LECTURE 4- *PRODUCTION, TECHNOLOGY AND COST FUNCTIONS (PRODUCTIVITY, TECHNOLOGICAL, TECHNICAL AND SCALE CHANGE)*

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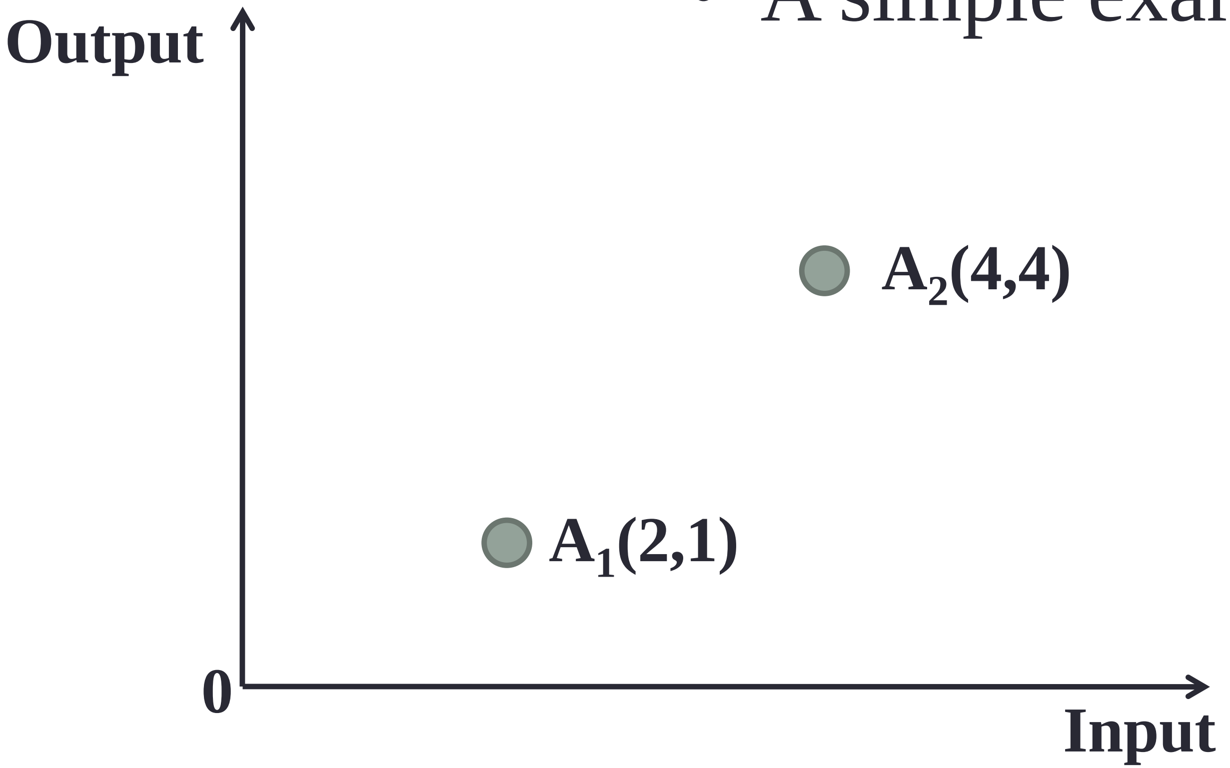
Malmquist Productivity Index

- In general, the TFP index in the simplest case is defined as the ratio of the output ratio to the input ratio for two periods. Productivity = Output / Input.
- Productivity (Growth) Index measures the Productivity changes over Time
- Malmquist (Productivity Growth) Index measures the productivity changes along with time variations and can be decomposed into changes in efficiency and technology.

Malmquist Productivity Index

- A simple example



- Productivity Index = (4/4)/(1/2) = 2
  Productivity is improved by 100%

Malmquist TFP Index-History

- Is so simple??
- Seminal papers by Nishizimu and Page (1982); Fare et al., (1994); Caves et al., (1982) using Aigner et al., (1968) LP methodologies.
- Fare et al (1994) took MPI of total factor productivity growth defined by Caves et al., (1982) and illustrated calculation using DEA based models.

Malmquist Productivity Index-Input Orientation I

- Malmquist Productivity Index (period t)

$$MPI_I^t(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{D_I^t(x^{t+1}, y^{t+1})}{D_I^t(x^t, y^t)}$$

Where Input based distance function at time t is defined by

$$D_I^t(x^t, y^t) = \max\{\theta \mid (x^t/\theta, y^t) \in P^t(x^t, y^t)\}$$

for Production Possibility Set $P(x^t, y^t)$

Input vector $x = \{x_1, x_2, x_3, \ldots, x_m\}$

Output vector $y = \{y_1, y_2, y_3, \ldots, y_n\}$

MPI_I^t is measured by production possibility set P^t at time t.

Malmquist Productivity Index-Input Orientation II

- Malmquist Productivity Index (period t+1)

And accordingly,

$$D_I^{t+1}(x^t, y^t) = \max\{\theta \mid (x^t/\theta, y^t) \in P^{t+1}(x^{t+1}, y^{t+1})\}$$

$$D_I^{t}(x^{t+1}, y^{t+1}) = \max\{\theta \mid (x^{t+1}/\theta, y^{t+1}) \in P^{t}(x^{t}, y^{t})\}$$

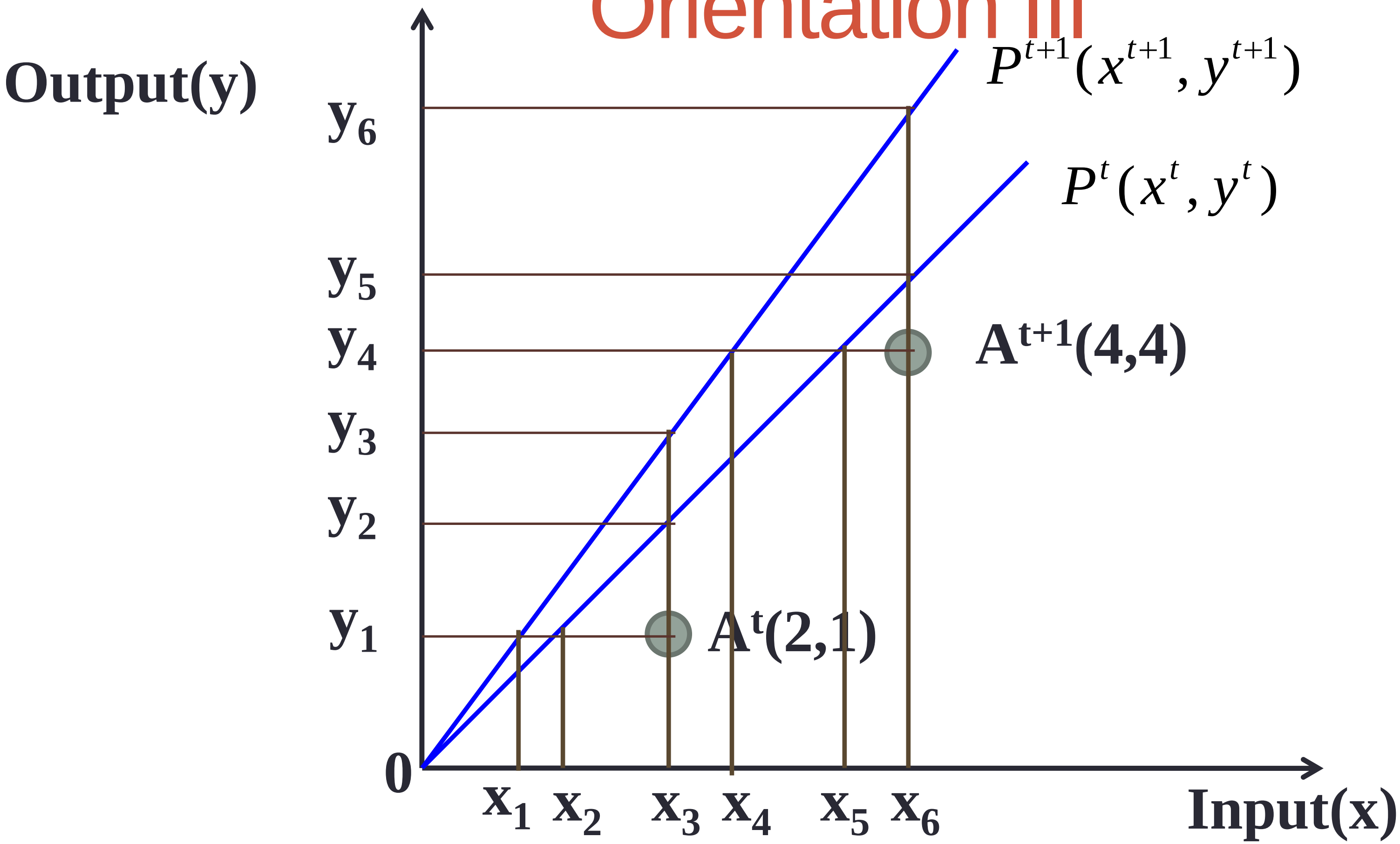
for cross period distance function.

Further, MPI_I^{t+1} can be defined as

Hicks Neutral??

$$MPI_I^{t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{D_I^{t+1}(x^{t+1}, y^{t+1})}{D_I^{t+1}(x^t, y^t)}$$

Malmquist Productivity Index-Input Orientation III



$$\text{Productivity change} = \frac{\overline{oy_4} / \overline{ox_6}}{\overline{oy_1} / \overline{ox_3}} = \frac{\overline{oy_4}}{\overline{oy_1}} \frac{\overline{ox_3}}{\overline{ox_6}} = \frac{4}{1}\frac{2}{4} = 2$$

Malmquist Productivity Index-Input Orientation IV

- Malmquist Productivity Index

$$D_I^t(x^t, y^t) = \overline{ox_2} / \overline{ox_3}$$

$$D_I^t(x^{t+1}, y^{t+1}) = \overline{ox_5} / \overline{ox_6}$$

$$M_I^t(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{D_I^t(x^{t+1}, y^{t+1})}{D_I^t(x^t, y^t)} = \frac{\overline{ox_5} / \overline{ox_6}}{\overline{ox_2} / \overline{ox_3}}$$

$$= \frac{\overline{ox_3}}{\overline{ox_6}} \frac{\overline{ox_5}}{\overline{ox_2}} = \frac{\overline{ox_3}}{\overline{ox_6}} \frac{\overline{oy_4}}{\overline{oy_1}} = \text{Productivity Change}$$

Malmquist Productivity Index-Output Orientation I

- Following Fare et al., (1994)

$$M_O^t(x^t, y^t, x^{t+1}, y^{t+1}) = \sqrt{\frac{D_O^t(x^{t+1}, y^{t+1})}{D_O^t(x^t, y^t)} \frac{D_O^{t+1}(x^{t+1}, y^{t+1})}{D_O^{t+1}(x^t, y^t)}}$$

- TFP decline if MPI<1 and TFP growth if MPI>1.
- Note that it is also the geometric mean of two TFP indices.

Malmquist Productivity Index-Output Orientation II

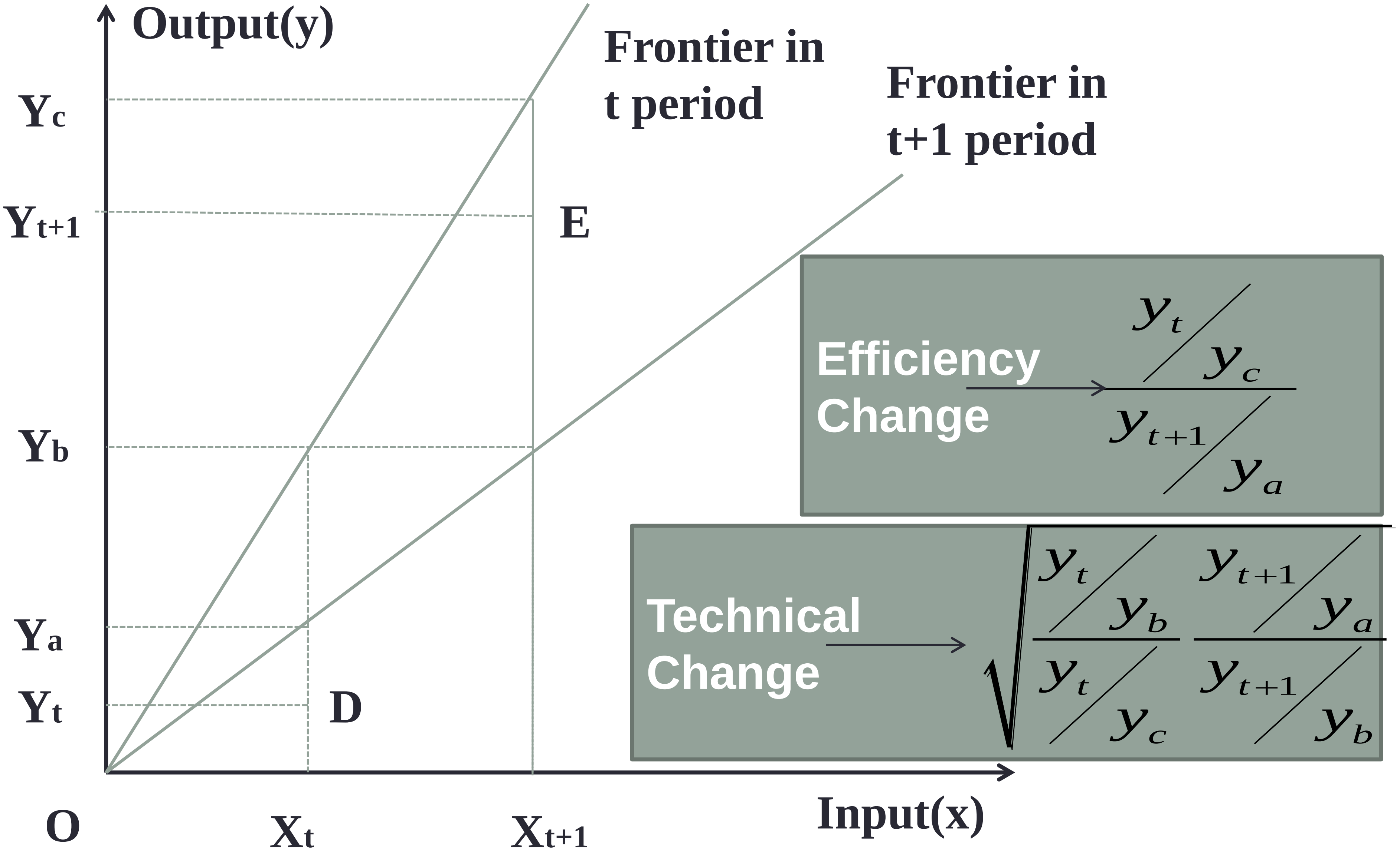
- An alternative way of writing:

$$M_O^t(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{D_O^{t+1}(x^{t+1}, y^{t+1})}{D_O^t(x^t, y^t)} \sqrt{\frac{D_O^t(x^{t+1}, y^{t+1})}{D_O^{t+1}(x^{t+1}, y^{t+1})} \frac{D_O^t(x^t, y^t)}{D_O^{t+1}(x^t, y^t)}}$$

Efficiency Change

Technical Change

Measuring MPI-graphical representation



Malmquist Productivity Index-Output Orientation-Scale Efficiency

$$M_0^t(x^t,y^t,x^{t+1},y^{t+1}) = \frac{D_0^{t+1}(x^{t+1},y^{t+1})}{D_0^t(x^t,y^t)} \sqrt{\frac{D_0^t(x^{t+1},y^{t+1})}{D_0^{t+1}(x^{t+1},y^{t+1})}\,\frac{D_0^t(x^t,y^t)}{D_0^{t+1}(x^t,y^t)}}\;\frac{\dfrac{\overline{D_0^{t+1}}(x^{t+1},y^{t+1})}{D_0^{t+1}(x^{t+1},y^{t+1})}}{\dfrac{\overline{D_0^t}(x^t,y^t)}{D_0^t(x^t,y^t)}}$$

Efficiency Change

Scale Efficiency Change

Technical Change

Notes on MPI

- It is the geometric mean of two MPI indexes.
- If the technology is Hicks neutral these indices are equivalent (Fare et al., 1994).
- The issue of transitivity isn’t of great importance
- Many authors provide alternative decompositions for TFP index (i.e Balk 2002; O’Donell 2015)

Estimation Methods for MPI calculation I

- Two basic methodologies DEA & SFA .
- In the case of DEA we have to calculate the corresponding distance functions to measure TFP for two periods. We leave this to programs like DEAP.
- In the SFA case we have to calculate efficiency change using the type

$$EFFCH = \frac{TE_i^{t+1}}{TE_i^{t}} \;\; \text{from} \;\; \mathrm{TE}_i^t = \frac{e^{x_i\beta - u_i}}{e^{x_i\beta}} = e^{-u_i}$$

Estimation Methods for MPI calculation II

- We need also estimation for technological change.

$$TECH = \sqrt{\left\{\left[1 + \frac{\partial\left(x_{it+1}, t+1, \beta\right)}{\partial t+1}\right] \times \left[1 + \frac{\partial\left(x_{it}, t, \beta\right)}{\partial t}\right]\right\}}$$

Olley-Pakes overview

- • A method for robust estimation of the production function
- allowing for
- – Endogeneity of some of the inputs
- – Selection (exit)
- – Unobserved (quasi-) permanent differences across firms
- • Main requirement (limitation) of their method:
- – There is a monotonic relationship between a firm-level decision
- variable (investment in this case) and the unobserved firm-level
- state variable “productivity.”
- – Exit is also conditioned on the unobserved productivity.
- • OP Method also useful if you have only one or two of
- these problems - somewhat more robust than some of
- the other techniques used in the past

Production function using Olley Pakes method

Four significant problems:

1. Substantial heterogeneity (different clusters or sectors)
2. Dynamics are important (within a firms residuals are serially correlated)
3. Exit and entry are pervasive
4. Endogeneity of inputs.
5. Simultaneity-Selection problem

(https://www.youtube.com/watch?v=e4DIobM9axk)

Production function using Olley Pakes method

Olley and Pakes (1996) introduced a semiparametric method that control for simultaneity and selection biases allowing to estimate the production function parameters consistently and obtain reliable productivity estimates.

They suggest a novel approach to addressing this simultaneity problem. They include in the estimation equation a proxy which they derive from a structural model of the optimizing firm. The proxy controls for the part of the error correlated with inputs by "annihilating" any variation that is possibly related to the productivity term.

http://www.stata-journal.com/sjpdf.html?articlenum=st0145

The question in OP paper

- • What was the effect of deregulation on productivity? Taking into account the following Initial conditions:
- Heterogeneity among plant
- Serial correlation in productivity within plant
– Induced lots of entry and exit
– Productivity increased
– Break down productivity increase
- Average productivity level
- Due to reallocation of labor
- Due to reallocation of assets to more productive plants

The question

Consider the Air transport sector. What is the effect of deregulation on European Air Transport sector the last 15 years for Europe?

- Initial conditions (Heterogeneity and serial correlation within air transport firms)
- Productivity increased or decreased?
- Induced lots of entry-exit

The Model I

Incumbent firms decide at the beginning of each period whether to continue participating in the market. If the firm exits, it receives a liquidation value of Φ dollars and never appears again. If it does not exit, it chooses variable inputs (such as labor, material, and energy) and a level of investment. Thus a production function can be referred as

$$Q_{it} = f\left(L_{it}, M_{it}, E_{it}, K_{it}, \mathrm{AGE}_{it}, \Omega_{it}\right)$$

$$Q_{it} = \beta_0 + \beta_l L_{it} + \beta_m M_{it} + \beta_e E_{it} + \beta_K K_{it} + \beta_a AGE_{it} + \Omega_{it} + \eta_{it}$$

$$Q_{it} = \beta_0 + \beta_l L_{it} + \beta_m M_{it} + \beta_e E_{it} + \beta_K K_{it} + \beta_a AGE_{it} + u_{it}$$

$$with\ I_{it} = g\left(K_{it}, AGE_{it}, \Omega_{it}\right) and\ \Omega_{it} = h\left(K_{it}, AGE_{it}, \mathrm{I}_{it}\right)$$

The Model II

Assume that future productivity is a function of current productivity and capital $E\left[\Omega_{it+1} \middle| \Omega_{it}, K_{it}\right], \Pi(\bullet) = f\left(K_{it}, \Omega_{it}\right)$

$$V_{it}\left(K_{it}, \mathrm{AGE}_{it}, \Omega_{it}\right) =$$

$$Max\left[\Phi, \mathrm{Sup}_{Iit\geq 0}\middle|\Pi_{it}\left(K_{it}, \mathrm{AGE}_{it}, \Omega_{it}\right) - C\left(I_{it}\right) + \rho \mathrm{E}\left\{V_{it+1}\left(K_{it+1}, \mathrm{AGE}_{it+1}, \Omega_{it+1}\right)\right\}\middle| J_{it}\right]$$

The previous Bellman equation implies that a firm exits the market if the liquidation value Φ exceeds that expected discounted returns. The exit rule is formed as:

$$X_{it} = \begin{cases} 1, \text{if } \ \Omega_{it} \geq \Omega_{it}\left(K_{it}, \alpha_{it}\right) \\ 0, \text{otherwise} \end{cases}$$

Moreover

$$I_{it} = g\left(K_{it}, AGE_{it}, \Omega_{it}\right)$$

The Model III

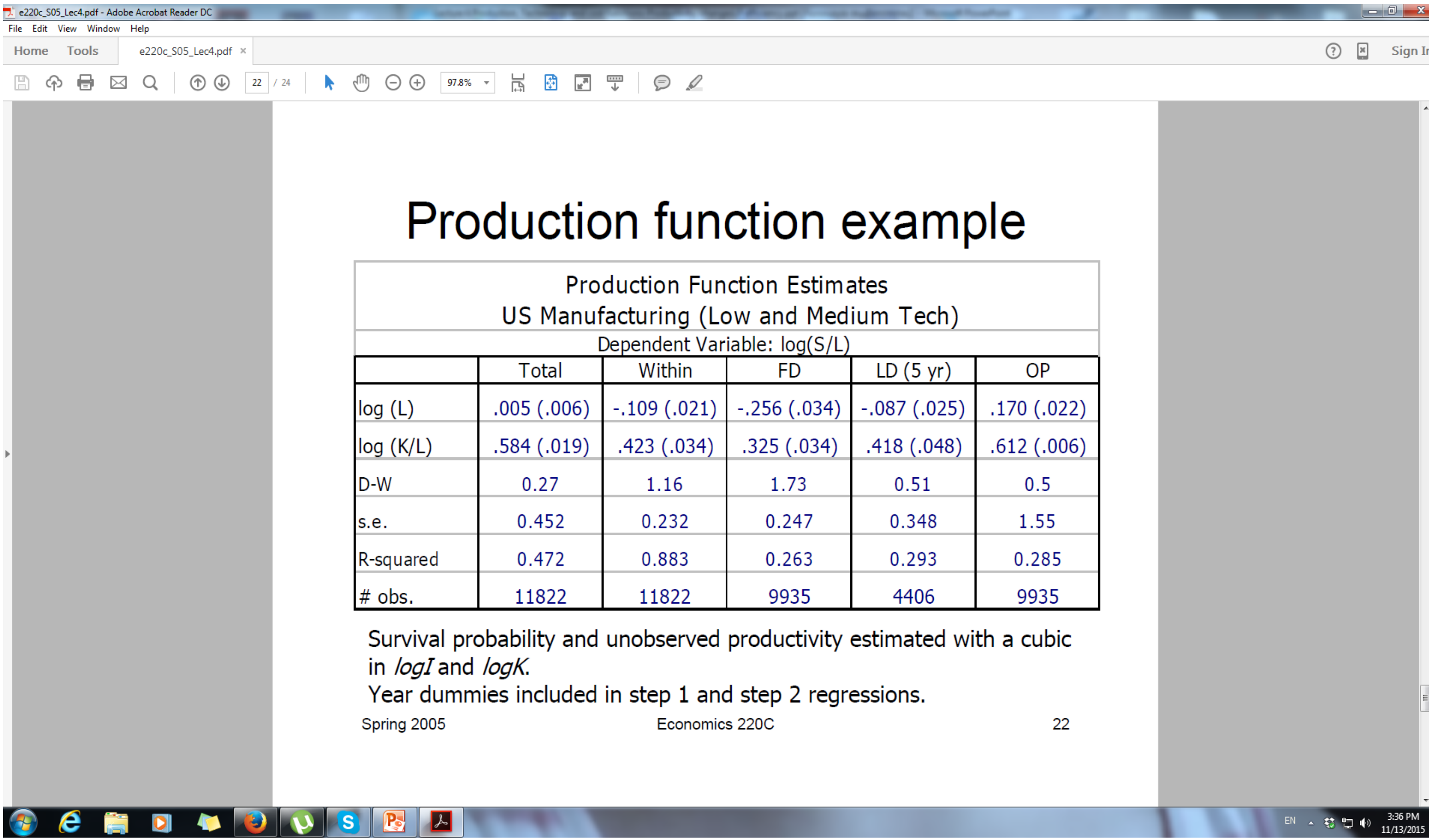
Having in our mind that

$$Q_{it} = \beta_0 + \beta_l L_{it} + \beta_m M_{it} + \beta_e E_{it} + \beta_K K_{it} + \beta_a AGE_{it} + \Omega_{it} + \eta_{it}$$
$$Q_{it} = \beta_0 + \beta_l L_{it} + \beta_m M_{it} + \beta_e E_{it} + \beta_K K_{it} + \beta_a AGE_{it} + u_{it}$$

We can solve $I_{it} = g\left(K_{it}, AGE_{it}, \Omega_{it}\right)$ as $\Omega_{it} = I_{it}^{-1}\left(I_{it}, K_{it}, AGE_{it}\right) = h\left(I_{it}, K_{it}, AGE_{it}\right)$ to control for simultaneity problem.

$$Q_{it} = \beta_0 + \beta_l L_{it} + \beta_m M_{it} + \beta_e E_{it} + \beta_K K_{it} + \beta_a AGE_{it} + \Omega_{it} + \eta_{it}$$
$$Q_{it} = \beta_0 + \beta_l L_{it} + \beta_m M_{it} + \beta_e E_{it} + \beta_K K_{it} + \beta_a AGE_{it} + u_{it}$$
$$Q_{it} = \beta_0 + \beta_l L_{it} + \beta_m M_{it} + \beta_e E_{it} + \varphi\left(I_{it}, K_{it}, AGE_{it}\right) + \eta_{it}, \text{ with}$$
$$\varphi\left(I_{it}, K_{it}, AGE_{it}\right) = \beta_0 + \beta_K K_{it} + \beta_a AGE_{it} + h\left(I_{it}, K_{it}, AGE_{it}\right)$$

Bronwyn H. Hall, Berkley, 2005



Production function example

Production Function Estimates US Manufacturing (Low and Medium Tech) Dependent Variable: log(S/L)					
	Total	Within	FD	LD (5 yr)	OP
log (L)	.005 (.006)	-.109 (.021)	-.256 (.034)	-.087 (.025)	.170 (.022)
log (K/L)	.584 (.019)	.423 (.034)	.325 (.034)	.418 (.048)	.612 (.006)
D-W	0.27	1.16	1.73	0.51	0.5
s.e.	0.452	0.232	0.247	0.348	1.55
R-squared	0.472	0.883	0.263	0.293	0.285
# obs.	11822	11822	9935	4406	9935

Survival probability and unobserved productivity estimated with a cubic in *logI* and *logK*.
Year dummies included in step 1 and step 2 regressions.

Malmquist Index using DEAP program

The things are much more different than in the DEA crs and vrs models case.
The key issue here is the creation of the correct file containing the data that you have. In your mind you must have the following structure.

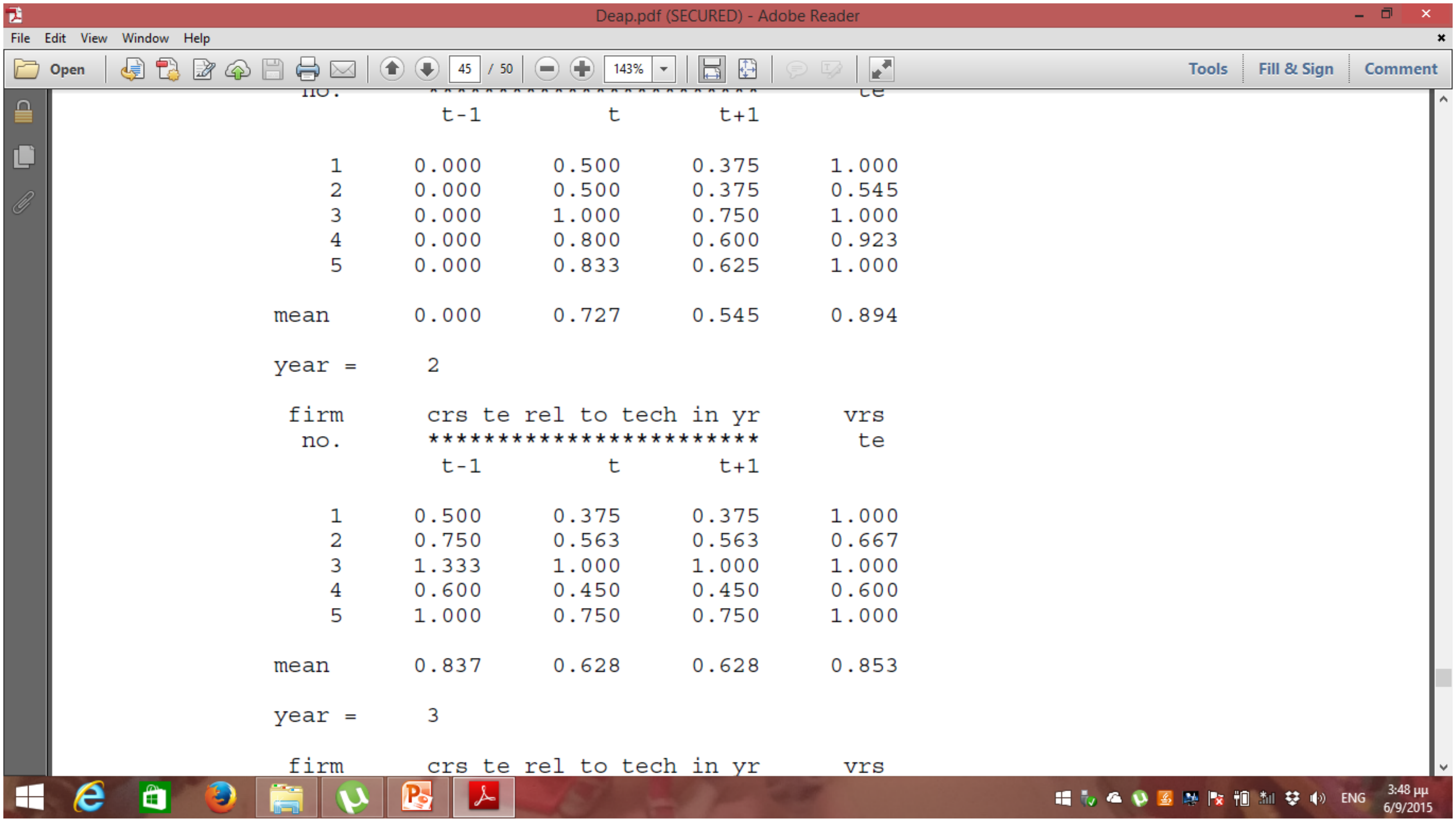
DMUs	Period	Input1	Input 2	Input 3	Output 1	Output 2
1	1					
2	1					
3	1					
1	2					
2	2					
3	2					
1	3					
2	3					
3	3					

Malmquist Index using DEAP program

The changes relative to the previous case is that we have to define periods and to have 2 for MPI.

eg1-dta.txt DATA FILE NAME
eg1-out.txt OUTPUT FILE NAME
5 NUMBER OF FIRMS
3 NUMBER OF TIME PERIODS
1 NUMBER OF OUTPUTS
2 NUMBER OF INPUTS
0 0=INPUT AND 1=OUTPUT ORIENTATED
0 0=CRS AND 1=VRS
2 0=DEA(MULTI-STAGE), 1=COST-DEA, 2=MALMQUIST-DEA, 3=DEA(1-STAGE), 4=DEA(2-STAGE)

Results




```
                  t-1          t         t+1

       1      0.000      0.500      0.375      1.000
       2      0.000      0.500      0.375      0.545
       3      0.000      1.000      0.750      1.000
       4      0.000      0.800      0.600      0.923
       5      0.000      0.833      0.625      1.000

 mean         0.000      0.727      0.545      0.894

 year =        2

  firm        crs te rel to tech in yr        vrs
   no.        ************************         te
                  t-1          t         t+1

       1      0.500      0.375      0.375      1.000
       2      0.750      0.563      0.563      0.667
       3      1.333      1.000      1.000      1.000
       4      0.600      0.450      0.450      0.600
       5      1.000      0.750      0.750      1.000

 mean         0.837      0.628      0.628      0.853

 year =        3

  firm        crs te rel to tech in yr        vrs
```

Results

Deap.pdf (SECURED) - Adobe Reader

firm no.	crs te rel to tech in yr t-1	t	t+1	vrs te
1	0.375	0.375	0.000	1.000
2	0.563	0.563	0.000	0.667
3	1.000	1.000	0.000	1.000
4	0.450	0.450	0.000	0.600
5	0.750	0.750	0.000	1.000
mean	0.628	0.628	0.000	0.853

[Note that t-1 in year 1 and t+1 in the final year are not defined]

MALMQUIST INDEX SUMMARY

year = 2

firm	effch	techch	pech	sech	tfpch
1	0.750	1.333	1.000	0.750	1.000
2	1.125	1.333	1.222	0.920	1.500
3	1.000	1.333	1.000	1.000	1.333
4	0.562	1.333	0.650	0.865	0.750
5	0.900	1.333	1.000	0.900	1.200

Malmquist Index using DEA Frontier

Decomposition of the input oriented geometric mean of Malmquist index using the concept of input oriented efficiency change and input oriented technical change

$$M_I^G = (ECH_I \cdot TCH_I^G) = (\frac{E_I^{t+1}(x^{t+1}, y^{t+1})}{E_I^t(x^t, y^t)}) \cdot [(\frac{E_I^t(x^t, y^t)}{E_I^{t+1}(x^t, y^t)}) \cdot (\frac{E_I^t(x^{t+1}, y^{t+1})}{E_I^{t+1}(x^{t+1}, y^{t+1})})]^{1/2} \quad (4)$$

Malmquist Index can be obtained from the DEA measure

MPI USING STATA

The User written command “malmq”

- Program Syntax

malmq ivars = ovars [if] [in] [, ort(in | out) period(*varname*) trace saving(filename)]

  - ort(in | out) specifies the orientation. The default is ort(in), meaning input-oriented DEA.
  - period(*varname*) identifies the time variable.
  - trace specifies to save all the sequences displayed in the Results window in the malmq.log file. The default is to save the final results in the malmq.log file.
  - saving(filename) specifies that the results be saved in filename.dta.

- See “malmq.ado” file for the details

Notes and Examples

- Notes
  - Updated “dea.ado”, “malm.ado” files
  - In terms of accuracy and computational efficiency?
    Current version is more focused on ‘accuracy’
  - Tested for 365DMU data set for *dea.ado* command and compared with other DEA programs.
  - Data : see “365dmu.dta” for *dea* command and “panel_data_for_malmquist_dea.dta” for *malmq* command.
  - Try the following commands
    - **dea i_total = o_licnese o_sic o_nsic o_dpatent o_fpatent, rts(crs) ort(i)**
    - **malmq i_AC = O_SPI O_CPI, ort(i) period(period)**

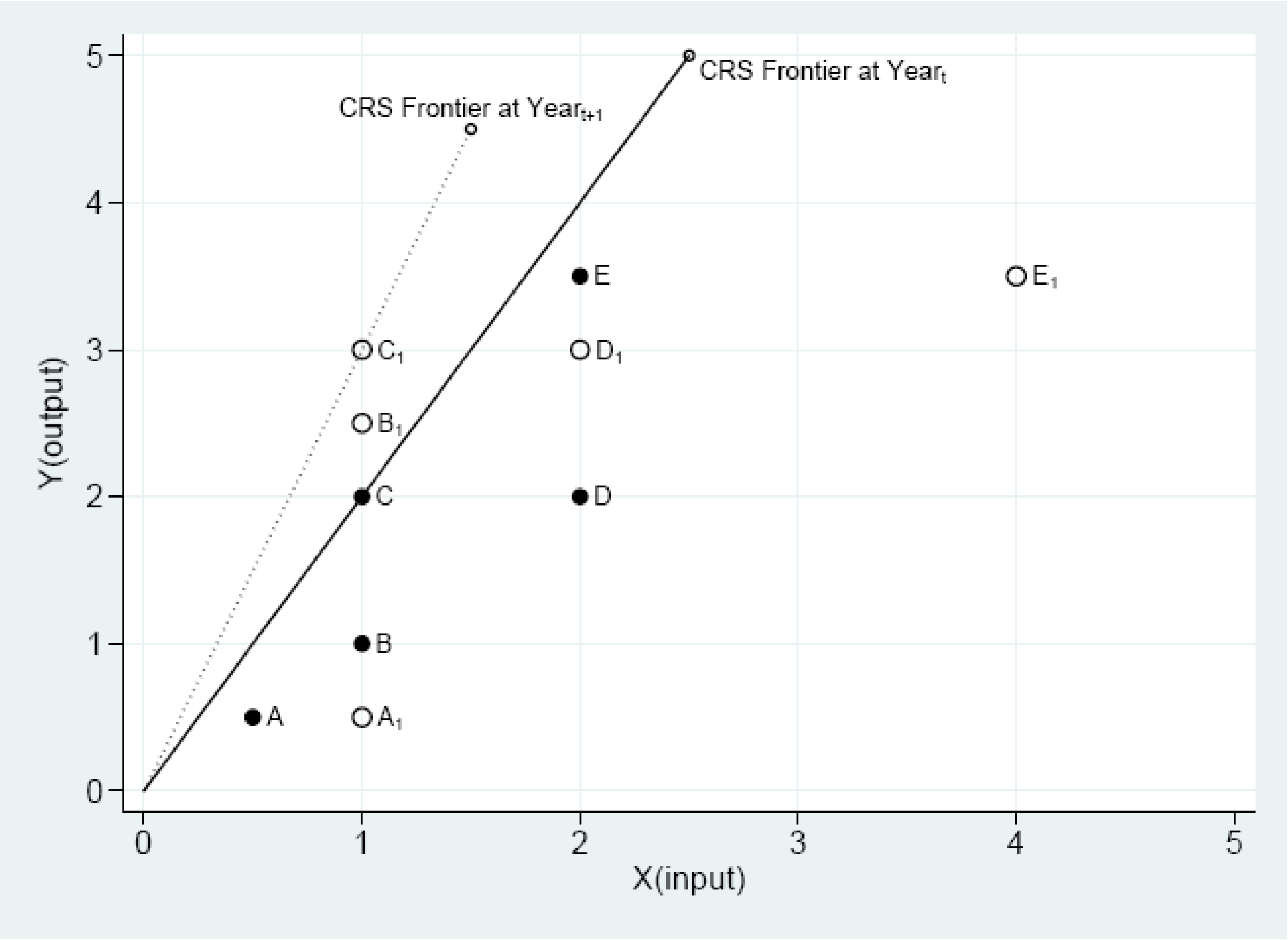
Notes and Examples

– Result

- For dea: Results including the messages “No Solution(LOOP grather than maxiter):[DMUi=119][LOOP=16001]CRS-IN-SI-PII”.
  - ✓ See “dea.log” file for details
  - ✓ Compare with results by other programs
- For malmq
  - ✓ see “malmquist.log” file for details
  - ✓ Compare with results by other programs

Malmquist Index using DEA Frontier

- Concepts of Malmquist Index using CRS Frontier



Malmquist Index using nonparaeff (R)

Calculate productivity growth index under the DEA framework.

Usage

```
faremalm2(dat = NULL, noutput = 1, id = "id", year = "year")
```


Arguments

`dat`	A data frame to be evaluated. The format of this data frame is data.frame(id, year, outputs, inputs). This data frame should have a balanced panel data form.
`noutput`	The number of outputs.
`id`	A column name for the producer index.
`year`	A column name for the time index.

Details

The Malmquist productivity growth index is calculated. For model specification, take a look at Fare et al. (1994).

Value

A data frame with (id: the id index of the original data. time: the time index of the original data. y's: original outputs x's: original inputs Dt2t2: $D^{t+1}(x^{t+1}, y^{t+1})$ Dtt2: $D^{t}(x^{t+1}, y^{t+1})$ Dt2t: $D^{t+1}(x^{t}, y^{t})$ ec: efficiency change tc: technical change pc: productivity change

Author(s)

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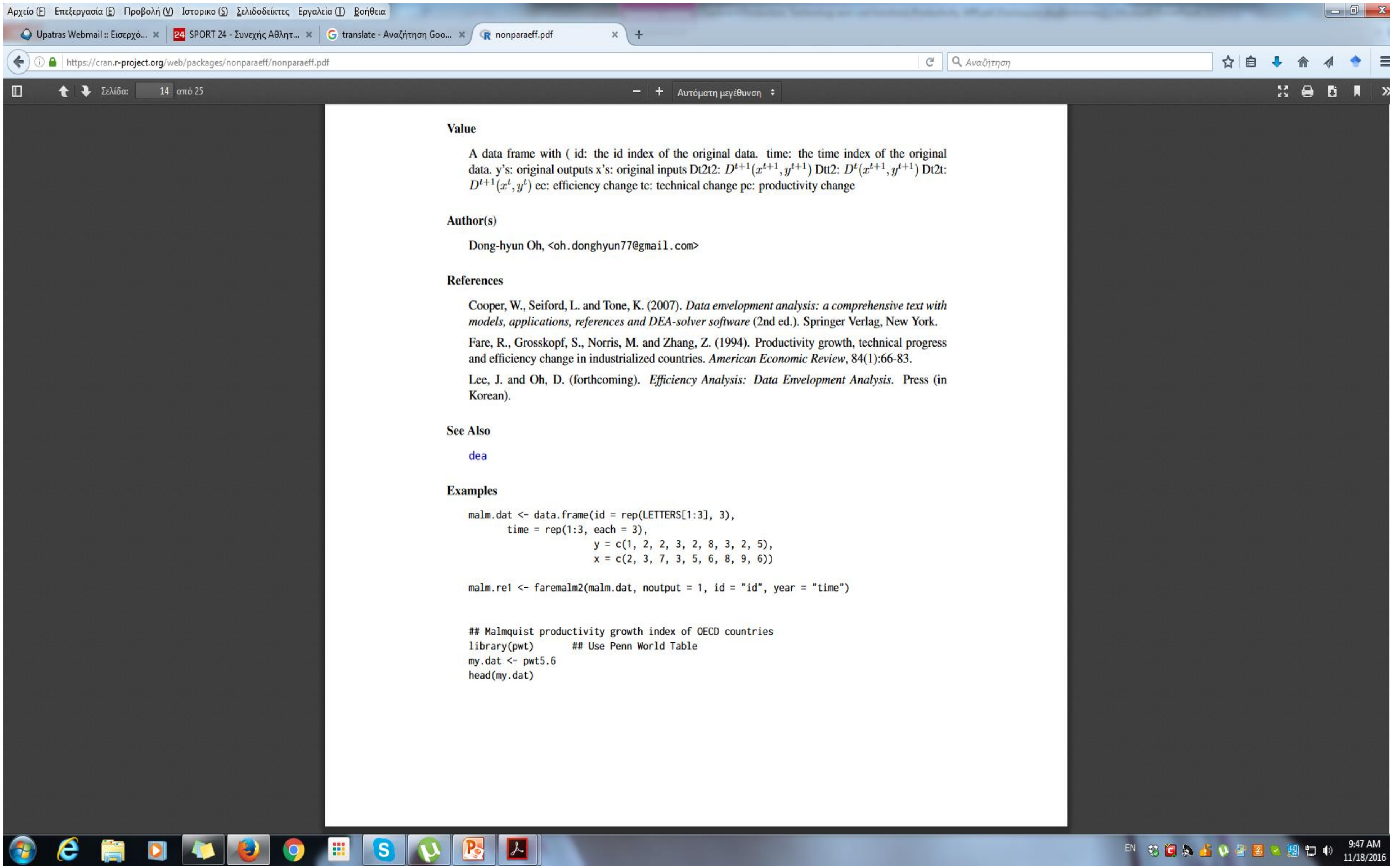
Malmquist Index using nonparaeff (R)



Value

A data frame with (id: the id index of the original data. time: the time index of the original data. y's: original outputs x's: original inputs Dt2t2: $D^{t+1}(x^{t+1}, y^{t+1})$ Dtt2: $D^{t}(x^{t+1}, y^{t+1})$ Dt2t: $D^{t+1}(x^{t}, y^{t})$ ec: efficiency change tc: technical change pc: productivity change

Author(s)

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Lee, J. and Oh, D. (forthcoming). *Efficiency Analysis: Data Envelopment Analysis*. Press (in Korean).

See Also

dea

Examples

```
malm.dat <- data.frame(id = rep(LETTERS[1:3], 3),
       time = rep(1:3, each = 3),
                       y = c(1, 2, 2, 3, 2, 8, 3, 2, 5),
                       x = c(2, 3, 7, 3, 5, 6, 8, 9, 6))

malm.re1 <- faremalm2(malm.dat, noutput = 1, id = "id", year = "time")


## Malmquist productivity growth index of OECD countries
library(pwt)      ## Use Penn World Table
my.dat <- pwt5.6
head(my.dat)
```

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