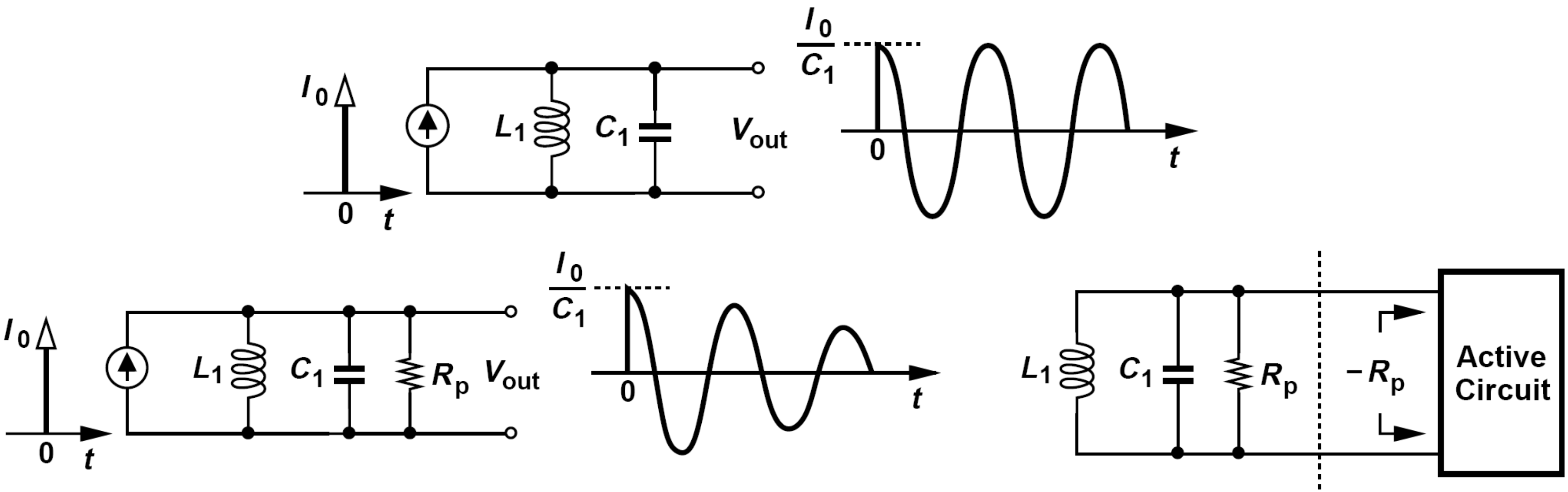
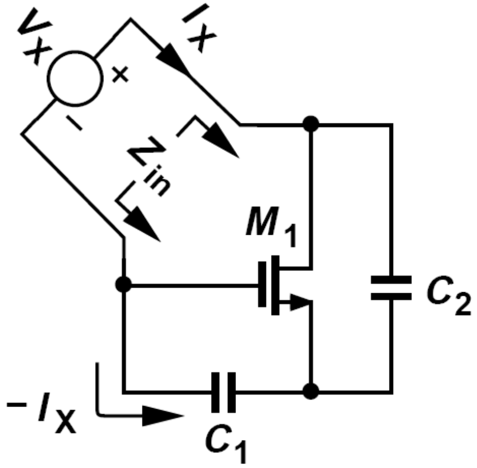
One-Port View of Oscillators

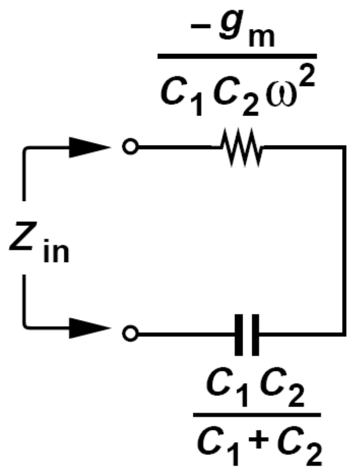
- **An alternative perspective views oscillators as two one-port components, namely, a lossy resonator and an active circuit that cancels the loss.**



- **If an active circuit replenishes the energy lost in each period, then the oscillation can be sustained.**
- **In fact, we predict that an active circuit exhibiting an input resistance of -R_p can be attached across the tank to cancel the effect of R_p.**

How Can a Circuit Present a Negative Input Resistance?





$$-\frac{I_X}{C_1 s} + V_X = \left(I_X + I_X \frac{g_m}{C_1 s}\right)\frac{1}{C_2 s}$$

$$\frac{V_X}{I_X}(s) = \frac{1}{C_1 s} + \frac{1}{C_2 s} + \frac{g_m}{C_1 C_2 s^2}$$

$$\frac{V_X}{I_X}(j\omega) = \frac{1}{jC_1\omega} + \frac{1}{jC_2\omega} - \frac{g_m}{C_1 C_2 \omega^2}$$

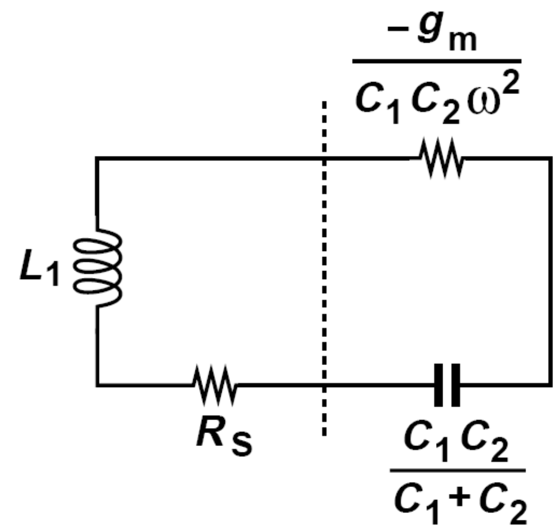
- The negative resistance varies with frequency.

Connection of Lossy Inductor to Negative-Resistance Circuit

- **Since the capacitive component in equation above can become part of the tank, we simply connect an inductor to the negative-resistance port.**

$$R_S = \frac{g_m}{C_1 C_2 \omega^2}$$

$$\omega_{osc} = \frac{1}{\sqrt{L_1 \frac{C_1 C_2}{C_1 + C_2}}}$$



Express the oscillation condition in terms of inductor's parallel equivalent resistance, R_p, rather than R_S.

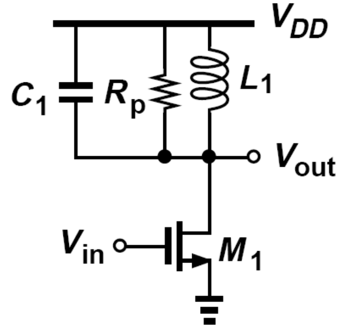
$$\frac{L_1 \omega}{R_S} \approx \frac{R_p}{L_1 \omega} \Rightarrow \frac{L_1^2 \omega^2}{R_p} = \frac{g_m}{C_1 C_2 \omega^2}$$

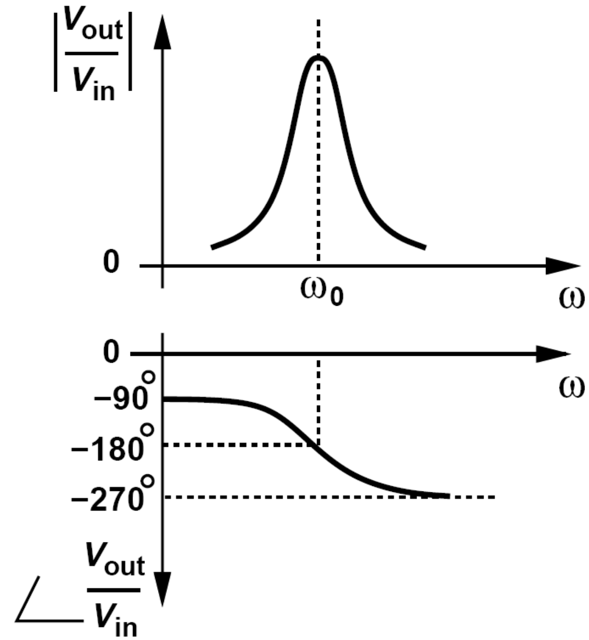
The startup condition:

$$g_m R_p = \frac{(C_1 + C_2)^2}{C_1 C_2} = \frac{C_1}{C_2} + \frac{C_2}{C_1} + 2$$

Tuned Oscillator

We wish to build a negative-feedback oscillatory system using "LC-tuned" amplifier stages.





$$\frac{V_X}{V_{in}}(j\omega) = \frac{-jg_m R_p L_1 \omega}{R_p(1 - L_1 C_1 \omega^2) + jL_1\omega}$$

At very low frequencies, L_1 dominates the load and

$$\frac{V_{out}}{V_{in}} \approx -g_m L_1 s$$

$|V_{out}/V_{in}|$ is very small and $\angle(V_{out}/V_{in})$ remains around -90°

At the resonance frequency

$$\frac{V_{out}}{V_{in}} = -g_m R_p$$

The phase shift from the input to the output is thus equal to 180°

At very high frequencies

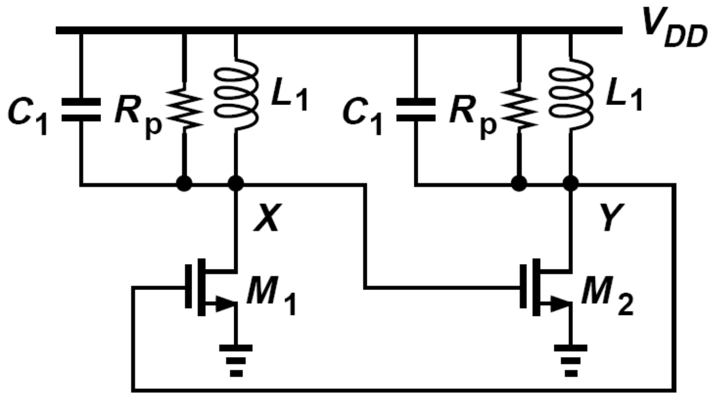
$$\frac{V_{out}}{V_{in}} \approx -g_m \frac{1}{C_1 s}$$

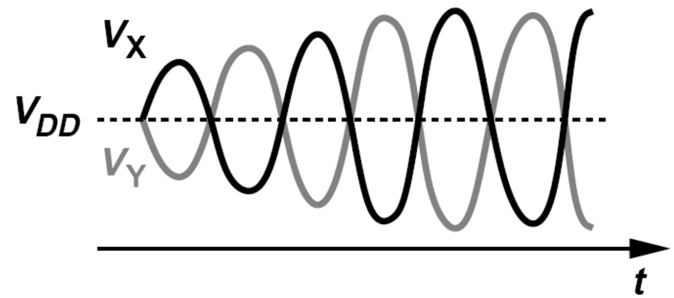
$|V_{out}/V_{in}|$ dinimishes $\angle(V_{out}/V_{in})$ approaches +90°

Cascade of Two Tuned Amplifiers in Feedback Loop

Can the circuit above oscillate if its input and output are shorted? No.

We recognize that the circuit provides a phase shift of 180° with possibly adequate gain (g_mR_p) at ω_0. We simply need to increase the phase shift to 360°.





$$(g_m R_p)^2 \geq 1$$

Assuming that the circuit above (left) oscillates, plot the voltage waveforms at *X* and *Y*.

Wave form is shown above (right). A unique attribute of inductive loads is that they can provide peak voltages above the supply. The growth of V_X and V_Y ceases when M_1 and M_2 enter the triode region for part of the period, reducing the loop gain.

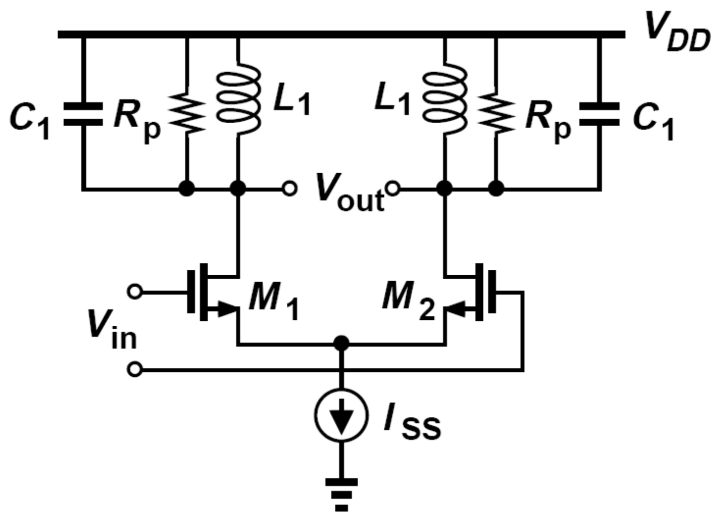
Example of Voltage Swings (Ⅰ)

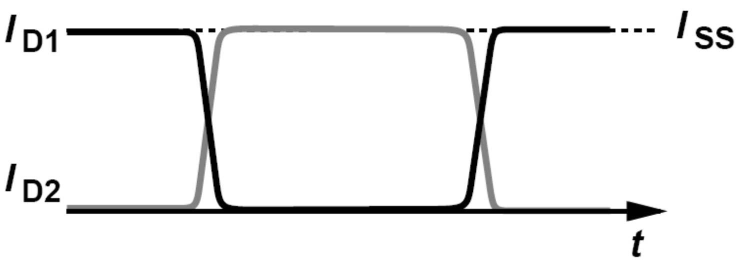
The inductively-loaded differential pair shown in figure below is driven by a large input sinusoid at

$$\omega_0 = 1/\sqrt{L_1 C_1}$$

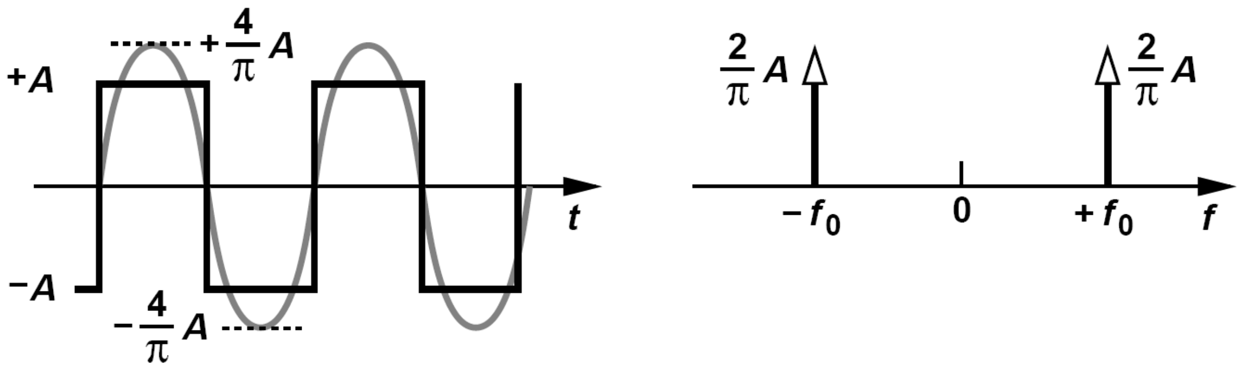
Plot the output waveforms and determine the output swing.

With large input swings, M_1 and M_2 experience complete switching in a short transition time, injecting nearly square current waveforms into the tanks. Each drain current waveform has an average of $I_{SS}/2$ and a peak amplitude of $I_{SS}/2$. The first harmonic of the current is multiplied by R_p whereas higher harmonics are attenuated by the tank selectivity.





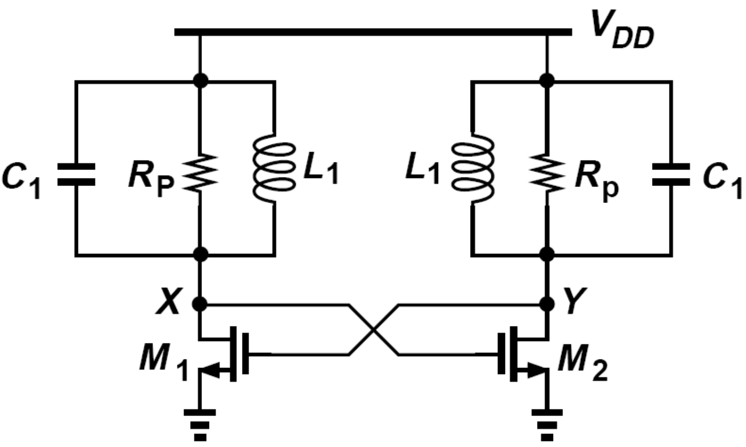
Example of Voltage Swings (Ⅱ)

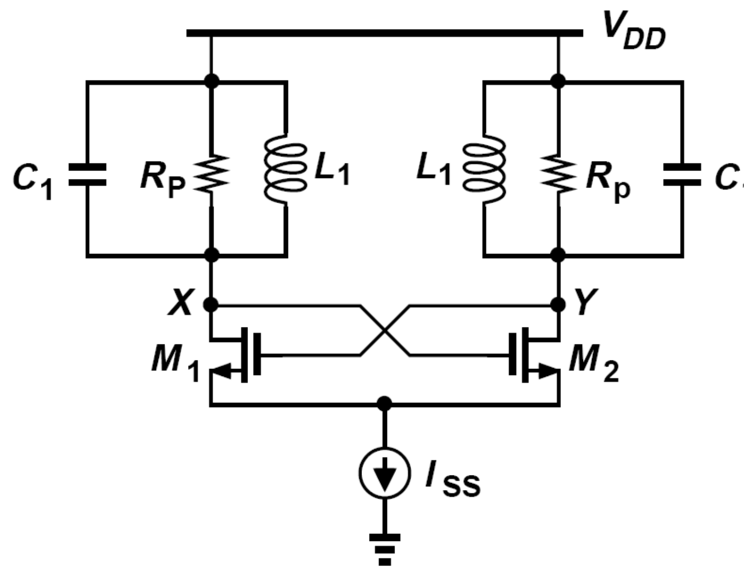


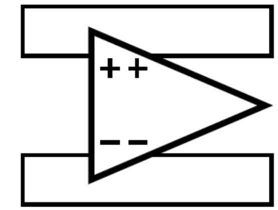
Recall from the Fourier expansion of a square wave of peak amplitude A (with 50% duty cycle) that the first harmonic exhibits a peak amplitude of (4/*π*)*A* (slightly greater than A). The peak single-ended output swing therefore yields a peak differential output swing of

$$V_{out} = \frac{4}{\pi} I_{SS} R_p$$

Cross-Coupled Oscillator







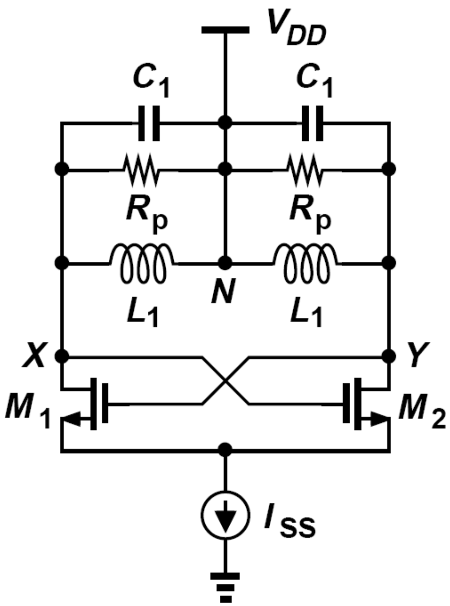
$$\omega_{osc} = \frac{1}{\sqrt{L_1(C_{GS2} + C_{DB1} + 4C_{GD} + C_1)}}$$

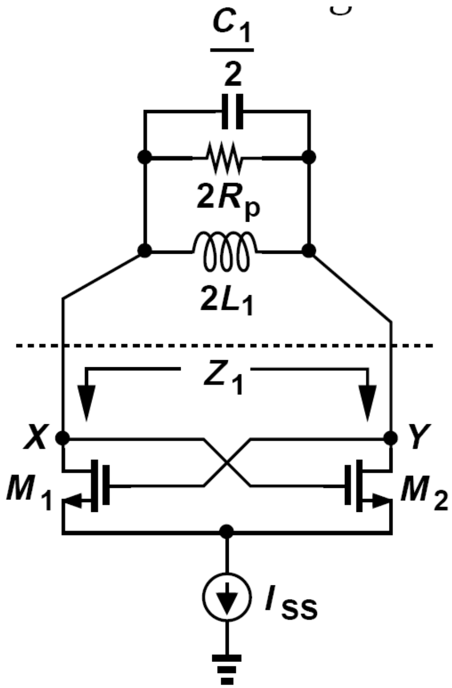
The oscillator above (left) suffers from poorly-defined bias currents. The circuit above (middle) is more robust and can be viewed as an inductively-loaded differential pair with positive feedback.

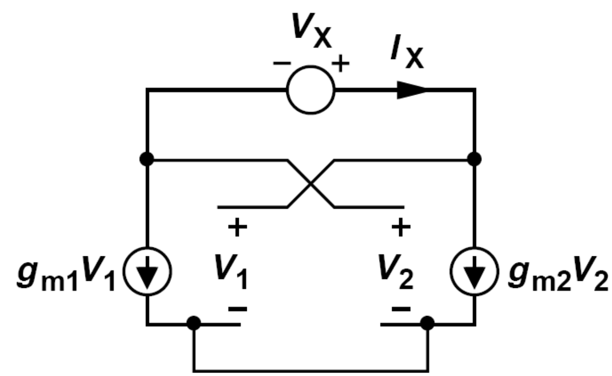
Compute the voltage swings in the circuit above (middle) if M_1 and M_2 experience complete current switching with abrupt edges.

$$V_{XY} \approx \frac{4}{\pi} I_{SS} R_p$$

One-Port View of Cross-Coupled Oscillator







$$I_X = -g_{m1}V_1 = g_{m2}V_2 \quad \Rightarrow \quad \frac{V_X}{I_X} = -\left(\frac{1}{g_{m1}} + \frac{1}{g_{m2}}\right)$$

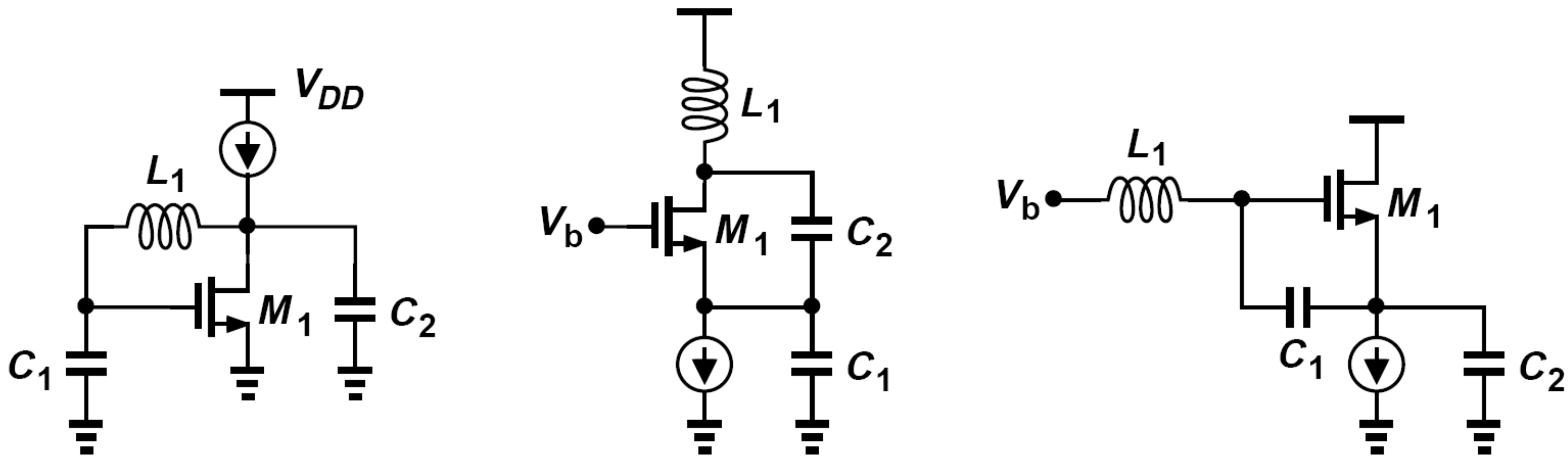
For $g_{m1} = g_{m2} = g_m$ $\quad \dfrac{V_X}{I_X} = -\dfrac{2}{g_m}$

For oscillation to occur, the negative resistance must cancel the loss of the tank:

$$\frac{2}{g_m} \leq 2R_p \quad \Rightarrow \quad g_m R_p \geq 1$$

Three-Point Oscillators

Three different oscillator topologies can be obtained by grounding each of the transistor terminals. Figures below depict the resulting circuits if the source, the gate, or the drain is (ac) grounded, respectively.



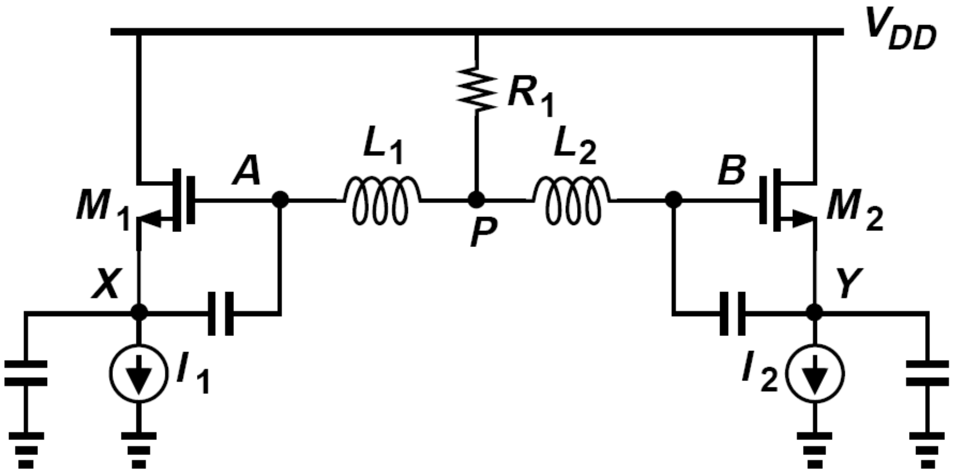
If $C_1 = C_2$, the transistor must provide sufficient transconductance to satisfy

$$g_m R_p \geq 4$$

- **The circuits above may fail to oscillate if the inductor *Q* is not very high.**

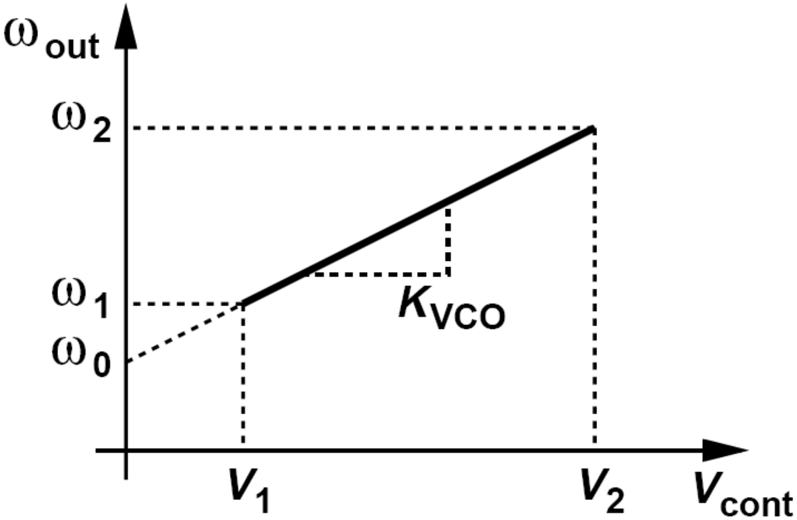
Differential Version of Three-Point Oscillators

- **Another drawback of the circuits shown above is that they produce only single-ended outputs. It is possible to couple two copies of one oscillator so that they operate differentially.**



- **If chosen properly, the resistor R_1 prohibits common-mode oscillation.**
- **Even with differential outputs, the circuit above may be inferior to the cross-coupled oscillator previous discussed —not only for the more stringent start-up condition but also because the noise of I_1 and I_2 directly corrupts the oscillation.**

Voltage-Controlled Oscillators: Characteristic



$$\omega_{out} = K_{VCO} V_{cont} + \omega_0$$

- **The output frequency varies from ω_1 to ω_2 (the required tuning range) as the control voltage, V_{cont}, goes from V_1 to V_2.**
- **The slope of the characteristic, K_{VCO}, is called the "gain" or "sensitivity" of the VCO and expressed in rad/Hz/V.**

Example: V_{DD} as the "Control Voltage"

As explained in previous example, the cross-coupled oscillator exhibits sensitivity to V_{DD}. Considering V_{DD} as the "control voltage," determine the gain.

If C_1 includes all circuit capacitances except C_{DB}

$$\omega_{osc} = \frac{1}{\sqrt{L_1(C_1 + C_{DB})}}$$

$$K_{VCO} = \frac{\partial \omega_{out}}{\partial V_{DD}} = \frac{\partial \omega_{osc}}{\partial C_{DB}} \cdot \frac{\partial C_{DB}}{\partial V_{DD}}$$

The junction capacitance is approximated as

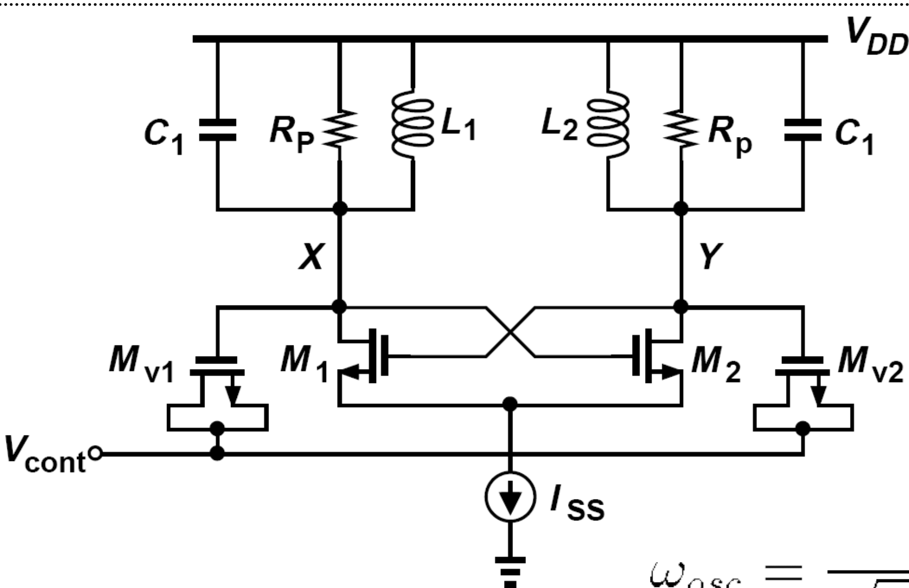
$$C_{DB} = \frac{C_{DB0}}{\left(1 + \frac{V_{DD}}{\phi_B}\right)^m}$$

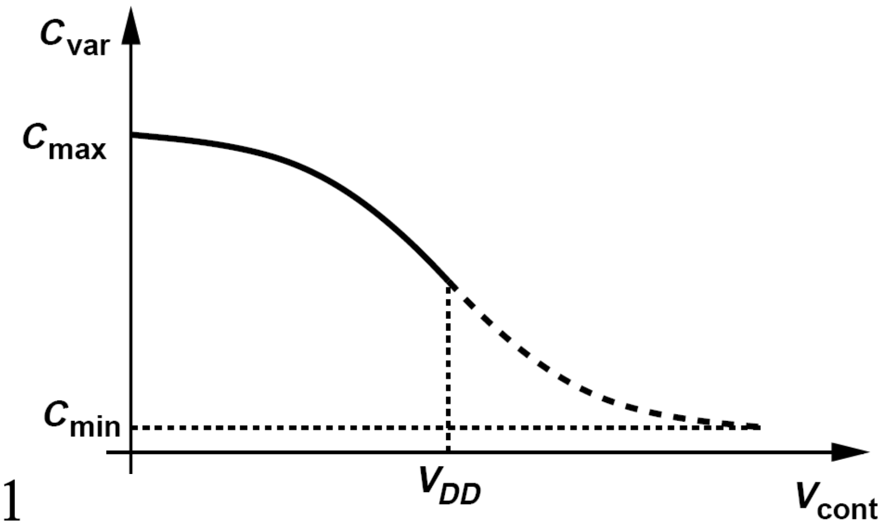
$$\Rightarrow K_{VCO} = \frac{-1}{2\sqrt{L_1}} \cdot \frac{1}{\sqrt{C_1 + C_{DB}}(C_1 + C_{DB})} \cdot \frac{-mC_{DB0}}{\phi_B\left(1 + \frac{V_{DD}}{\phi_B}\right)^{m+1}}$$

$$= \frac{C_{DB}}{C_1 + C_{DB}} \cdot \frac{m}{2\phi_B + 2V_{DD}} \omega_{osc}.$$

VCO Using MOS Varactors

- **Since it is difficult to vary the inductance electronically, we only vary the capacitance by means of a varactor.**
- **MOS varactors are more commonly used than *pn* junctions, especially in low-voltage design.**



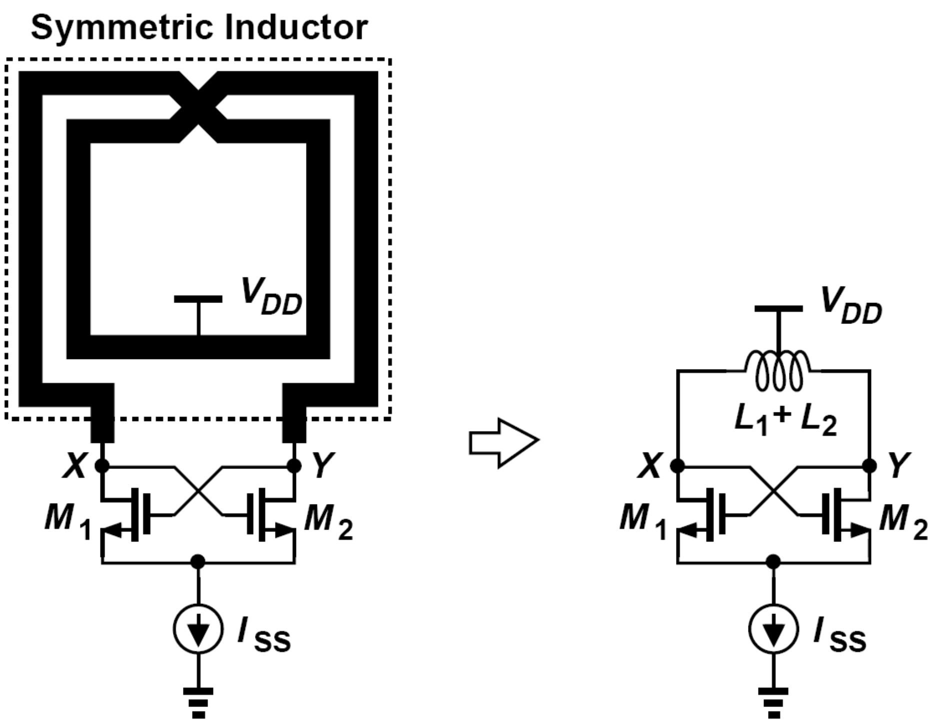


$$\omega_{osc} = \frac{1}{\sqrt{L_1(C_1 + C_{var})}}$$

- **First, the varactors are stressed for part of the period if V_{cont} is near ground and V_X (or V_Y) rises significantly above V_{DD}.**
- **Second, only about half of C_{max} - C_{min} is utilized in the tuning.**

Oscillator Using Symmetric Inductor

- **Symmetric spiral inductors excited by differential waveforms exhibit a higher *Q* than their single-ended counterparts.**



The symmetric inductor above has a value of 2 nH and a *Q* of 10 at 10 GHz. What is the minimum required transconductance of M_1 and M_2 to guarantee start-up?

$$g_{m1,2} \geq (630\ \Omega)^{-1}$$

Tuning Range Limitations

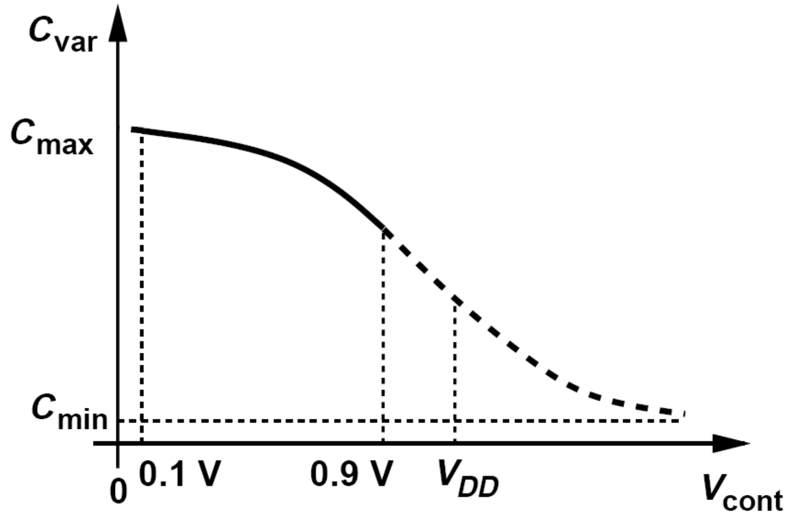
We make a crude approximation, $C_{var} << C_1$, and

$$\omega_{osc} \approx \frac{1}{\sqrt{L_1 C_1}}\left(1 - \frac{C_{var}}{2C_1}\right)$$

If the varactor capacitance varies from C_{var1} to C_{var2}, then the tuning range is given by

$$\Delta\omega_{osc} \approx \frac{1}{\sqrt{L_1 C_1}} \frac{C_{var2} - C_{var1}}{2C_1}$$

- **The tuning range trades with the overall tank *Q*.**
- **Another limitation on C_{var2} - C_{var1} arises from the available range for the control voltage of the oscillator, $V_{cont.}$**

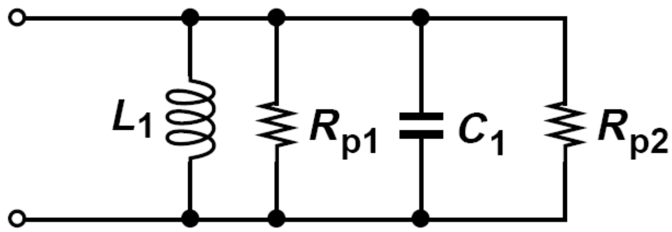


Effect of Varactor *Q:* Tank Consisting of Lossy Inductor and Capacitor

A lossy inductor and a lossy capacitor form a parallel tank. Determine the overall *Q* in terms of the quality factor of each.

The loss of an inductor or a capacitor can be modeled by a parallel resistance (for a narrow frequency range). We therefore construct the tank as shown below, where the inductor and capacitor *Q*'s are respectively given by:

$$Q_L = \frac{R_{p1}}{L_1\omega}$$
$$Q_C = R_{p2}C_1\omega$$

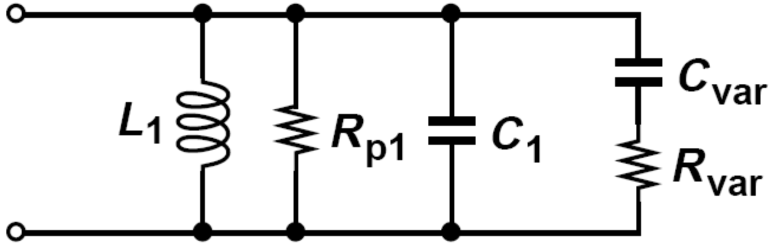


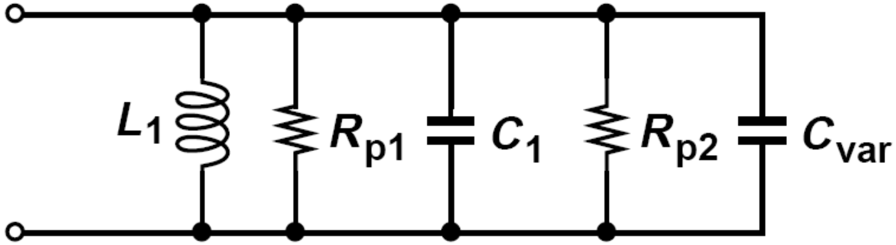
Merging R_{p1} and R_{p2} yields the overall *Q*:

$$Q_{tot} = \frac{R_{p1}R_{p2}}{R_{p1}+R_{p2}} \cdot \frac{1}{L_1\omega}$$
$$= \frac{1}{\frac{L_1\omega}{R_{p1}} + \frac{L_1\omega}{R_{p2}}}$$
$$= \frac{1}{\frac{L_1\omega}{R_{p1}} + \frac{1}{R_{p2}C_1\omega}}.$$

$$\Rightarrow \quad \frac{1}{Q_{tot}} = \frac{1}{Q_L} + \frac{1}{Q_C}$$

Tank Using Lossy Varactor





Transforming the series combination of C_{var} and R_{var} to a parallel combination

$$R_{p2} = \frac{1}{C_{var}^2 \omega^2 R_{var}}$$

The Q associated with C_1+C_{var} is equal to

$$\begin{aligned} Q_C &= R_{P2}(C_1 + C_{var})\omega \\ &= \frac{C_1 + C_{var}}{C_{var}^2 \omega R_{var}}. \end{aligned}$$

$$\Rightarrow \quad Q_C = \left(1 + \frac{C_1}{C_{var}}\right) Q_{var}$$

The overall tank Q is therefore given by

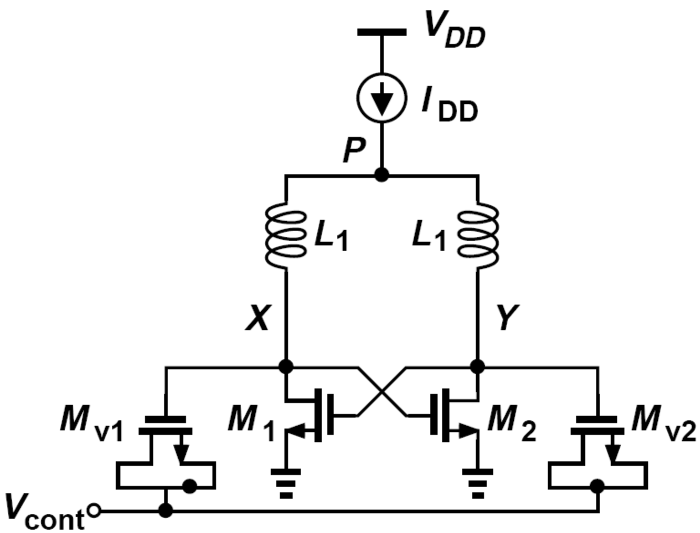
$$\frac{1}{Q_{tot}} = \frac{1}{Q_L} + \frac{1}{\left(1 + \frac{C_1}{C_{var}}\right) Q_{var}}$$

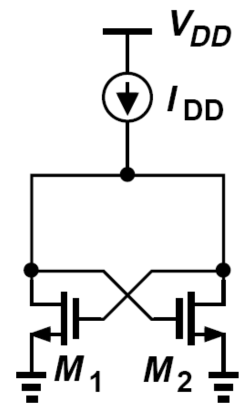
Equation above can be generalized if the tank consists of an ideal capacitor, C_1, and lossy capacitors, C_2-C_n, that exhibit a series resistance of R_2-R_n, respectively.

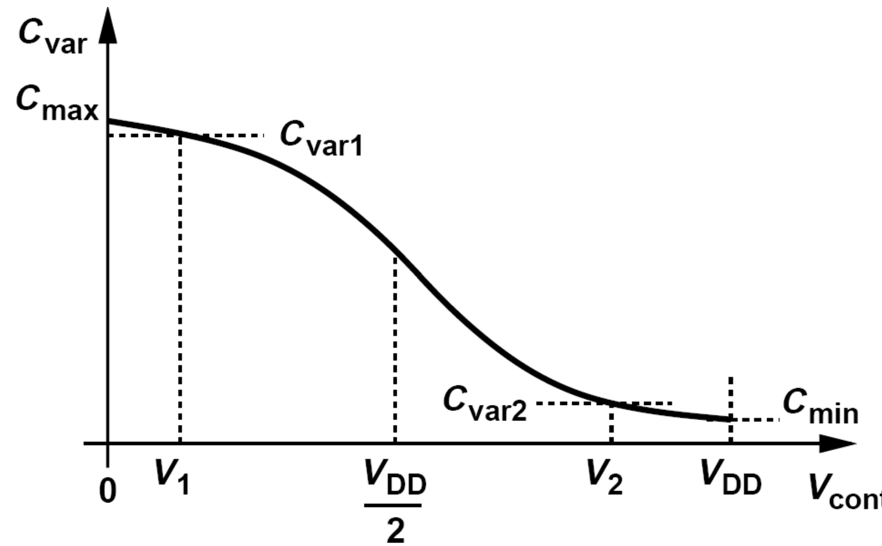
$$\frac{1}{Q_{tot}} = \frac{1}{Q_L} + \frac{C_2}{C_{tot}} \frac{1}{Q_2} + \cdots + \frac{C_n}{C_{tot}} \frac{1}{Q_n}$$

LC VCOs with Wide Tuning Range: VCOs with Continuous Tuning

- **We seek oscillator topologies that allow both positive and negative (average) voltages across the varactors, utilizing almost the entire range from C_{min} to C_{max}.**







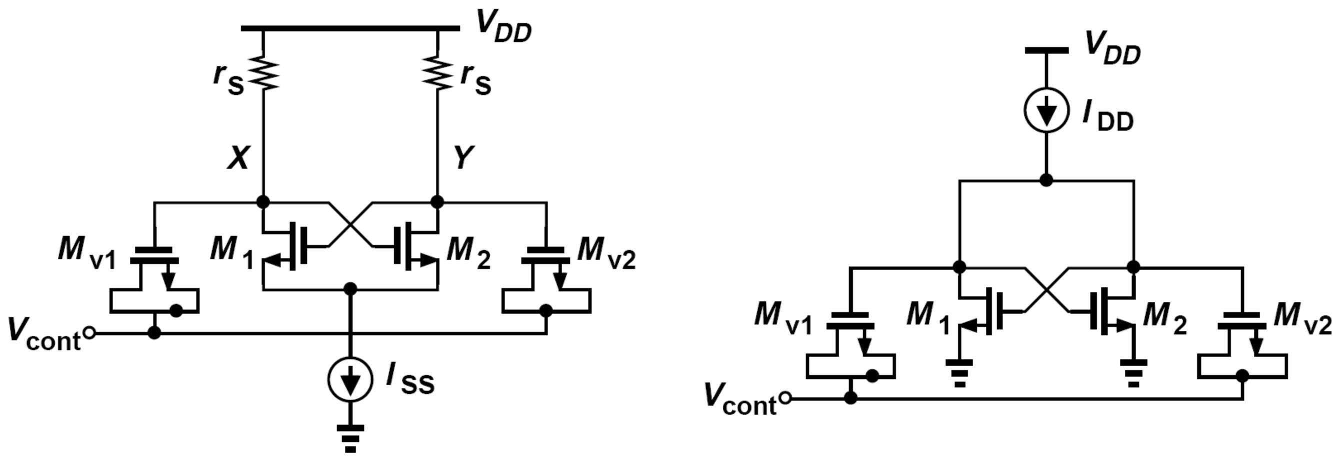
The CM level is simply given by the gate-source voltage of a diode-connected transistor carrying a current of $I_{DD}/2$.

$$V_{GS1,2} = \sqrt{\frac{I_{DD}}{\mu_n C_{ox}(W/L)}} + V_{TH}$$

We select the transistor dimensions such that the CM level is approximately equal to $V_{DD}/2$. Consequently, as V_{cont} varies from 0 to V_{DD}, the gate-source voltage of the varactors, $V_{GS,var}$, goes from $+V_{DD}/2$ to $-V_{DD}/2$,

Output CM Dependence on Bias Current

The tail or top bias current in the above oscillators is changed by DI. Determine the change in the voltage across the varactors.



Each inductor contains a small low-frequency resistance, r_s . If I_{SS} changes by ΔI, the output CM level changes by $\Delta V_{CM} = (\Delta I/2)r_s$, and so does the voltage across each varactor. In the top-biased circuit, on the other hand, a change of ΔI flows through two diode-connected transistors, producing an output CM change of $\Delta V_{CM} = (\Delta I/2)(1/g_m)$. Since $1/g_m$ is typically in the range of a few hundred ohms, the top-biased topology suffers from a much higher varactor voltage modulation.

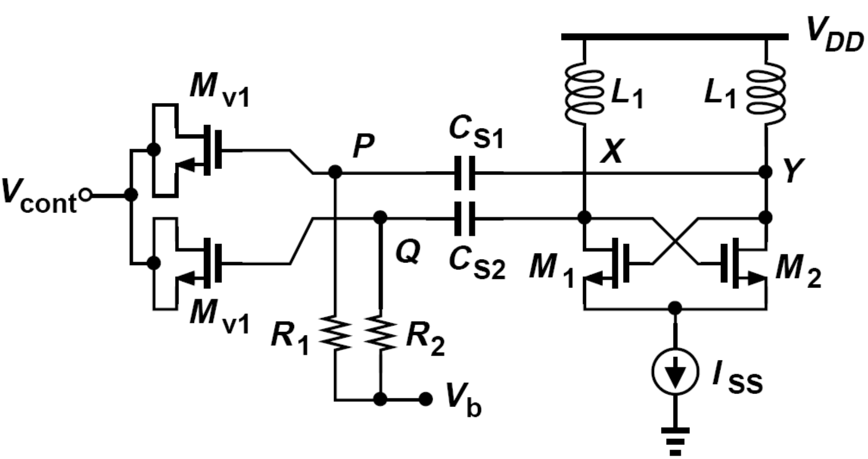
What is the change in the oscillation frequency in the above example?

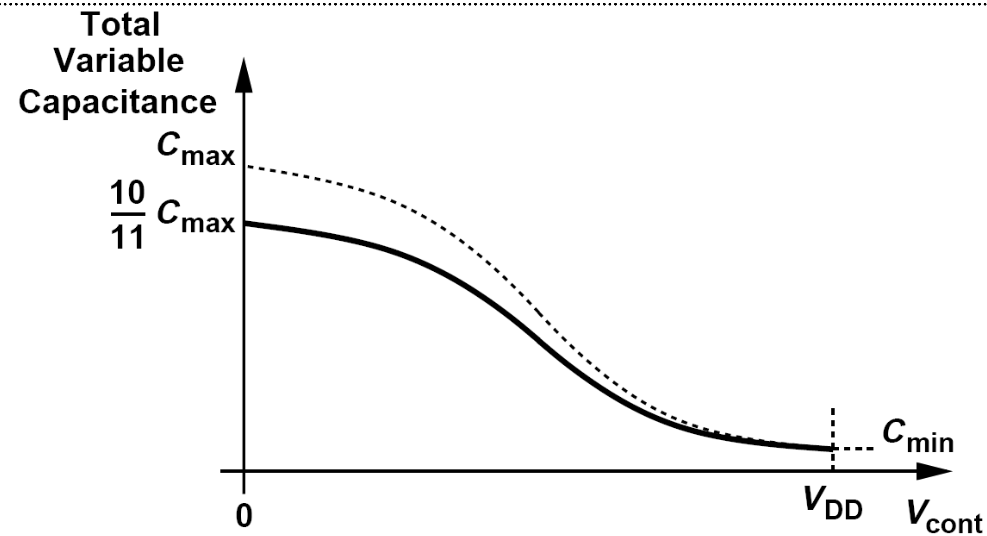
Since a CM change at *X* and *Y* is indistinguishable from a change in V_{cont}, we have

$$\begin{aligned}\Delta\omega &= K_{VCO}\Delta V_{CM} \\ &= K_{VCO}\frac{\Delta I}{2}r_s \quad \text{or} \quad K_{VCO}\frac{\Delta I}{2}\frac{1}{g_m}\end{aligned}$$

VCO Using Capacitor Coupling to Varactors

- In order to avoid varactor modulation due to the noise of the bias current source, we return to the tail-biased topology but employ ac coupling between the varactors and the core so as to allow positive and negative voltages across the varactors.



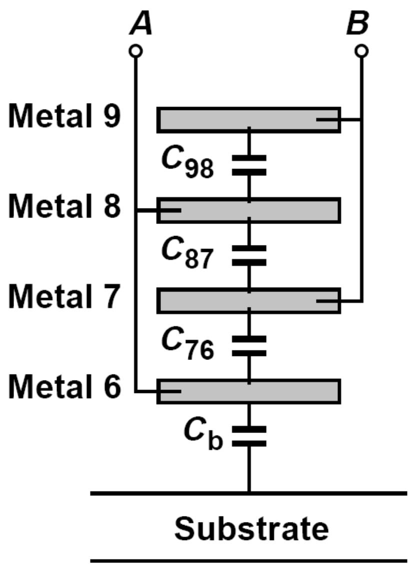


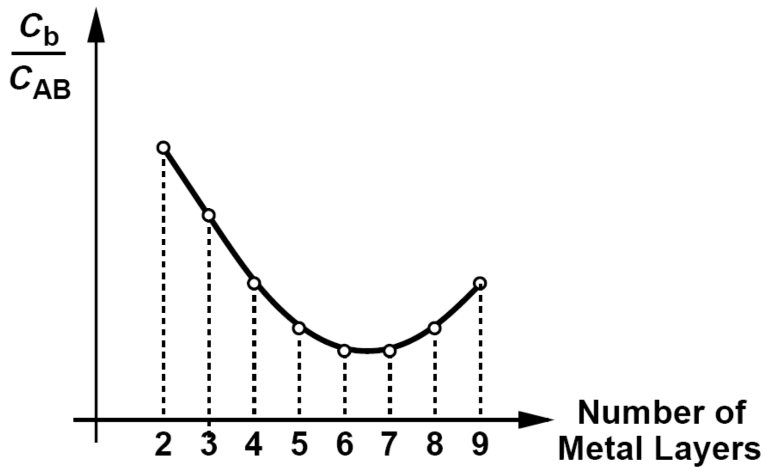
- The principal drawback of the above circuit stems from the parasitics of the coupling capacitors.

$$\Delta\omega_{os} \approx \frac{1}{\sqrt{L_1 C_1}} \cdot \frac{1}{2C_1} \cdot \frac{C_S^2(C_{var2} - C_{var1})}{(C_S + C_{var2})(C_S + C_{var1})}$$

VCO Using Capacitor Coupling to Varactors: Parasitic Capacitances to the Substrate

- The choice of $C_S = 10C_{max}$ reduces the capacitance range by 10% but introduces substantial parasitic capacitances at X and Y or at P and Q because integrated capacitors suffer from parasitic capacitances to the substrate.





- C_b/C_{AB} typically exceeds 5%.

VCO Using Capacitor Coupling to Varactors: Effect of the Parasitics of C_{S1} and C_{S2}

- A larger C_1 further limits the tuning range.

$$\Delta\omega_{osc} \approx \frac{1}{\sqrt{L_1(C_1+0.5C_{max})}} \times \frac{1}{2(C_1+0.5C_{max})} \times \frac{C_S^2(C_{max}-0.5C_{max})}{(10C_{max}+C_{max})(10C_{max}+0.5C_{max})}$$

$$\approx \frac{1}{\sqrt{L_1(C_1+0.5C_{max})}} \times \frac{0.43C_{max}}{2(C_1+0.5C_{max})}.$$

The VCO above is designed for a tuning range of 10% without the series effect of C_S and parallel effect of C_b. If $C_S = 10C_{max}$, $C_{max} = 2C_{min}$, and $C_b = 0.05C_S$, determine the actual tuning range.

Without the effects of C_S and C_b $\Delta\omega_{osc} \approx \frac{1}{\sqrt{L_1C_1}} \frac{0.5C_{max}}{2C_1}$

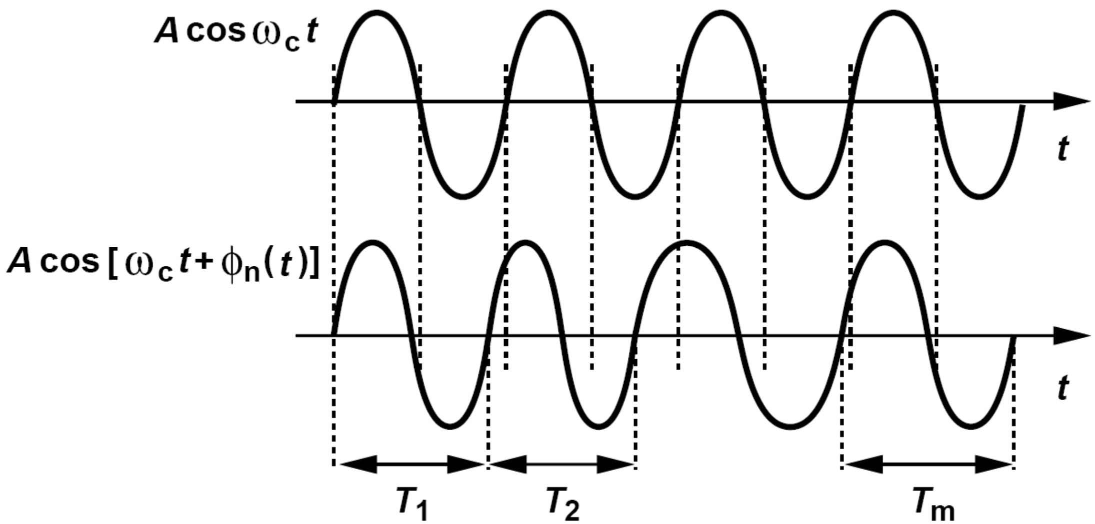
For this range to reach 10% of the center frequency, we have $C_{max} = \frac{2}{5}C_1$

With the effects of C_S and C_b

$$\Delta\omega_{osc} \approx \frac{1}{\sqrt{L_1(1.2C_1)}} \times \frac{0.43}{6}$$

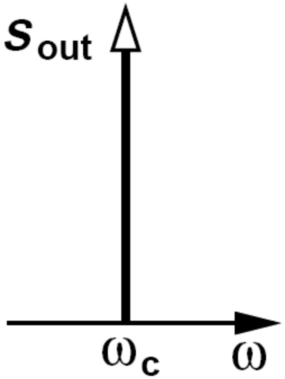
$$\approx \frac{7.2\%}{\sqrt{1.2L_1C_1}}.$$

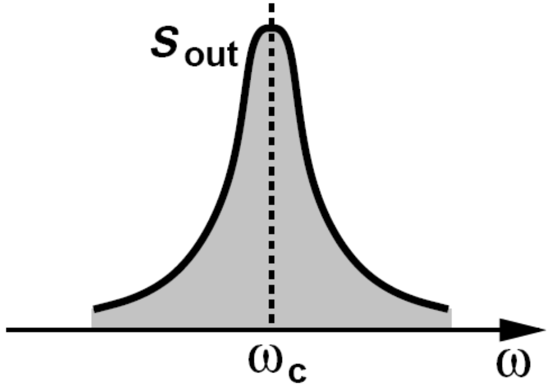
Phase Noise: Basic Concepts



- The noise of the oscillator devices randomly perturbs the zero crossings. To model this perturbation, we write $x(t) = A\cos[\omega_c t + \Phi_n(t)]$, The term $\Phi_n(t)$ is called the "phase noise."

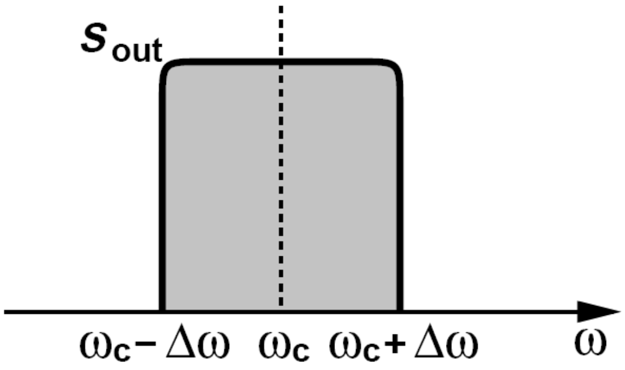
- From another perspective, the frequency experiences random variations, i.e., it departs from ω_c occasionally.





Phase Noise: Declining Phase Noise "Skirts"

Explain why the broadened impulse cannot assume the shape shown below.

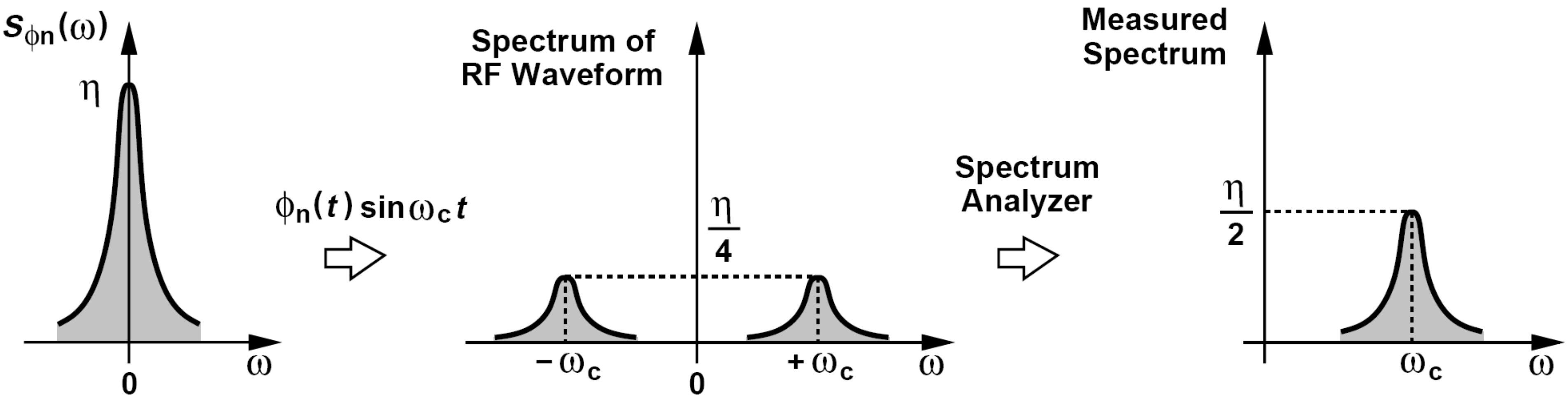


This spectrum occurs if the oscillator frequency has equal probability of appearing anywhere between $\omega_c - \Delta\omega$ and $\omega_c + \Delta\omega$. However, we intuitively expect that the oscillator prefers ω_c to other frequencies, thus spending lesser time at frequencies that are farther from ω_c. This explains the declining phase noise "skirts".

The spectrum can be related to the time-domain expression.

$$\begin{aligned} x(t) &= A\cos[\omega_c t + \phi_n(t)] \\ &\approx A\cos\omega_c t - A\sin\omega_c t \sin[\phi_n(t)] \\ &\approx A\cos\omega_c t - A\phi_n(t)\sin\omega_c t. \end{aligned}$$

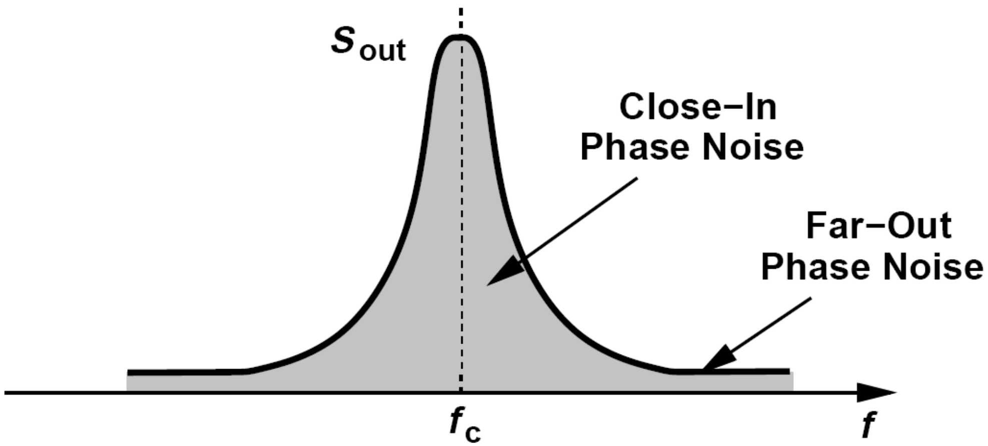
Various Factors of 4 and 2



- (1) since $\Phi_n(t)$ in equation above is multiplied by sin $\omega_c t$, its power spectral density, $S_{\phi n}$, is multiplied by 1/4 as it is translated to $\pm\omega_c$;
- (2) A spectrum analyzer measuring the resulting spectrum folds the negative frequency spectrum atop the positive-frequency spectrum, raising the spectral density by a factor of 2.

How is the Phase Noise Quantified?

- Since the phase noise falls at frequencies farther from ω_c, it must be specified at a certain "frequency offset," i.e., a certain difference with respect to ω_c.
- We consider a 1-Hz bandwidth of the spectrum at an offset of Δf, measure the power in this bandwidth, and normalize the result to the "carrier power", called "dB with respect to the carrier".

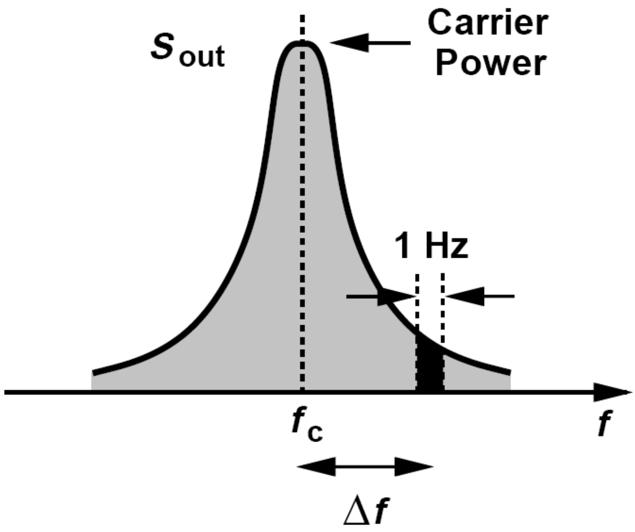


- In practice, the phase noise reaches a constant floor at large frequency offsets (beyond a few megahertz).
- We call the regions near and far from the carrier the "close-in" and the "far-out" phase noise, respectively.

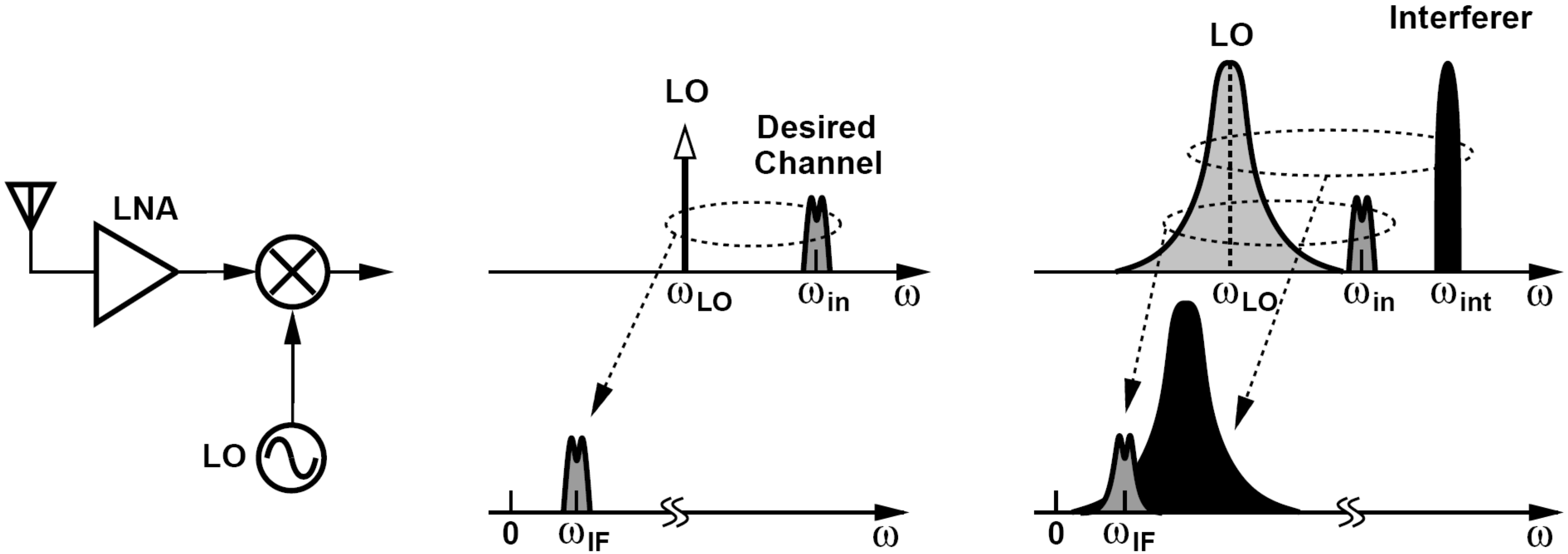
Specification of Phase Noise

At high carrier frequencies, it is difficult to measure the noise power in a 1-Hz bandwidth. Suppose a spectrum analyzer measures a noise power of -70 dBm in a 1-kHz bandwidth at 1-MHz offset. How much is the phase noise at this offset if the average oscillator output power is -2 dBm?

Since a 1-kHz bandwidth carries 10 log(1000 Hz) = 30 dB higher noise than a 1-Hz bandwidth, we conclude that the noise power in 1 Hz is equal to -100 dBm. Normalized to the carrier power, this value translates to a phase noise of -98 dBc/Hz.



Effect of Phase Noise: Reciprocal Mixing



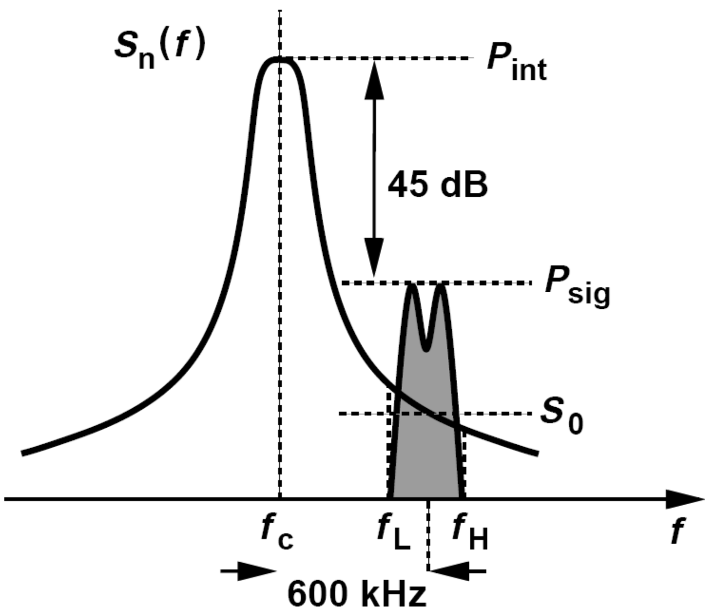
- Referring to the ideal case depicted above (middle), we observe that the desired channel is convolved with the impulse at ω_{LO}, yielding an IF signal at $\omega_{IF} = \omega_{in} - \omega_{LO}$.
- Now, suppose the LO suffers from phase noise and the desired signal is accompanied by a large interferer. The convolution of the desired signal and the interferer with the noisy LO spectrum results in a broadened downconverted interferer whose noise skirt corrupts the desired IF signal.
- This phenomenon is called “reciprocal mixing.”

Example of Reciprocal Mixing

A GSM receiver must withstand an interferer located three channels away from the desired channel and 45 dB higher. Estimate the maximum tolerable phase noise of the LO if the corruption due to reciprocal mixing must remain 15 dB below the desired signal.

The total noise power introduced by the interferer in the desired channel is equal to

$$P_{n,tot} = \int_{f_L}^{f_H} S_n(f)df$$



For simplicity, we assume $S_n(f)$ is relatively flat in this bandwidth and equal to S_0,

$$\text{SNR} = \frac{P_{sig}}{S_0(f_H - f_L)}$$

which must be at least 15 dB.

$$10\log\frac{S_0}{P_{sig}} = -15\text{ dB} - 10\log(f_H - f_L)$$

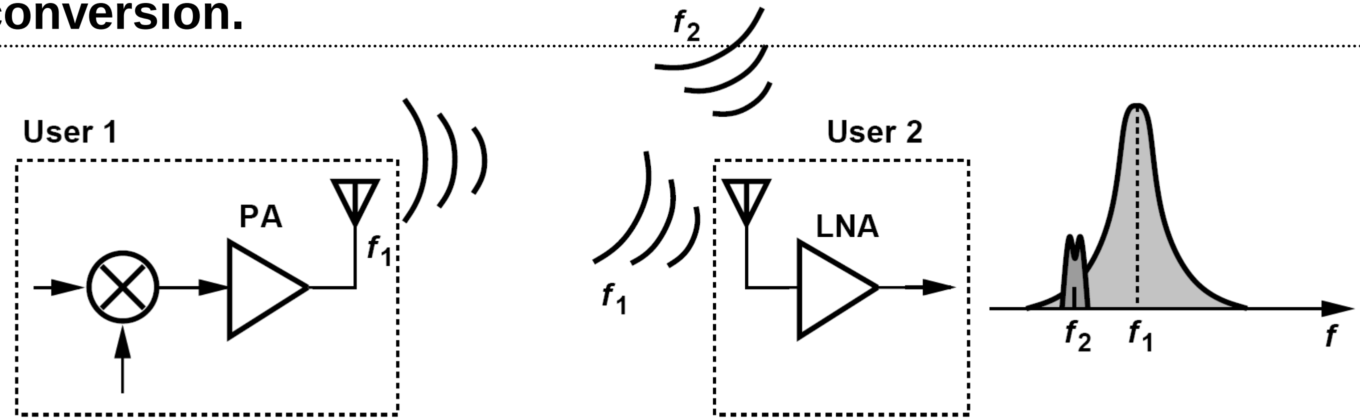
$$10\log\frac{S_0}{P_{int}} = -15\text{ dB} - 10\log(f_H - f_L) - 45\text{ dB}$$

If f_H - f_L = 200 kHz, then

$$10\log\frac{S_0}{P_{int}} = -113\text{ dBc/Hz at } 600\text{–kHz offset}$$

Received Noise due to Phase Noise of an Unwanted Signal

- In figure below, two users are located in close proximity, with user #1 transmitting a high-power signal at f_1 and user #2 receiving this signal and a weak signal at f_2. If f_1 and f_2 are only a few channels apart, the phase noise skirt masking the signal received by user #2 greatly corrupts it even *before* downconversion.



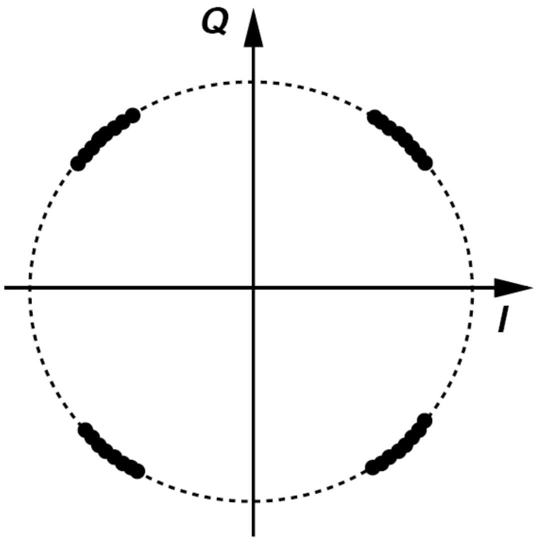
A student reasons that, if the interferer at f_1 above is so large that its phase noise corrupts the reception by user #2, then it also heavily compresses the receiver of user #2. Is this true?

Not necessarily. An interferer, say, 50 dB above the desired signal produces phase noise skirts that are not negligible. For example, the desired signal may have a level of -90 dBm and the interferer, -40 dBm. Since most receivers' 1-dB compression point is well above -40 dBm, user #2's receiver experiences no desensitization, but the phenomenon above is still critical.

Corruption of a QPSK Signal due to Phase Noise

- **Since the phase noise is indistinguishable from phase (or frequency) modulation, the mixing of the signal with a noisy LO in the TX or RX path corrupts the information carried by the signal.**

$$x_{QPSK}(t) = A\cos\left[\omega_c t + (2k+1)\frac{\pi}{4} + \phi_n(t)\right] \quad k = 0, \cdots, 3$$

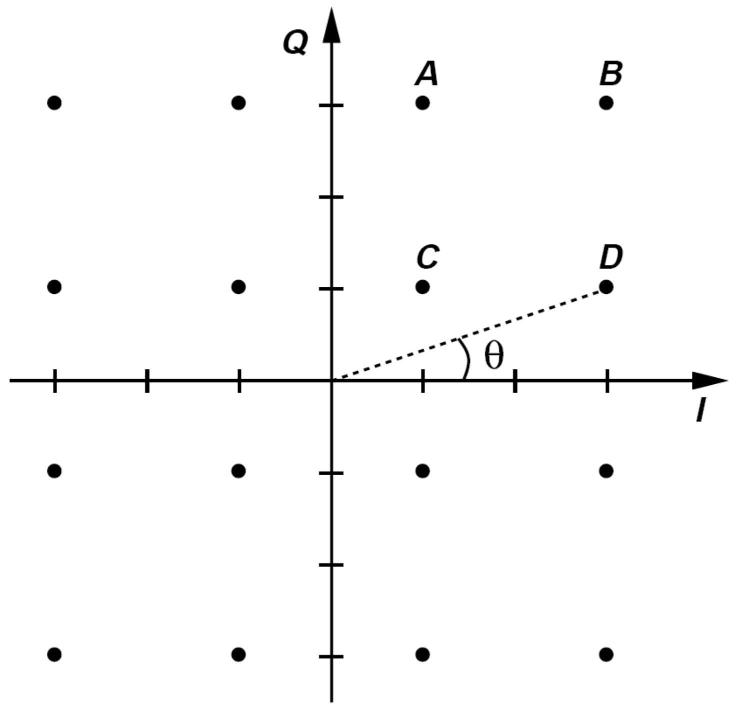


- **The constellation points experience only random rotation around the origin. If large enough, phase noise and other nonidealities move a constellation point to another quadrant, creating an error.**

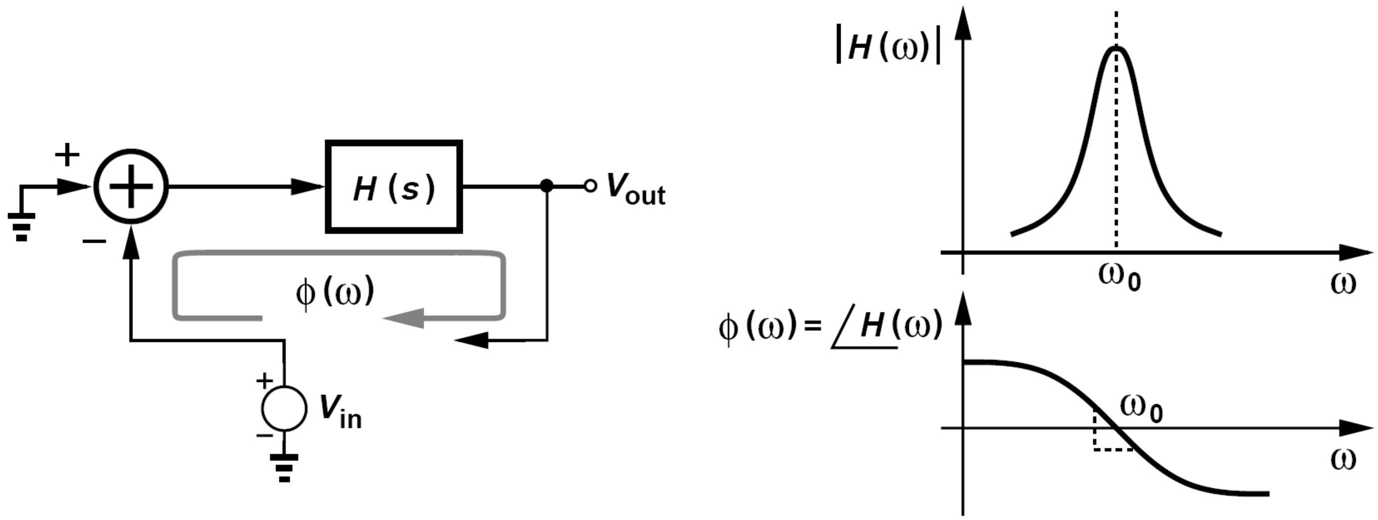
Phase Noise Corruption on 16-QAM Constellation

Which points in a 16-QAM constellation are most sensitive to phase noise?

Consider the four points in the top right quadrant. Points *B* and *C* can tolerate a rotation of 45° before they move to adjacent quadrants. Points *A* and *D*, on the other hand, can rotate by only $\theta = \tan^{-1}(1/3) = 18.4°$. Thus, the eight outer points near the *I* and *Q* axes are most sensitive to phase noise.



Analysis of Phase Noise: Approach I --- Q of an Oscillator



- **Another definition of the Q that is especially well-suited to oscillators is shown above, where the circuit is viewed as a feedback system and the phase of the open-loop transfer function, is examined at the resonance frequency.**

$$Q = \frac{\omega_0}{2}\left|\frac{d\phi}{d\omega}\right|$$

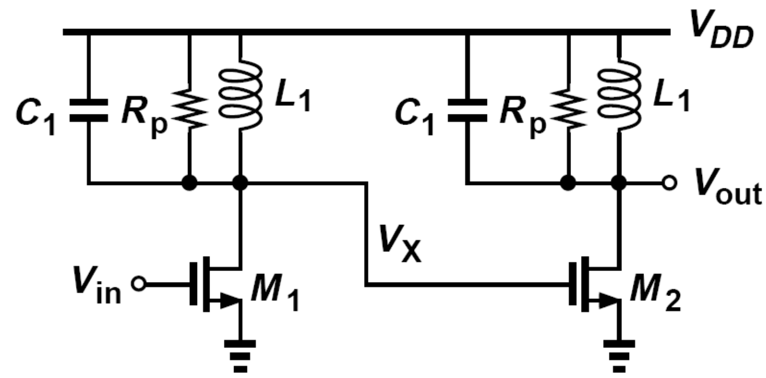
- **Oscillators with a high open-loop Q tend to spend less time at frequencies other than ω_0.**

Open-Loop Model of a Cross-Coupled Oscillator

Compute the open-loop *Q* of a cross-coupled LC oscillator.

Since at *s* = *jω*,

$$\frac{V_X}{V_{in}}(j\omega) = \frac{-jg_m R_p L_1 \omega}{R_p(1 - L_1 C_1 \omega^2) + jL_1\omega}$$



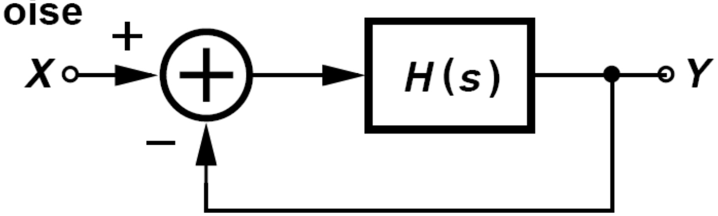
We have

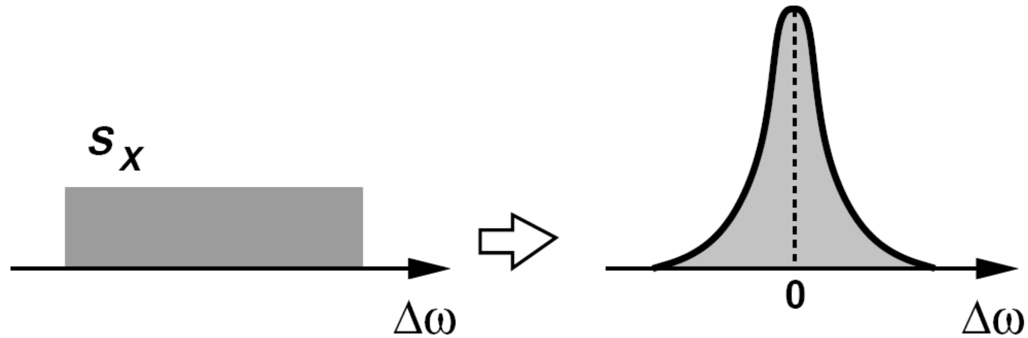
$$\angle H(j\omega) = 2\left[-\frac{\pi}{2} - \tan^{-1}\frac{L_1\omega}{R_p(1 - L_1C_1\omega^2)}\right]$$

$$\left|\frac{\omega_0}{2}\frac{d\angle H(j\omega)}{d\omega}\right|\Bigg|_{\omega 0} = 2R_pC_1\omega_0 = 2Q_{tank},$$

This result is to be expected: the cascade of frequency-selective stages makes the phase transition sharper than that of one stage.

Noise Shaping in Oscillators(I)





$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + H(s)}$$

In the vicinity of the oscillation frequency, we can approximate *H(jω)* with the first two terms in its Taylor series:

$$H(j\omega) \approx H(j\omega_0) + \Delta\omega \frac{dH}{d\omega}$$

If $H(j\omega_0)$ = -1 and $\Delta\omega dH/d\omega$ << 1,

$$\frac{Y}{X}(j\omega_0 + j\Delta\omega) \approx \frac{-1}{\Delta\omega \frac{dH}{d\omega}}$$

The noise spectrum is "shaped" by

$$\left|\frac{Y}{X}(j\omega_0 + j\Delta\omega)\right|^2 = \frac{1}{\Delta\omega^2 \left|\frac{dH}{d\omega}\right|^2}$$

Noise Shaping in Oscillators (Ⅱ)

To determine the shape of |*dH/dω*|^2, we write *H(jω)* in polar form, and differentiate with respect to *ω*,

$$\frac{dH}{d\omega} = \left(\frac{d|H|}{d\omega} + j|H|\frac{d\phi}{d\omega}\right)\exp(j\phi)$$

$$|\frac{dH}{d\omega}|^2 = |\frac{d|H|}{d\omega}|^2 + |\frac{d\phi}{d\omega}|^2|H|^2$$

Note that (a) in an LC oscillator, the term |*d*|*H*|/*dω*|^2 is much less than |*dΦ/dω*|^2 in the vicinity of the resonance frequency, and (b) |*H*| is close to unity for steady oscillations.

$$|\frac{Y}{X}(j\omega_0 + j\Delta\omega)|^2 = \frac{1}{\frac{\omega_0^2}{4}|\frac{d\phi}{d\omega}|^2}\frac{\omega_0^2}{4\Delta\omega^2}$$

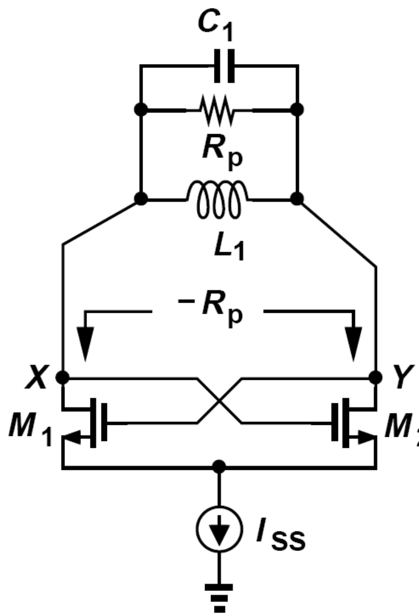
$$|\frac{Y}{X}(j\omega_0 + j\Delta\omega)|^2 = \frac{1}{4Q^2}\left(\frac{\omega_0}{\Delta\omega}\right)^2$$

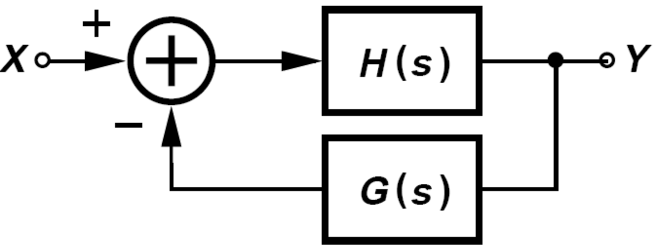
Known as "Leeson's Equation", this result reaffirms our intuition that the open-loop *Q* signifies how much the oscillator rejects the noise.

Apparently Infinite Q in an Oscillator

A student designs the cross-coupled oscillator below with $2/g_m = 2Rp$, reasoning that the tank now has infinite Q and hence the oscillator produces no phase noise! Explain the flaw in this argument.

The Q in equation above is the open-loop Q, i.e., $\omega_0/2$ times the slope of the phase of the open-loop transfer function, which was calculated in previous example. The "closed-loop" Q does not carry much meaning.





If the feedback path has a transfer function G(s), then

$$|\frac{Y}{X}(j\omega_0 + j\Delta\omega)|^2 = \frac{1}{4Q^2}\left(\frac{\omega_0}{\Delta\omega}\right)^2 |\frac{1}{G(j\omega_0)}|^2$$

$$Q = \frac{\omega_0}{2}|\frac{d(GH)}{d\omega}|$$

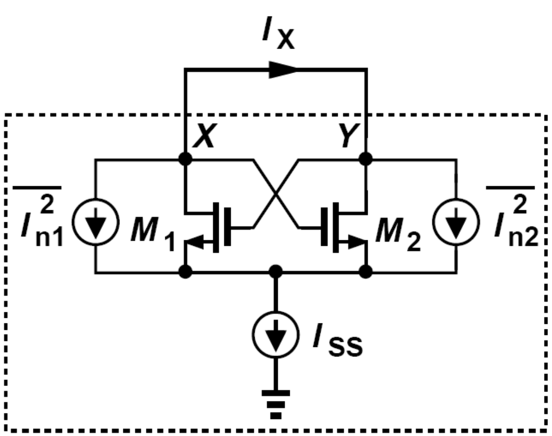
Linear Model (I)

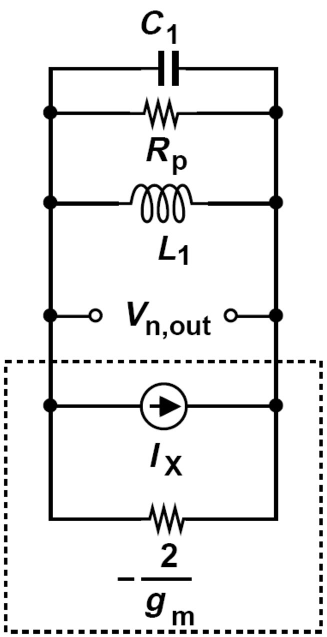
- The small-signal (linear) model may ignore some important effects, e.g., the noise of the tail current source, or face other difficulties.

Compute the total noise injected to the differential output of the cross-coupled oscillator when the transistors are in equilibrium. Note that the two-sided spectral density of the drain current noise is equal to $\overline{I_n^2} = 2kT\gamma g_m$.

The output noise is obtained as

$$\overline{V_{n,out}^2} = \left(\overline{I_X^2} + \frac{2kT}{R_p}\right) \frac{R^2 L_1^2 \omega^2}{R^2(1 - L_1 C_1 \omega^2)^2 + L_1^2 \omega^2}$$

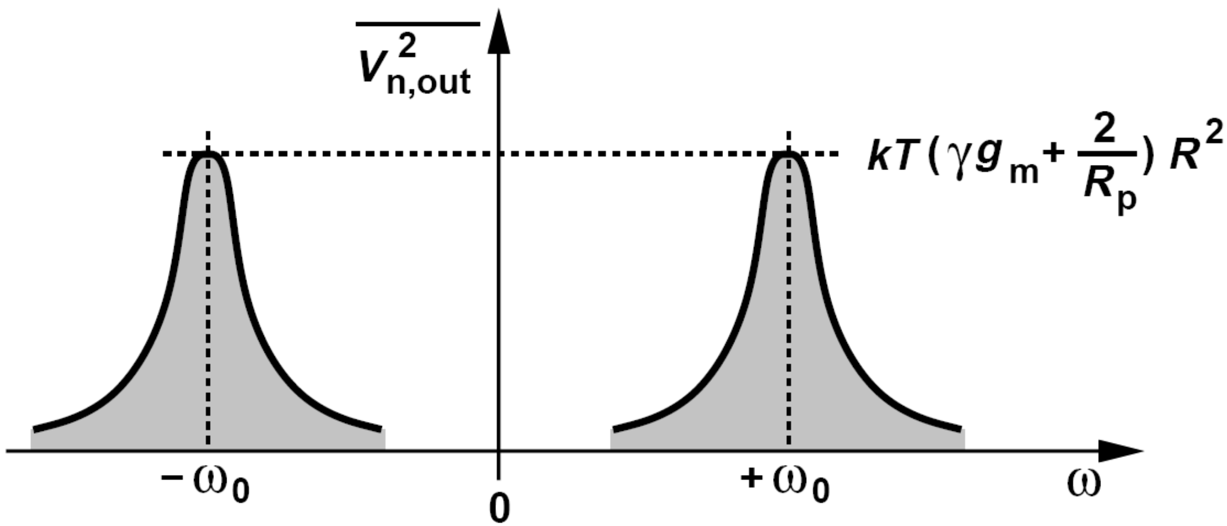




Linear Model (Ⅱ)

Since I_{n1} and I_{n2} are uncorrelated

$$\overline{V_{n,out}^2} = \left(kT\gamma g_m + \frac{2kT}{R_p}\right)\frac{R^2L_1^2\omega^2}{R^2(1-L_1C_1\omega^2)^2+L_1^2\omega^2}$$

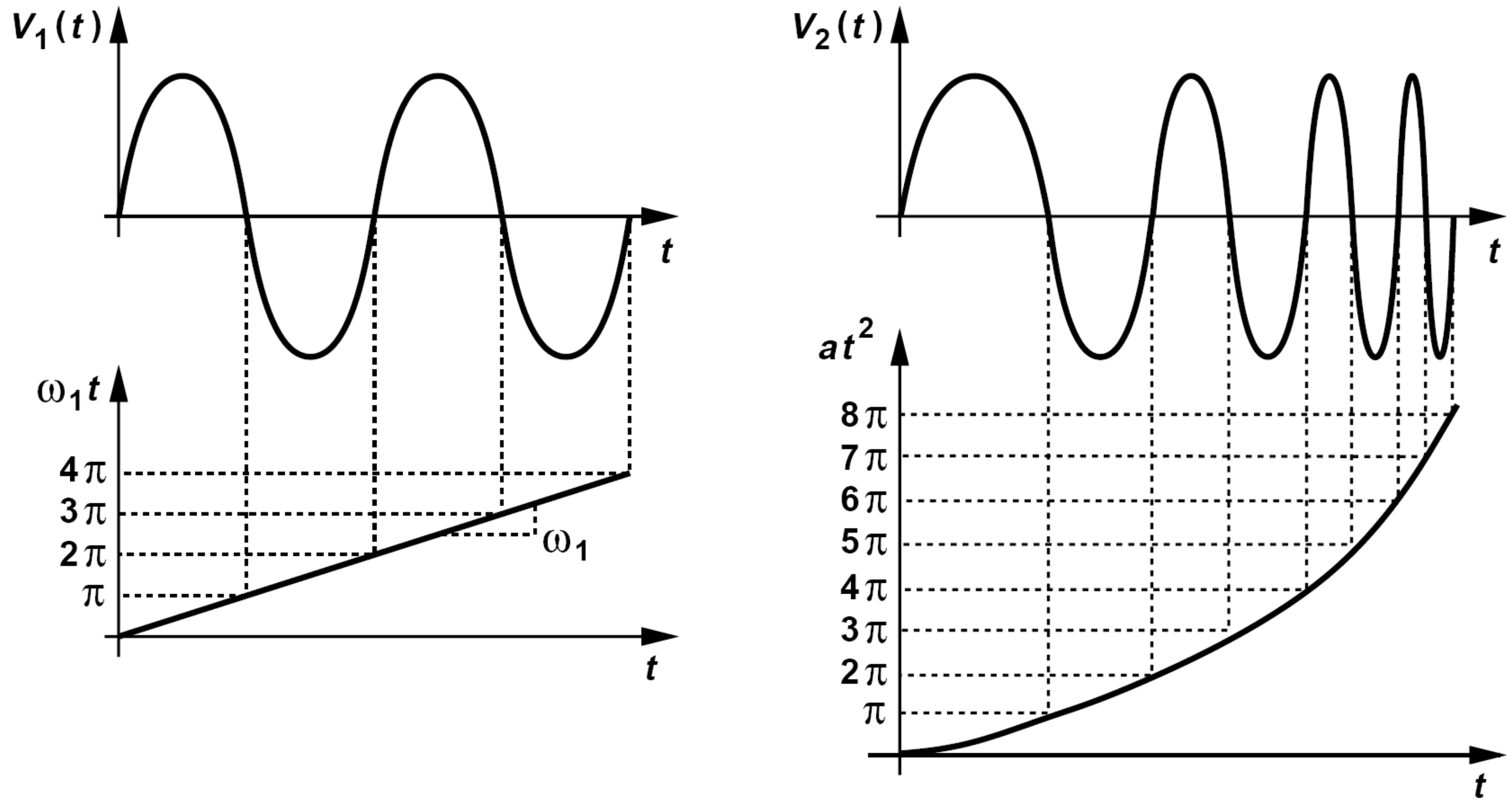


Unfortunately, this result contradicts Leeson's equation. g_m is typically quite higher than $2/R_p$ and hence $R \neq \infty$.

Mathematical Model of VCOs: Linear and Quadrature Growth of Phase with Time

Plot the waveforms for $V_1(t) = V_0 \sin\omega_1 t$ and $V_2(t) = V_0 \sin(at^2)$.

To plot these waveforms carefully, we must determine the time instants at which the argument of the sine reaches integer multiples of π. For $V_1(t)$, the argument, $\omega_1 t$, rises linearly with time, crossing $k\pi$ at $t = \pi k/\omega_1$. For $V_2(t)$, on the other hand, the argument rises increasingly faster with time, crossing $k\pi$ more frequently.

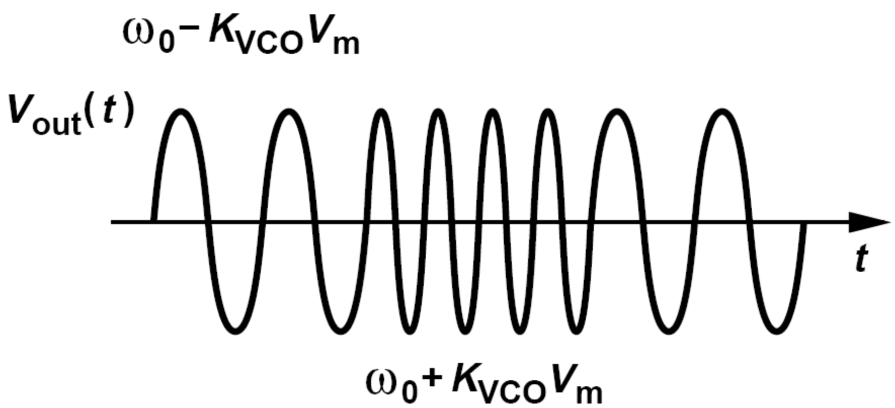


Example of Mathematical Model of VCOs (I)

Since a sinusoid of constant frequency !1 can be expressed as V0 cos !1t, a student surmises that the output waveform of a VCO can be written as

$$V_{out}(t) = V_0 \cos \omega_{out} t$$
$$= V_0 \cos(\omega_0 + K_{VCO} V_{cont}) t$$

Explain why this is incorrect.

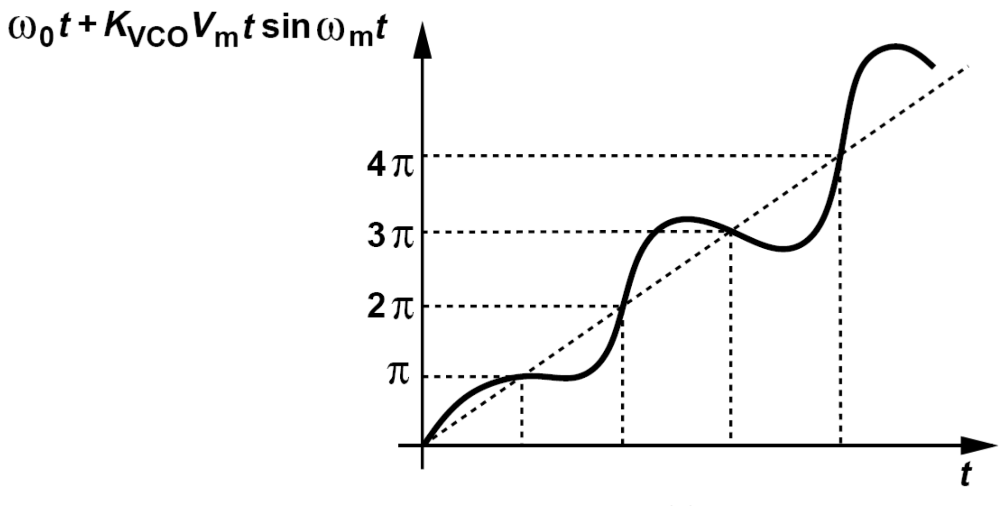


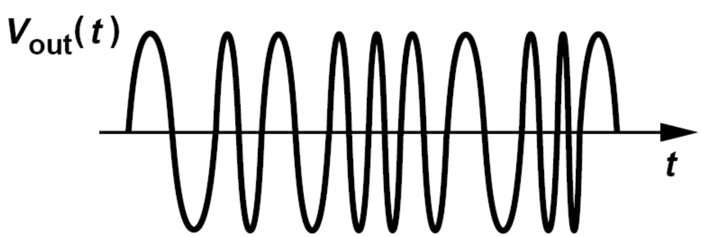
As an example, suppose $V_{cont} = V_m \sin \omega_m t$, i.e., the frequency of the oscillator is modulated periodically. Intuitively, we expect the output waveform frequency periodically swings between $\omega_0 + K_{VCO} V_m$ and $\omega_0 - K_{VCO} V_m$, i.e., has a "peak deviation" of $\pm K_{VCO} V_m$. However, the student's expression yields

$$V_{out}(t) = V_0 \cos[\omega_0 t + K_{VCO} V_m (\sin \omega_m t) t]$$

Example of Mathematical Model of VCOs (Ⅱ)

We plot the overall argument and draw horizontal lines corresponding to *kπ*.





The intersection of each horizontal line with the phase plot signifies the zero crossings of $V_{out}(t)$. Thus, $V_{out}(t)$ appears as shown above. The key point here is that the VCO frequency is not modulated periodically.

VCO as a Frequency Modulator

Let us now consider an unmodulated sinusoid, $V_1(t) = V_0 \sin \omega_1 t$. Called the "total phase," the argument of the sine, $\omega_1 t$, varies linearly with time in this case, exhibiting a slope of ω_1.

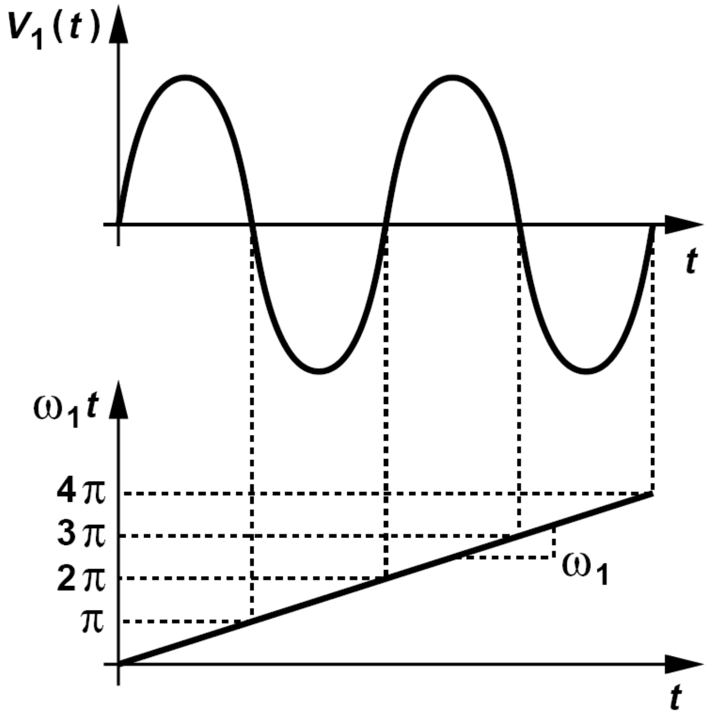
Define the instantaneous frequency as the time derivative of the phase:

$$\omega = \frac{d\phi}{dt}$$

$$\Rightarrow \quad \phi = \int \omega dt + \phi_0$$

Since a VCO exhibits an output frequency given by $\omega_0 + K_{VCO}V_{cont}$, we can express its output waveform as

$$V_{out}(t) = V_0 \cos(\int \omega_{out} dt) = V_0 \cos(\omega_0 t + K_{VCO} \int V_{cont} dt)$$



- **A VCO is simply a frequency modulator. For example, the narrow-band FM approximation holds here as well.**

Frequency Modulation by a Square Wave

A VCO experiences a small square-wave disturbance on its control voltage. Determine the output spectrum.

We expand the square wave in its Fourier series,

$$V_{cont}(t) = a\left(\frac{4}{\pi}\cos\omega_m t - \frac{1}{3}\frac{4}{\pi}\cos 3\omega_m t + \cdots\right)$$

$$\Rightarrow V_{out}(t) = V_0\cos\left[\omega_0 t - K_{VCO}a\left(\frac{1}{\omega_m}\frac{4}{\pi}\sin\omega_m t - \frac{1}{9\omega_m}\frac{4}{\pi}\sin 3\omega_m t + \cdots\right)\right]$$

If $4K_{VCO}a/(\pi\omega_m)$ << 1 rad, then the narrow-band FM approximation applies:

$$V_{out}(t) \approx V_0\cos\omega_0 t + \left[K_{VCO}a\left(\frac{1}{\omega_m}\frac{4}{\pi}\sin\omega_m t - \frac{1}{9\omega_m}\frac{4}{\pi}\sin 3\omega_m t + \cdots\right)\right]V_0\sin\omega_0 t$$