



ΠΑΝΕΠΙΣΤΗΜΙΟ  
ΠΑΤΡΩΝ  
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# Introduction in measuring Efficiency using Data Envelopment Analysis under different technology assumptions. Examples in R and STATA.

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# Overview

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## ① *What is DEA?*

DEA is the non parametric mathematical programming approach to frontier estimation.

## ② *Simplier?*

DEA is a popular and simple method to obtain efficiency scores for firms. The firms under analysis are compared to the most efficient firm, which most often is a synthetic firm obtained as a linear combination of reference firms.

The method is non parametric as no assumptions on functional relations between inputs and outputs have to be made.

## ③ The aim of efficient production is characterized by maximizing output given a certain amount of input or by minimizing input given a certain amount of output.

## ④ We do not compare actual output with theoretical output but with maximum observed output.

## ⑤ The most efficient DMU is regarded as the benchmark for calculating the relative efficiency of DMUs.

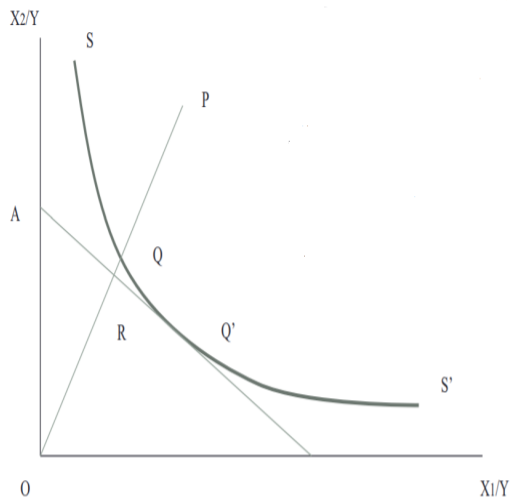
## Technical & Allocative Efficiency

- **Technical Efficiency** is the ability of a firm to obtain maximal output for a given set of inputs.
- **Allocative Efficiency** is the ability of a firm to use the inputs in optimal proportions, given their respective prices.

## Orientation, CRS - VRS

- **Input orientation:** "By how much can input quantities be proportionally reduced without changing the output quantities produced?"
- **Output orientation:** "By how much can output quantities be proportionally increased without changing the input quantities used?"
- **CRS - VRS** assumption

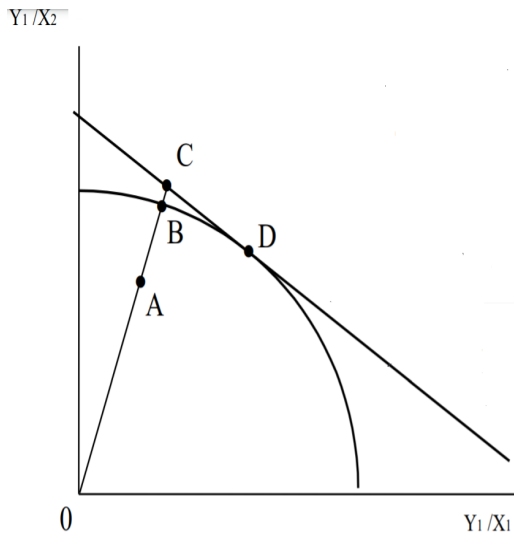
# Input Oriented Measures



# Input Oriented Measures

- 2 inputs ( $X_1, X_2$ ) & 1 output ( $Y$ ), with CRS. AA' depicts the input price ratio line
- firm uses quantity of inputs defined by point P, to produce a unit of output
- QP technical inefficiency, or the amount by which all inputs could be proportionately reduced without a reduction in output  
 $\Rightarrow TE = OQ/OP$
- RQ represents the reduction in production costs that would occur if production were to occur at the allocatively and (and technically) efficient point Q', instead of the technically efficient, but allocatively inefficient, point Q

# Output Oriented Measures



# Output Oriented Measures

- 2 inputs ( $X_1, X_2$ ) & 1 output ( $Y$ ), with CRS.  $ZZ'$  depicts the unit production possibility curve i.e. it is the upper bound of production possibilities
- point A corresponds to an inefficient firm. AB distance is the technical inefficiency, which is the amount by which outputs could be increased without requiring extra inputs  
 $\Rightarrow TE = OA/OB$
- DD' line is the isorevenue line (price information), thus allocative efficiency is  $AE = OB/OC$



# One Input - One Output

- `install.packages(" Benchmarking" )`
- `library(Benchmarking)`
- `emp <- matrix(1:5)`
- `sales <- matrix (c(1,3,5,2,4))`
- `nam <- LETTERS [1:5]`
- `maxc <- cmax(sales/emp)`
- `eff <- sales/emp / max`
- `tab <- t(round(cbind(emp, sales, round(sales/emp, 3), eff), 3))`
- `colnames(tab) <- nam`
- `rownames(tab) <- c(" employees" , " sales" , " sales/employees" , " efficiency" )`
- `View(tab)`
- `dea.plot(emp, sales, RTS=crs, ORIENTATION=inout, pch=19,cex=0.8,txt=LETTERS[1:length(emp)],las=1)`

# One Input - One Output

- $x$  represents labor
- $y$  represents sales
- We assume CRS
- Ratio of output to input

	A	B	C	D	E
$x$	1.000	2.000	3.000	4.000	5.000
$y$	1.000	3.000	2.000	5.000	4.000
$y/x$	1.000	<b>1.500</b>	0.667	1.250	0.800
Efficiency	0.667	1.000	0.444	0.833	0.533

# Two Inputs - One Output

- `x1 <- c(4,7,8,5,2)`
- `x2 <- c(3,2,1,5,5)`
- `y <- rep(1,5)`
- `X <- cbind(x1,x2)`
- `Y <- matrix(y)`
- `e <- dea(X,Y,RTS=crs,ORIENTATION=in,SLACK=T)`
- `eff(e)`
- `plot <- dea.plot.isoquant(x1,x2,txt=LETTERS[1:5],  
xlim=c(0,10),pch=19,cex=0.8)`

- findit dea
- net install st0193
- generate dm\_u = \_n
- dea ivars = ovars, rts(crs, vrs, drs, nirs), ort(in, out), stage(1, 2)
  - rts specifies the returns to scale.
  - ort specifies the orientation. Default is input orientated DEA.
  - stage specifies the way to identify all efficiency slacks. The default is stage(2), meaning the two-stage DEA.

- [1] *Lee, C., & Ji, Y. B.* (2009). **Data envelopment analysis in Stata.** Stata Journal.
- [2] *Behr, A.* (2015). **Production and efficiency analysis with R.** Berlin: Springer.
- [3] *Coelli, T.* (1996). **A guide to DEAP version 2.1: a data envelopment analysis (computer) program.** Centre for Efficiency and Productivity Analysis, University of New England, Australia.