Physics 152H Course Notes for Mon 10/1 and Wed 10/3 2001

Readings: Serway: Ch 23, Secs. 4, 6, 7; H,R&W: Ch 23, Secs. 1-4, and 8.

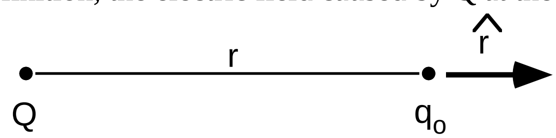
Electric Field. Consider a set of point charges, q_1, q_2, ..., q_n fixed in space. If a test charge, q_o, is placed at a point in space and it experiences an electric force, $\mathbf{F}_{qo}$, caused by the presence of the charges, q_1, q_2, ..., q_n, then the electric field caused by the charges, q_1, q_2, ..., q_n , at that point in space is defined by:

$$\vec{E} \equiv \frac{\vec{F}_{q_o}}{q_o} \ .$$

NOTE:
1. $\vec{E}$ is a vector, and hence has a magnitude and a direction.
2. The electric field, $\vec{E}$, as defined above is caused by the charges, q_1, q_2, ..., q_n. It is not caused by the test charge, q_o.
3. An electric field, $\vec{E}$, caused by the charges, q_1, q_2, ..., q_n, will exist at the position of the test charge, q_o, whether or not the test charge is there.
4. If you know the $\vec{E}$ field at a point in space caused by the charges, q_1, q_2, ..., q_n,, then you could find the force on any charge, q, at that point by taking the product: $F_q = qE$

Electric Field Due to a Point Charge. Let's assume that a charge Q is near another charge, q_o, in a region of space (see diagram). By definition, the electric field caused by Q at the position of q_o is given by:

$$\vec{E}_Q = \frac{\vec{F}_{q_o}}{q_o} = \frac{\left(\frac{kQq_o}{r^2}\hat{r}\right)}{q_o} = \frac{kQ}{r^2}\hat{r}$$



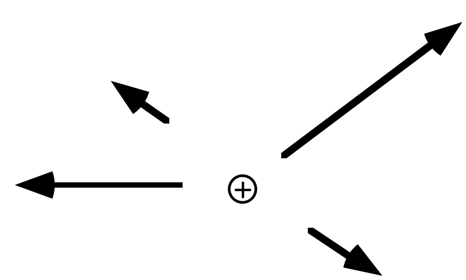
where the unit vector, $\hat{r}$, always points away from Q.

NOTE:
1. If Q is +, then E_Q points away from Q; if Q is -, E_Q points toward Q.
2. The length (i.e., magnitude) of $\vec{E}$ diminishes as you get farther from Q according to the inverse square of the distance $(1/r^2)$.

If there are several charges in space, then the $\vec{E}$ field at a point in space caused by these charges is given by applying the definition of E together with superposition. That is, compute the electric field due to each charge at the point in space that is of interest to you, and add up all these electric field vectors to find the total electric field. The electric field caused by N point charges can be written as:

$$\vec{E} = k\sum_{i=1}^{N} \frac{q_i}{r_i^2}\hat{r}_i \ .$$

At the right is a figure showing the electric field vectors at different points in space for a + charge.



Example 1: Look back at the example worked out in your notes for Mon. 10/1-Wed. 10/3. Convince yourself that the electric fields due to the two charges, q, at position (0,b) for parts a and c of that problem are:

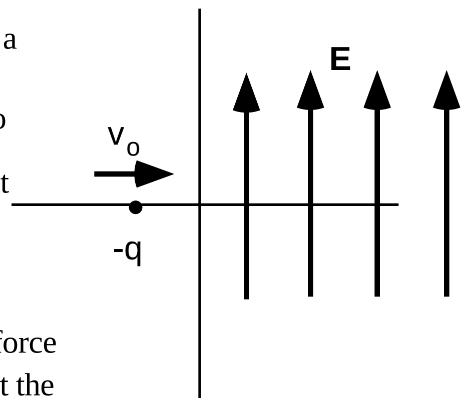
a) $\vec{E} = \dfrac{2kqb}{\left(a^2+b^2\right)^{3/2}}\hat{j}$ c) $\vec{E} = \dfrac{2kqa}{\left(a^2+b^2\right)^{3/2}}\hat{i}$

Other than at , ∞, where would the electric field due to the two small charges be 0 for parts a and c of that problem?

<u>Solution</u>: For part a), the elctric field due to the two qs is 0 at the origin. For part c), there is no place other than ∞ where the total electric field due to the + and - q is 0.

Example 2: A negative charge, -q, of mass, m, traveling horizontally with a speed v_o enters a region of space where there is a constant electric field, $E\hat{j}$, directed in the positive y direction (see diagram). Assume the gravitational force is very small compared to the electrical force.

a) Find an expression for the y-position of the charge after it travels a distance d in the x-direction.
b) How long did it take the charge to travel to the position described in part a)?



<u>Strategy for solving both a) and b)</u>: The charge, -q, experiences a force in the $-\hat{j}$ direction given by Coulomb's Law (recall we are told that the Coulomb force is much larger than the gravitational force). This force, according to Newton's Second Law, causes the charge to accelerate in the $-\hat{j}$ direction. The velocity in the x-direction remains constant at $v_o\hat{i}$ since there is no acceleration in that direction. Hence, apply Newton's Second Law to find the acceleration, and use the kinematic equations to determine both the y-position of the particle after it travels a horizontal distance, d, and the time it took to get there.

a) and b) <u>Solution</u>. The force on the charge is: $\vec{F}_{-q} = -qE_o\hat{j}$. By Newton's Second Law, $\vec{F}_{net} = m\vec{a}$, the acceleration of the charge under the influence of this force is:

$$\vec{a}_{-q} = 0\hat{i} - \frac{qE_o}{m}\hat{j} \quad \text{(Note that the acceleration is in the -y-direction)}$$

To find the time it took the charge to travel a distance d along the x-direction, simply apply the following kinematic equation, noting that a_x and x_o are 0.

$$x = x_o + v_{o,x}t + \frac{1}{2}a_x t^2 = 0 + v_o t + 0 \quad \Rightarrow \quad t = \frac{d}{v_o}$$

To find the y-position (at the time above) when the particle is at an x-position equal to d, apply the same kinematic equation, noting that now there is an acceleration in the y-direction:

$$y = y_o + v_{o,y}t + \frac{1}{2}a_y t^2 = 0 + 0 + \frac{1}{2}\left(-\frac{qE_o}{m}\right)\left(\frac{d}{v_o}\right)^2 = -\frac{qd^2E_o}{2mv_o^2}$$