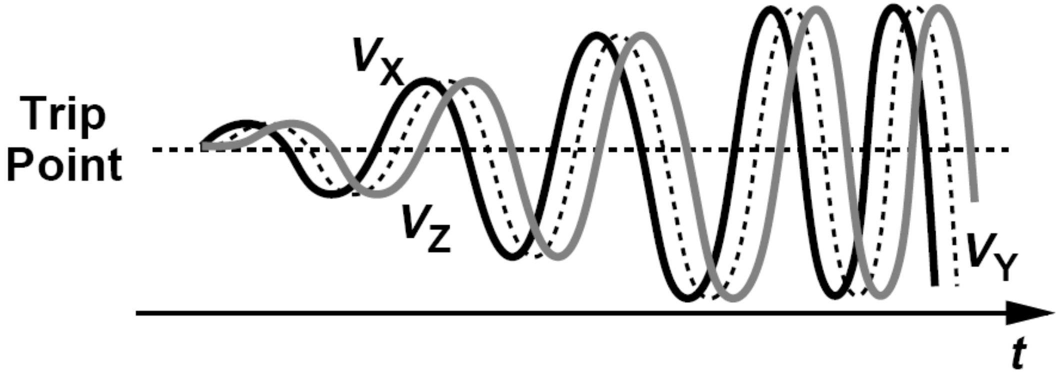
$$z(0) = \frac{1}{K}\frac{dy}{dt}|_{t=0} = -A\sin\phi_1 = z_0$$

$$\Rightarrow \quad \tan\phi_0 = -\frac{z_0}{y_0}, \qquad A = \sqrt{z_0^2 + y_0^2}$$

Ring Oscillator

- Other oscillators may begin to oscillate at a frequency at which the loop gain is higher than unity, thereby experiencing an exponential growth in their output amplitude.
- The growth eventually stops due to the saturating behavior of the amplifier(s) in the loop.





- Each stage operates as an amplifier, leading to an oscillation frequency at which each inverter contributes a frequency-dependent phase shift of 60° .

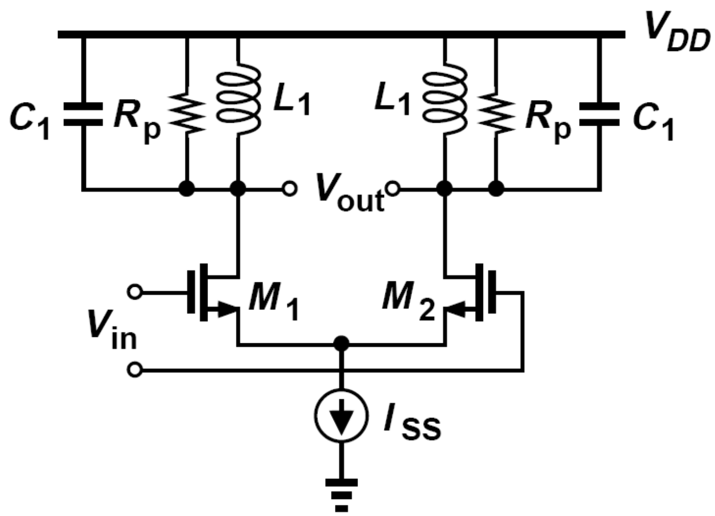
Example of Voltage Swings (Ⅰ)

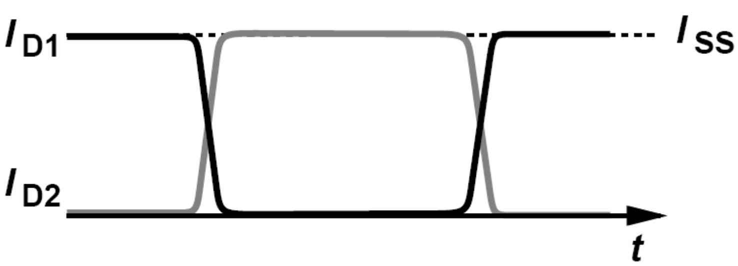
The inductively-loaded differential pair shown in figure below is driven by a large input sinusoid at

$$\omega_0 = 1/\sqrt{L_1 C_1}$$

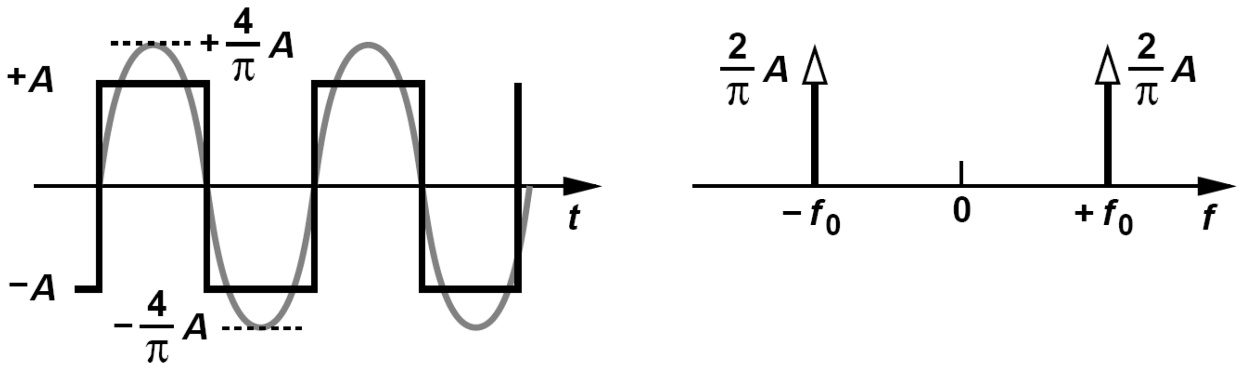
Plot the output waveforms and determine the output swing.

With large input swings, M_1 and M_2 experience complete switching in a short transition time, injecting nearly square current waveforms into the tanks. Each drain current waveform has an average of $I_{SS}/2$ and a peak amplitude of $I_{SS}/2$. The first harmonic of the current is multiplied by R_p whereas higher harmonics are attenuated by the tank selectivity.





Example of Voltage Swings (Ⅱ)



Recall from the Fourier expansion of a square wave of peak amplitude A (with 50% duty cycle) that the first harmonic exhibits a peak amplitude of (4/*π*)*A* (slightly greater than A). The peak single-ended output swing therefore yields a peak differential output swing of

$$V_{out} = \frac{4}{\pi} I_{SS} R_p$$