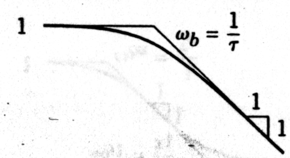
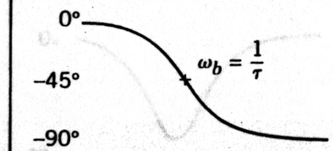
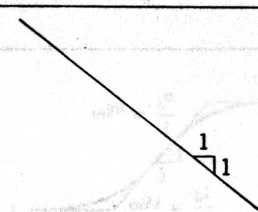
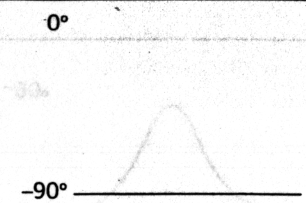
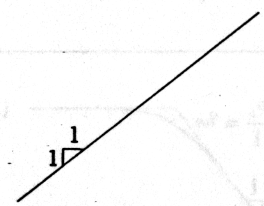
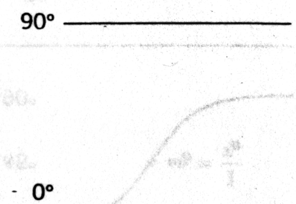
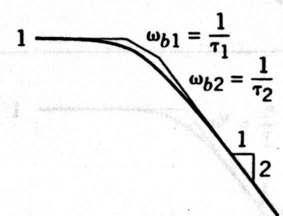
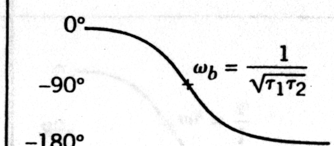
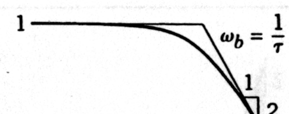
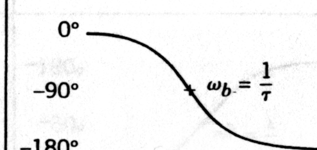
Table 13.4 Frequency Response Characteristics of Important Process Transfer Functions

Transfer Function	$G(s)$	$AR = \lvert G(j\omega)\rvert$	Plot of log AR_N vs. log ω	$\phi = \angle G(j\omega)$	Plot of ϕ vs. log ω
1. First-order	$\dfrac{K}{\tau s + 1}$	$\dfrac{K}{\sqrt{(\omega\tau)^2 + 1}}$		$-\tan^{-1}(\omega\tau)$	
2. Integrator	$\dfrac{K}{s}$	$\dfrac{K}{\omega}$		$-90°$	
3. Derivative	Ks	$K\omega$		$+90°$	
4. Overdamped second-order	$\dfrac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\dfrac{K}{\sqrt{(\omega\tau_1)^2 + 1}\sqrt{(\omega\tau_2)^2 + 1}}$		$-\tan^{-1}(\omega\tau_1) - \tan^{-1}(\omega\tau_2)$	
5. Critically damped second-order	$\dfrac{K}{(\tau s + 1)^2}$	$\dfrac{K}{(\omega\tau)^2 + 1}$		$-2\tan^{-1}(\omega\tau)$	

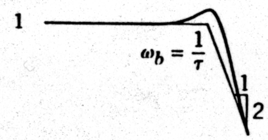
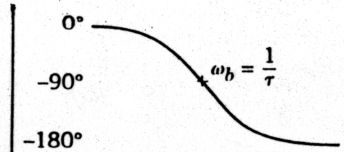
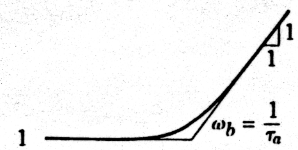
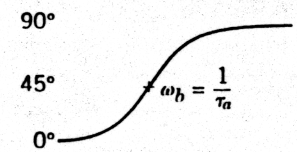
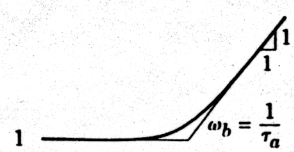
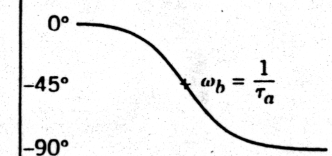
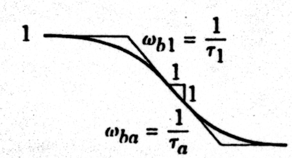
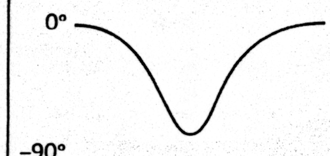
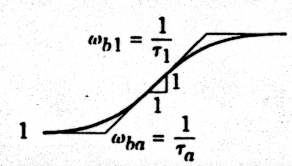
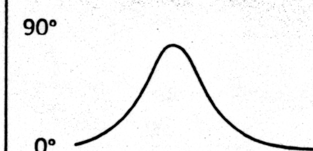
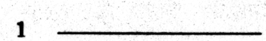
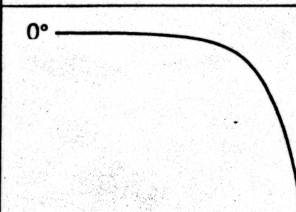
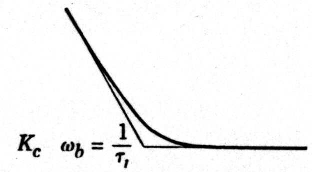
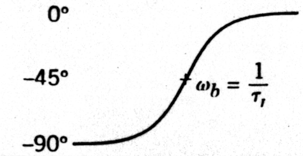
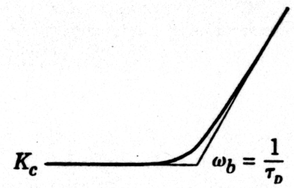
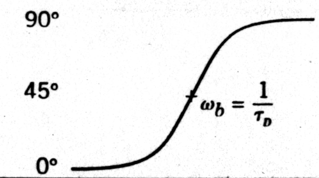
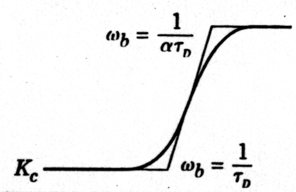
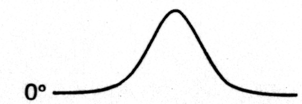
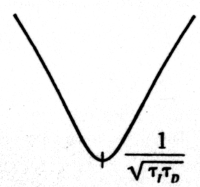
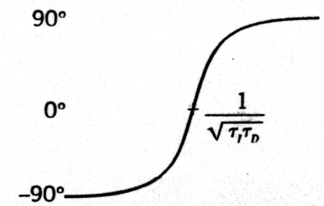
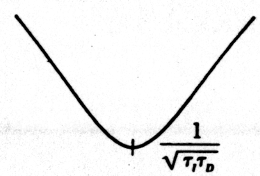
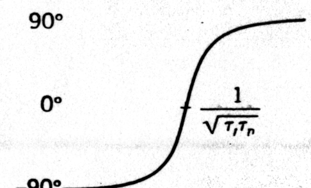
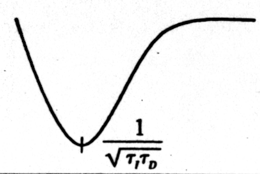
6. **Underdamped second-order**	$\dfrac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\dfrac{K}{\sqrt{(1-(\omega\tau)^2)^2 + (2\zeta\omega\tau)^2}}$		$-\tan^{-1}\left[\dfrac{2\zeta\omega\tau}{1-(\omega\tau)^2}\right]$	
7. **Left-half plane (positive) zero**	$K(\tau_a s + 1)$	$K\sqrt{(\omega\tau_a)^2 + 1}$		$+\tan^{-1}(\omega\tau_a)$	
8. **Right-half plane (negative) zero**	$-\tau_a s + 1$	$K\sqrt{(\omega\tau_a)^2 + 1}$		$-\tan^{-1}(\omega\tau_a)$	
9. **Lead–lag unit ($\tau_a < \tau_1$)**	$K\dfrac{\tau_a s + 1}{\tau_1 s + 1}$	$K\dfrac{\sqrt{(\omega\tau_a)^2 + 1}}{\sqrt{(\omega\tau_1)^2 + 1}}$		$+\tan^{-1}(\omega\tau_a) - \tan^{-1}(\omega\tau_1)$	
10. **Lead–lag unit ($\tau_a > \tau_1$)**	$K\dfrac{\tau_a s + 1}{\tau_1 s + 1}$	$K\dfrac{\sqrt{(\omega\tau_a)^2 + 1}}{\sqrt{(\omega\tau_1)^2 + 1}}$		$+\tan^{-1}(\omega\tau_a) - \tan^{-1}(\omega\tau_2)$	
11. **Time delay**	$Ke^{-\theta s}$	K		$-\omega\tau$	

Table 13.5 Frequency Response Characteristics of Important Controller Transfer Functions

Controller	$G_c(s)$	$AR = \|G_c(j\omega)\|$	Plot of log AR_N vs. log ω	$\phi = \angle G_c(j\omega)$	Plot of ϕ vs. log ω
1. PI	$K_c\left(1 + \frac{1}{\tau_I s}\right) = K_c\left(\frac{\tau_I s + 1}{\tau_I s}\right)$	$K_c\left(\frac{\sqrt{(\omega\tau_I)^2 + 1}}{\omega\tau_I}\right)$		$\tan^{-1}(\omega\tau_I) - 90°$	
2. Ideal PD	$K_c(\tau_D s + 1)$	$K_c\sqrt{(\omega\tau_D)^2 + 1}$		$\tan^{-1}(\omega\tau_D)$	
3. PD with Derivative Filter	$K_c\left(\frac{\tau_D s + 1}{\alpha\tau_D s + 1}\right)$	$K_c\sqrt{\frac{(\omega\tau_D)^2 + 1}{(\alpha\omega\tau_D)^2 + 1}}$		$\tan^{-1}(\omega\tau_D) - \tan^{-1}(\alpha\omega\tau_D)$	
4. Parallel PID	$K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$	$K_c\sqrt{\left(\omega\tau_D - \frac{1}{\omega\tau_I}\right)^2 + 1}$		$\tan^{-1}\left(\omega\tau_D - \frac{1}{\omega\tau_I}\right)$	
5. Series PID	$K_c\left(\frac{\tau_I s + 1}{\tau_I s}\right)(\tau_D s + 1)$	$K_c\frac{\sqrt{(\omega\tau_I)^2 + 1}}{\omega\tau_I}\sqrt{(\omega\tau_D)^2 + 1}$		$\tan^{-1}(\omega\tau_I) + \tan^{-1}(\omega\tau_D) - 90°$	
6. Series PID with Filter	$K_c\left(\frac{\tau_I s + 1}{\tau_I s}\right)\left(\frac{\tau_D s + 1}{\alpha\tau_D s + 1}\right)$	$K_c\left(\frac{\sqrt{(\omega\tau_I)^2 + 1}}{\omega\tau_I}\right)\sqrt{\frac{(\omega\tau_D)^2 + 1}{(\alpha\omega\tau_D)^2 + 1}}$		$\tan^{-1}(\omega\tau_I) + \tan^{-1}(\omega\tau_D) - \tan^{-1}(\alpha\omega\tau_D) - 90°$