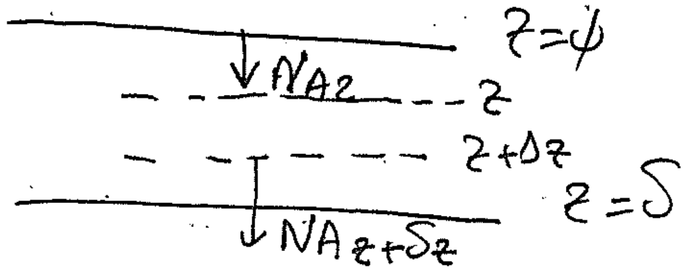
Q3



Assumptions : * 1-D diffusion, pseudo-steady state
* T, P const → C, D_{AB} : const
* Stagnant biofilm, $N_{Bz}=\phi$
* $x_A \ll 1$, conc of A in biofilm is very low

Mass Balance

$$S.N_{Az}\big|_{z+\Delta z} - S N_{Az}\big|_{z} = -K_o S \delta z$$

divide by $S.\delta z$ $\Longrightarrow$ limit $\delta z \to 0$

$$\frac{dN_A}{dz} = -K_o \quad (1)$$

where K_o is zeroth order rate constant

Fick's law

$$N_{Az} = -cD_{AB}\frac{dx_A}{dz} + \cancel{x_A}^{\phi}\left(N_{Az} + \cancel{N_{Bz}}^{\phi}\right)$$

$$\longrightarrow N_{Az} = -cD_{AB}\frac{dx_A}{dz} \longrightarrow N_{Az} = -D_{AB}\frac{dc_A}{dz} \quad (2)$$

$$(1) \xrightarrow{(2)} \frac{d}{dz}\left(-D_{AB}\frac{dc_A}{dz}\right) = -K_o \longrightarrow$$

$$\longrightarrow \frac{d^2 c_A}{dz^2} = \frac{K_o}{D_{AB}} \quad (3)$$

BCs
BC1 : $C_A = C_{Ao}$ at $z = 0$
BC2 : $\frac{dC_A}{dz} = 0$ at $z = \delta$ (Barrier)

(3) $\xrightarrow[\text{twice}]{\text{integrate}}$ $\frac{dC_A}{dz} = \frac{k_o}{D_{AB}} z + C_1 \longrightarrow$

$\longrightarrow C_A = \frac{k_o}{2D_{AB}} z^2 + C_1 z + C_2$ (4)

From BC1 : (4) $\longrightarrow C_2 = C_{Ao}$ (5)
from BC2 : (4) $\longrightarrow C_1 = -\frac{k_o \delta}{D_{AB}}$ (6)

(a) from eqn (4) $\xrightarrow[(6)]{(5)}$ $\boxed{C_A = \frac{k_o}{2D_{AB}} z^2 - \frac{k_o \delta}{D_{AB}} z + C_{Ao} \quad (7)}$

(b) (2) $\longrightarrow N_{Az} = -D_{AB} \frac{dC_A}{dz}$
and (4) $\frac{dC_A}{dz} = \frac{k_o z}{D_{AB}} + C_1$ $\Longrightarrow$

$\Longrightarrow N_{Az} = -D_{AB} \left\{ \frac{k_o z}{D_{AB}} + C_1 \right\} \overset{(6)}{\Longrightarrow}$

$N_{Az} = -D_{AB} \left\{ \frac{k_o z}{D_{AB}} - \frac{k_o \delta}{D_{AB}} \right\} \longrightarrow$

$\longrightarrow N_{Az} = -\frac{D_{AB}}{D_{AB}} k_o (z - \delta) \Rightarrow \boxed{N_{Az} = k_o(\delta - z) \quad (8)}$

Therefore, the flux at $z=0$ would be:

$$\boxed{N_{Az}\big|_{z=0} = k_o\delta}$$

(c) Diffusion depth: $z = x_\delta$ where C_A becomes zero

$$(7) \xrightarrow[z=x_\delta]{C_A=0} 0 = \frac{k_o}{2D_{AB}}x_\delta^2 - \frac{k_o\delta}{D_{AB}}x_\delta + C_{Ao} \rightarrow$$

$$\rightarrow \frac{k_o}{2D_{AB}}x_\delta^2 - \frac{k_o\delta}{D_{AB}}x_\delta + C_{Ao} = 0$$

* Need to solve the above quadratic equation of the form: $\alpha x^2 + \beta x + \gamma = 0$

$$\Delta = \beta^2 - 4\alpha\gamma \rightarrow \Delta = \left(\frac{k_o\delta}{D_{AB}}\right)^2 - \frac{2k_o}{D_{AB}}C_{Ao}$$

and $$x_{\delta_{1,2}} = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha} \longrightarrow$$

$$\rightarrow x_{\delta_{1,2}} = \delta \pm \sqrt{\delta^2 - 2C_{Ao}\frac{D_{AB}}{k_o}}$$

and finally: $$\boxed{x_\delta = \delta - \sqrt{\delta^2 - 2C_{Ao}D_{AB}/k_o}}$$

This is the root with physical meaning since x_δ must be $< \delta$