A Model for Technical Inefficiency Effects in a Stochastic Frontier Production Function for Panel Data
Published by: G.E. Battese, T.J. Coelli

Griva Anastasia, Mavropoulos Antonios, Panteli Antiopi, Vasiliki Spanou, Tsichlia Afroditi

University of Patras
Department of Economics

Friday, November 13th 2015

Theory of the Model

Abstract

A stochastic frontier production function is defined for panel data on firms, in which the non-negative technical inefficiency effects are assumed to be a function of firm-specific variables and time.(Aigner DJ, Lovell CAK, Schmidt P (1977) Formulation and estimation of stochastic frontier production function models. Journal of Econometrics 6:21-37).
The empirical model is obtained by using a ten years' data on paddy farmers from an Indian village.

Introduction

The stochastic frontier production function postulates the existence of technical inefficiencies of production of firms involved in producing a particular output.
These papers adopt a two-stage approach, in which:

- The first stage involves the specification and estimation of the stochastic frontier production function and the prediction of the technical inefficiency effects, under the assumption that these inefficiency effects are identically distributed.
- The second stage involves the specification of a regression model for the predicted technical inefficiency effects, which contradicts the assumption of identically distributed inefficiency effects in the stochastic frontier.

Inefficiency Frontier Model for Panel Data

$lnY_{it} = x_{it}\beta + V_{it} - U_{it}$ (1)

This equation specifies the stochastic frontier production function in terms of the original production values.

- Y_{it} denotes the production at the t-th observation $(t = 1, 2, ..., T)$ for the i-th firm $(i = 1, 2, ..., N)$
- x_{it} is a (1 x k) vector of values of known functions of inputs of production and other explanatory variables associated with the i-th firm at the t-th observation
- β is a (k x 1) vector of unknown parameters to be estimated
- $V_{it}s$ are assumed to be iid N(0,$\sigma_\nu{}^2$) random errors, independently distributed of the $U_{it}s$

The technical inefficiency effect

The explanatory variables in the inefficiency model may include some input variables in the stochastic frontier, provided the inefficiency effects are stochastic. If the first z-variable has value one and the coefficients of all other z-variables are zero, then this case represents the model specified in Stevenson (1980) and Battese and Coelli (1988, 1992).

Whereas If all elements of the f-vector are equal to zero, then the technical inefficiency effects are not related to the z-variables and so the half-normal distribution originally specified is obtained. If interactions between firm-specific variables and input variables are included as zvariables, then a non-neutral stochastic frontier, is obtained.

The technical inefficiency effect, U_i

Griva Anastasia, Mavropoulos Antonios, Panteli Antiopi, Vasiliki Spanou, Tsichlia Afroditi

$U_i = z_{it}\delta + W_{it}$
where the random variable, W_{it}, is defined by the truncation of the normal distribution with zero mean and variance,σ^2, such that the point of truncation is $-z_{it}\delta$, i.e.,$W_{it} >= -z_{it}\delta$. These assumptions are consistent with U, being a non-negative truncation of the $N(-z_{it}\delta, \sigma^2)$-distribution.

Technical Inefficiency Production function

The technical efficiency of production for the i-th firm at the t-th observation is defined by equation:
$TE_{it} = \exp(-U_{it}) = \exp(z_{it}\delta - W_{it})$
The assumption that the U,s and the V - ts are independently distributed for all t = 1, 2,..., T, and i = 1, 2, ..., N, is obviously a simplifying, but restrictive, condition. Alternative models are required to account for possible correlated structures of the technical inefficiency effects and the random errors in the frontier.The method of maximum likelihood is proposed for simultaneous estimation of the parameters of the stochastic frontier and the model for the technical inefficiency effects.

Empirical Application

Information on the age and years of schooling for 14 paddy farmers from Aurepalle are used to explain the differences in the inefficiency effects among the farmers.
A total of 125 observations are involved for a ten-year period from 1975-76 to 1984-85. The stochastic frontier production function is estimated to be:

- $\ln(Y_{it}) = \beta_0 + \beta_1 \; ln(Land_{it}) + \beta_2(PILand_{it}) + \beta_3 \ln(Labour_{it}) + \beta_4 \ln(Bullocks_{it}) + \beta_5 \ln[Max(Costs_{it}, 1 - D_{it})] + \beta_6(Year_{it}) + V_{it} - U_{it}$

where the technical inefficiency effects are assumed to be defined by:

- $U_{it} = \delta_0 + \delta_1(Age_{it}) + \delta_2(Schooling_{it}) + \delta_3(Year_{it}) + W_i$

Estimated Model

Maximum-likelihood estimates of the parameters of the model:
Stochastic Frontier:

- $\ln(Y) = \underset{(0.60)}{2.86} + \underset{(0.12)}{0.37}\ ln(Land) + \underset{(0.21)}{0.38}(PILand) + \underset{(0.12)}{0.85}\ln(Labour) - \underset{(0.11)}{0.33}\ln(Bullocks) + \underset{(0.031)}{0.071}\ln(Costs) + \underset{(0.60)}{0.014}(Year)$

Inneficiency Model:

- $U = -\underset{(2.8)}{1.5} + \underset{(0.034)}{0.035}Age - \underset{0.077)}{0.006}Schooling - \underset{(0.60)}{0.57}Year$

Empirical Analysis

Table 1. Tests of hypotheses for parameters of the inefficiency frontier model for paddy farmers in Aurepalle, $\gamma = \underset{(0.047)}{0.952}$

Null Hypothesis	x^2-value	Test Statistic*
$H_0 : \gamma = \delta_0 = \cdots = \delta_3 = 0$	12.59	29.99*
$H_0 : \gamma = 0$	7.82	26.97*
$H_0 : \delta_1 = \delta_2 = \delta_3 = 0$	7.82	10.69*

* An asterisk on the value of the test statistic indicates that it exceeds the 95th percentile for the corresponding x2-distribution and so the null hypothesis is rejected.

The estimate for the variance parameter, γ, is close to one, which indicates that the inefficiency effects are likely to be highly significant in the analysis of the value of output of the farmers.

Generalized likelihood-ratio tests of null hypotheses, that the inefficiency effects are absent or that they have simpler distributions, are presented in Table 1.
The first null hypothesis, which specifies that the inefficiency effects are absent from the model, is strongly rejected. The second null hypothesis, which specifies that the inefficiency effects are not stochastic, 6 is also strongly rejected. The third null hypothesis, considered in Table 1, specifies that the inefficiency effects are not a linear function of the age and schooling of the farmers and the year of observation. This null hypothesis is also rejected at the 5 o level of significance. This indicates that the joint effects of these three explanatory variables on the inefficiencies of production is significant although the individual effects of one or more of the variables may not be statistically significant.

Conclusions

The results indicate that the model for the technical inefficiency effects, involving a constant term, age and schooling of farmers and year of observation, is a significant component in the stochastic frontier production function. The application also illustrates that the model specification permits the estimation of both technical change and time-varying technical inefficiency, given that inefficiency effects are stochastic and have a known distribution.