

# Applied Microeconometrics (L3): Instrumental Variables Regression

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# Overview

1 The IV Estimator

2 Example

3 Inference

# Single Regressor & Single Instrument

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

- Aim in IV regression: breakdown  $X$  into
  - a part that might be correlated with  $u$
  - a part that is not correlated with  $u$
- Why?
  - in order get unbiased estimate of  $\beta_1$
- How?
  - using an instrumental variable  $Z_i$  uncorrelated with  $u_i$
  - $Z_i$  is considered to be an exogenous variable
  - Note: Endogenous variable: “determined within the system”

# Conditions for a valid instrument

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

- Instrument relevance:  $\text{corr}(Z_i, X_i) \neq 0$
- Instrument exogeneity:  $\text{corr}(Z_i, X_i) = 0$

How can you use  $Z_i$  to estimate  $\beta_1$ ?

# The IV Estimator, one $X$ and one $Z$

## Two Stage Least Squares (2SLS)

- First stage: isolates the part of  $X$  that is uncorrelated with  $u$ 
  - regress  $X$  on  $Z$  using OLS:  $X_i = \pi_0 + \pi_1 Z_i + v_i$
  - compute the predicted values:  $\hat{X}_i = \hat{\pi}_0 + \hat{\pi}_1 Z_i$
- Second stage: replace  $X_i$  by  $\hat{X}_i$  in the regression of interest
  - regress  $Y$  on  $\hat{X}_i$  using OLS:  $Y_i = \beta_0 + \beta_1 \hat{X}_i + u_i$
  - resulting estimator is called the “Two Stage Least Squares” (2SLS) estimator:  $\hat{\beta}_1^{2SLS}$
  - $\hat{\beta}_1^{2SLS}$  is a consistent estimator of  $\beta_1$

# Example: Supply and demand for butter

Estimate demand elasticities for agricultural goods

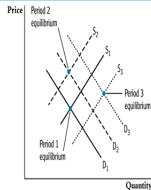
$$\ln(Q_i) = \beta_0 + \beta_1 \ln(P_i) + u_i$$

- $\beta_1$  = price elasticity of butter = percent change in quantity for a 1% change in price
- Data: observations on price and quantity of butter for different years
- The OLS regression of  $\ln(Q_i)$  on  $\ln(P_i)$  suffers from simultaneous causality bias
- (!) price and quantity are determined by the interaction of demand and supply

# Interaction of demand and supply

FIGURE 10.1

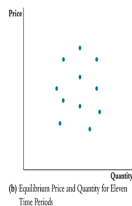
(a) Price and quantity are determined by the intersection of the supply and demand curves. The equilibrium in the first period is determined by the intersection of the demand curve  $D_1$  and the supply curve  $S_1$ . Equilibrium in the second period is the intersection of  $D_2$  and  $S_2$ , and equilibrium in the third period is the intersection of  $D_3$  and  $S_3$ .



(a) Demand and Supply in Three Time Periods

FIGURE 10.1

(b) This scatterplot shows equilibrium price and quantity in eleven different time periods. The demand and supply curves are hidden. Can you determine the demand and supply curves from the points on the scatterplot?



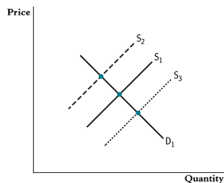
(b) Equilibrium Price and Quantity for Eleven Time Periods

Source: J.H. Stock and M.W. Watson, Introduction to Econometrics (first edition), Addison-Wesley, 2003

# What would you get if only supply shifted?

**FIGURE 10.1**

(c) When the supply curve shifts from  $S_1$  to  $S_2$  to  $S_3$  but the demand curve remains at  $D_1$ , the equilibrium prices and quantities trace out the demand curve.



(c) Equilibrium Price and Quantity When Only the Supply Curve Shifts

Source: J.H. Stock and M.W. Watson, Introduction to Econometrics (first edition), Addison-Wesley, 2003

- (1): 2SLS estimates the demand curve by isolating shifts in price and quantity that arise from shifts in supply
- (2):  $Z$  is a variable that shifts supply but not demand.



# Example: Supply and demand for butter

$$\ln(Q_i) = \beta_0 + \beta_1 \ln(P_i) + u_i$$

- Let  $Z$ =rainfall (rain) in dairy-producing regions
- Is  $Z$  a valid instrument?
  - Relevant?  $\text{corr}(\text{rain}_i, \ln(P_i)) \neq 0$   
Plausibly: insufficient rainfall means less grazing means less butter
  - Exogenous?  $\text{corr}(\text{rain}_i, u_i) = 0$   
Plausibly: whether it rains in dairy-producing regions should not affect demand

## 2SLS in the supply-demand example

$$\ln(Q_i) = \beta_0 + \beta_1 \ln(P_i) + u_i, \quad Z_i = \text{rain}_i, \quad i = 1, \dots, R$$

- First stage: isolates changes in log price that arise from supply
  - regress  $\ln(P_i)$  on  $\text{rain}_i$  using OLS:  $\ln(P_i) = \pi_0 + \pi_1 \text{rain}_i + v_i$
  - compute the predicted values:  $\ln(\hat{P}_i)$
- Second stage
  - regress  $\ln(Q_i)$  on  $\ln(\hat{P}_i)$  using OLS:  $\ln(Q_i) = \beta_0 + \beta_1 \ln(\hat{P}_i) + u_i$
  - resulting estimator  $\hat{\beta}_1^{2SLS}$
  - The regression counterpart of using shifts in the supply curve to trace out the demand curve

# Inference

- 1 Statistical inference proceeds in the usual way
- 2 The justification is (as usual) based on large samples
- 3 This all assumes that the instruments are valid (vs. weak instruments)
- 4 Standard errors
  - The OLS standard errors from the second stage regression are not right they do not take into account the estimation in the first stage ( $\hat{X}_i$  is estimated)
  - Instead, use a single specialized command that computes the 2SLS estimator and the correct SEs
  - as usual, use heteroskedasticity-robust SEs