Περιβαλλοντική Βιοτεχνολογία-Environmental Biotechnology

Ενότητα 4: Reactors

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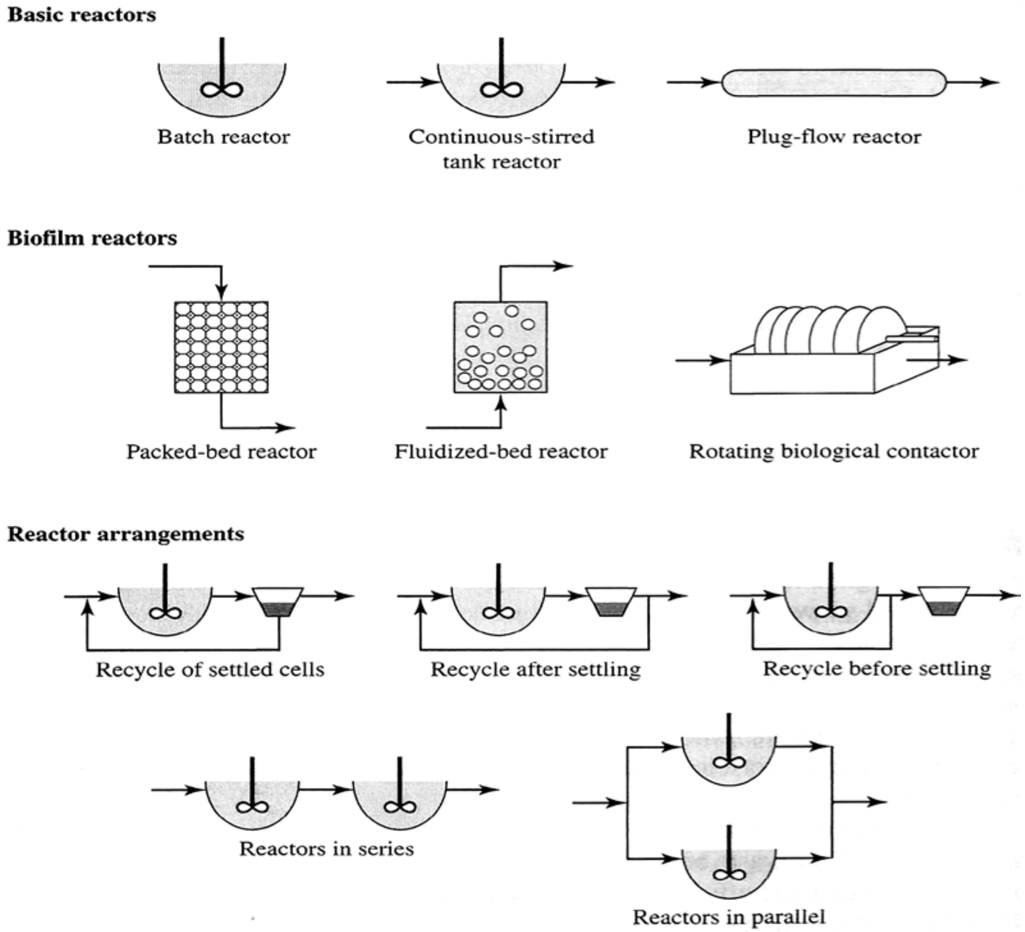
Πολυτεχνική Σχολή

Τμήμα Χημικών Μηχανικών

Types of reactors



- Suspended-floc
- Dispersed growth
- Fixed-film
- Attached growth
- Immobilized cells

<u>Factors for selecting the reactor:</u>

- Physical and chemical characteristics of waste
- Concentration of pollutants
- Presence or absence of oxygen
- Qualitative characteristics of runoff
- Climatic operating conditions
- Number of processes involved
- Experience of technical system operation
- Costs for operating time

Batch reactors

Mass rate of substrate accumulation in control volume =

rate of mass in - rate of mass out + rate of mass generation

(rate of mass in = 0; rate of mass out = 0)

While the microorganisms are consuming substrate, no substrate is added or removed from the batch reactor. Thus, over this time period the mass of substrate accumulating in the reactor equals the mass of substrate generated within the reactor. On the other hand, the substrate is consumed or destroyed by the microorganisms, and generation has a negative sign.

$$V\frac{ds}{dt} = Vr_{ut}$$

Commonly, the rate of substrate utilization is assumed to following Monod kinetics. With this substitution, we obtain:

$$V\frac{ds}{dt} = V\left(-\frac{\hat{q}S}{K+S}X_a\right)$$

$$\frac{ds}{dt} = -\frac{\hat{q}S}{K+S}X_a$$

Batch reactors

Mass rate of organism accumulation in control volume =

rate of mass in - rate of mass out + rate of mass generation

= 0 = 0

With μ being the net specific growth rate of organisms, the mathematical form is similar to previous equation:

$$V\frac{dX_a}{dt} = V(\mu X_a)$$

$$V\frac{dX_a}{dt} = V\left(\hat{\mu}\frac{S}{K+S} - b\right)X_a \Rightarrow \frac{dX_a}{dt} = \left(\hat{\mu}\frac{S}{K+S} - b\right)X_a$$

In the absence of decay, the organism concentration at any time equals the initial concentration, X_a^0, plus that which results from substrate consumption during that time, *$Y\,\Delta S$, or:*

$$X_a = X_a^0 + Y\Delta S \quad or \quad X_a = X_a^0 + Y(S^0 - S)$$

Batch reactors

One ordinary differential equation is obtained: $$\frac{ds}{dt} = -\frac{\hat{q}S}{K+S}\left[X_a^0 + Y(S^0 - S)\right]$$

This equation can be integrated to yield:

$$t = \frac{1}{\hat{q}}\left\{\left(\frac{K}{X_a^0 + YS^0} + \frac{1}{Y}\right)\ln\left(X_a^0 + YS^0 - YS\right) - \left(\frac{K}{X_a^0 + YS^0}\right)\ln\frac{SX_a^0}{S^0} - \frac{1}{Y}\ln X_a^0\right\}$$

It would be desirable to have an equation that explicitly gives *S* as a function of *t*, but because of the complexity of the equation, this is not possible. There is a computer spreadsheet which can be very useful for solving for S when t is known.

CSTR reactors with recirculation

Effect of recirculation in the system performance

$$QS^0 + Q^r S = Q^i S^i \quad and \quad QX_a^0 + Q^r X_a^r = Q^i X_a^i$$

From which:

$$S^i = \frac{QS^0 + Q^r S}{Q^i} \quad and \quad X_a^i = \frac{QX_a^0 + Q^r X_a^r}{Q^i}$$

Also:

$$Q^i = Q + Q^r$$

Now, if we do the mass balance for substrate around the reactor control volume, we obtain for the steady-state case:

$$0 = Q^i S^i - Q^i S + r_{ut} V$$

Then, by making appropriate substitutions and simplifying we obtain:

$$0 = Q(S^0 - S) + r_{ut} V$$

Simple recycle for a CSTR does not change substrate removal compared with that obtained without recycle. A mass balance on microorganisms can be performed similarly, and the result is the same: Organism concentrations within the reactor and in the reactor effluent are not affected by effluent recycle, since the same mass flow that leaves the reactor returns to the reactor. We will see, however, that this is not the case with a plug-flow reactor, where concentrations are not the same everywhere.

Plug-flow reactors

Substrate

$$\Delta V\frac{\Delta S}{\Delta t} = QS - Q(S + \Delta S) + r_{ut}\Delta V$$

Active microorganisms

$$\Delta V\frac{\Delta X_a}{\Delta t} = QX_a - Q(X_a + \Delta X_a) + r_{net}\Delta V$$

Substrate at steady state

$$(A = \Delta V / \Delta z \quad and \quad u = Q / A)$$

$$u\frac{\Delta S}{\Delta z} = r_{ut}$$

Active microorganisms at steady state

$$u\frac{\Delta X_a}{\Delta z} = r_{net}$$

Plug-flow reactors

Substrate at steady state with Monod kinetics:

$$u\frac{dS}{dz} = -\hat{q}\frac{S}{K+S}X_a$$

Active microorganisms at steady state with growth and decay:

$$u\frac{dX_a}{dz} = Y\hat{q}\frac{S}{K+S}X_a - bX_a$$

This series of equations cannot be solved analytically, and so numerical approaches must be used. However, if we again ignore organism decay (*b* = 0), then an analytical solution is possible.

$$u\frac{dX_a}{dz} = -uY\frac{ds}{dz}$$

We cancel *u* and take integrals to obtain:

$$\int_{X_a^0}^{X_a} dX_a = -Y\int_{S^0}^{S} dS$$

Integrating gives:

$$X_a = X_a^0 + Y(S^0 - S)$$

A differential equation with only two variables, *S* and *z*:

$$u\frac{dS}{dz} = -\hat{q}\frac{S}{K+S}\left[X_a^0 + Y(S^0 - S)\right]$$

Plug-flow reactors

The ratio *dz/u* has dimensions of time and equals the differential time, *dt*, for an element of water to move along the reactor a distance *dz*. Substituting *dt* for *dz/u* in previous equation yields a differential equation that is exactly the same as for the batch reactor.

$$\frac{z}{u}=\frac{1}{\hat{q}}\left\{\left(\frac{K}{X_a^0+YS^0}+\frac{1}{Y}\right)\ln\left\{X_a^0+YS^0-YS\right\}-\left(\frac{K}{X_a^0+YS^0}\right)\ln\frac{SX_a^0}{S^0}-\frac{1}{Y}\ln X_a^0\right\}$$

We obtain an expression for the effluent concentration from the batch reactor by letting z = L. We also note that L/u is equal to *V/Q*, the hydraulic detention time, ϑ, for the reactor. With these substitutions, the following solution, with ϑ replacing t, is:

$$\theta=\frac{1}{\hat{q}}\left\{\left(\frac{K}{X_a^0+YS^0}+\frac{1}{Y}\right)\ln\left\{X_a^0+YS^0-YS^e\right\}-\left(\frac{K}{X_a^0+YS^0}\right)\ln\frac{S^eX_a^0}{S^0}-\frac{1}{Y}\ln X_a^0\right\}$$

Plug-flow reactors with recirculation

$$S^i = \frac{QS^0 + Q^r S}{Q^i} \quad and \quad X_a^i = \frac{QX_a^0 + Q^r X_a^r}{Q^i}$$

$$and \quad Q^i = Q + Q^r$$

It should be noted that $X^r = X^e$ and $S = S^e$.

For the case in which *b*= 0, an integrated form of an equation relating the effluent concentrations as a function of detention time can be obtained. In this manner, the effluent concentration of X_a can be obtained:

$$X_a^e = X_a^0 + Y(S^0 - S^e)$$

With this, the series of equations can be combined and integrated to give:

$$\frac{V}{Q^i} = \frac{1}{\hat{q}}\left\{\left(\frac{K}{X_a^0 + YS^i} + \frac{1}{Y}\right)\ln\left\{X_a^i + YS^i - YS^e\right\} - \left(\frac{K}{X_a^i + YS^i}\right)\ln\frac{S^e X_a^i}{S^i} - \frac{1}{Y}\ln X_a^i\right\}$$

Of interest is the impact of recycle on the performance of a PFR. We define the recycle ratio *R*, as:

$$R = \frac{Q^r}{Q}$$

and the detention time, θ, as:

$$\vartheta = \frac{V}{Q} = \frac{V(1+R)}{Q^i}$$

CSTR by sedimentation and biomass recirculation

In order to develop mass balances for the reactor, we again need some simplifying assumptions. **Assumptions** that we make here as a first exercise are: **(1)** biodegradation of the substrates takes place in the reactor only, no biological reactions take place in the settling tank, and the biomass in the settler is insignificant; **(2)** no active microorganisms are in the influent to the reactor .(X_a^0= 0); and **(3)** the substrate is soluble so that it cannot settle out in the settling tank.

$$V\frac{dX_a}{dt} = 0 - (Q^e X_a^e + Q^w X_a^w) + \left[Y(-r_{ut})V - bX_a V\right]$$

Likewise, a mass balance for substrate gives:

$$V\frac{dS}{dt} = Q^0 S^0 - (Q^e S^e + Q^w S^w) + r_{ut} V$$

$$\vartheta_x = \frac{X_a V}{Q^e X_a^e + Q^w X_a^w}$$

We can rearrange the above **equation** for the steady-state case to give:

$$\frac{Q^e X_a^e + Q^w X_a^w}{X_a V} = \frac{Y(-r_{ut})}{X_a} - b$$

CSTR by sedimentation and biomass recirculation

$$\frac{1}{\vartheta_x} = \frac{Y(-r_{ut})}{X_a} - b$$

This equation is general for a CSTR with settling and recycle and can be applied whatever the form of the biological reaction, rut, may be. If we assume that it takes the usual form of the Monod reaction, then we obtain the following expression:

$$\frac{1}{\vartheta_x} = Y\frac{\hat{q}S}{K+S} - b$$

We solve this equation explicitly for S:

$$S = K\frac{1+b\vartheta_x}{\vartheta_x(Y\hat{q}-b)-1}$$

The final equation is identical with equation, which was developed for the chemostat without settling and recycle. So then, what is unique about the CSTR with settling and microorganism recycle? The answer is that the retention time of the microorganisms in the system (ϑ_x) is separated from the hydraulic detention time (ϑ). Thus, one can have a large ϑ_x, in order to obtain high efficiency of substrate removal, and at the sanle time have a small ϑ, which translates into a small reactor volume.

CSTR by sedimentation and biomass recirculation

$$X_a = \vartheta_x \frac{Y(-r_{ut})}{1+b\vartheta_x}$$

$$-r_{ut} = \frac{Q^0 S^0 - Q^e S^e - Q^w S^w}{V}$$

This equation is another general representation, this time of the utilization rate in terms of reactor characteristics and performance.

For the CSTR, we see that the substrate concentration in the reactor, S, is equal to the concentration in the effluent S^e and in the waste sludge line, S^W, since no reaction occurs in the settling tank. Also, through mass balance, $Q^e + Q^w = Q^0$. With these substitutions into previous equation, we obtain:

$$-r_{ut} = \frac{Q^0 (S^0 - S)}{V} = \frac{(S^0 - S)}{\vartheta}$$

$$X_a = \frac{\vartheta_x}{\vartheta} \frac{Y(S^0 - S)}{1+b\vartheta_x}$$

$$\frac{\vartheta_x}{\vartheta} = solids\ concentration\ ratio$$

CSTR by sedimentation and biomass recirculation

At steady state, the mass rate of **active biomass production** (r_{abp}, M/T) must just equal the rate at which biomass leaves the system from the effluent stream and the waste stream:

$$r_{abp} = Q^e X_a^e + Q^w X_a^w$$

↓

$$r_{abp} = \frac{X_a V}{\vartheta_x}$$

CSTR by sedimentation and biomass recirculation

For those who are still troubled by this concept, we can develop another equation without using ϑ_x as the master variable. We proceed this time by considering the control volume around the reactor. At steady state, a mass balance for substrate leads to:

$$V\frac{dS}{dt} = Q^i S^i - Q^i S + r_{ut} V = 0$$

If Monod kinetics apply and then substituting

$$\frac{\hat{q} S}{K+S} X_a = \frac{Q^i (S^i - S)}{V}$$

We can define V/Q^i to equal the hydraulic detention time in the reactor itself (ϑ_r)

$$S = \frac{S^i}{1 + \dfrac{\hat{q} X_a \vartheta_r}{K}} = \frac{S^0}{1 + \dfrac{\hat{q} X_a \vartheta}{K}} \qquad (S << K)$$

PFR by sedimentation and biomass recirculation

We can assume X_a to be constant $\bar{X}_a$ throughout the reactor, making integration of previous equation for the steady-state case much easier. The result is:

$$u\frac{dS}{dz} = -\hat{q}\frac{S}{K+S}X_a$$

$$\vartheta_r = \frac{1}{\hat{q}\bar{X}_a}\left[K\ln\left(\frac{S^i}{S}\right) + (S^i - S)\right]$$

We would like to relate treatment efficiency to ϑ_x, since it is much easier to control ϑ_x than it is to measure X_a. We construct a mass balance for active microorganisms around the entire reactor, which provides the same results as for the CSTR with settling and recycle, repeated here for convenience:

$$\frac{1}{\vartheta_x} - \frac{Y(-\bar{r}_{ut})}{\bar{X}_a} - b$$

PFR by sedimentation and biomass recirculation

$$\frac{1}{\vartheta_x} = \frac{YQ^0(S^0 - S)}{\bar{X}_a V} - b$$

$$\vartheta_r = \frac{1}{\hat{q}\bar{X}_a}\left[K \ln(\frac{S^i}{S}) + (S^i - S)\right]$$

Recognizing that $\vartheta_r = V/(Q^0 + Q^r)$, substituting this value into equation, solving for $\bar{X}_a$ V, and then substituting this previous equation give:

$$\frac{1}{\vartheta_x} = \frac{\hat{q}Y(S^0 - S)}{(S^0 - S) + eK} - b$$

in which

$$e = (1+R)\ln\left[(S^0 + RS)/(1+R)/S\right]$$

When R < 1, *e* approximately equals ln(S°/S):

$$\frac{1}{\vartheta_x} = \frac{\hat{q}Y(S^0 - S)}{(S^0 - S) + K\ln\frac{S^0}{S}} - b, \qquad R < 1$$

References

The images where their origin is not mentioned are derived from the book:

Environmental Biotechnology : Principles and Applications,

Bruce E. Rittmann and Perry L. McCarty,

McGraw-Hill Series in Water Resources and Environmental Engineering

Τέλος Ενότητας

Χρηματοδότηση

- Το παρόν εκπαιδευτικό υλικό έχει αναπτυχθεί στο πλαίσιο του εκπαιδευτικού έργου του διδάσκοντα.
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Σημειώματα

Σημείωμα Ιστορικού Εκδόσεων Έργου

Το παρόν έργο αποτελεί την έκδοση 1.0.0.

Σημείωμα Αναφοράς

Σημείωμα Αδειοδότησης