

Table 3.1 Laplace Transforms for Various Time-Domain Functions^a

| $f(t)$ | $F(s)$ |
|---|--|
| 1. $\delta(t)$ (unit impulse) | 1 |
| 2. $S(t)$ (unit step) | $\frac{1}{s}$ |
| 3. t (ramp) | $\frac{1}{s^2}$ |
| 4. t^{n-1} | $\frac{(n-1)!}{s^n}$ |
| 5. e^{-bt} | $\frac{1}{s+b}$ |
| 6. $\frac{1}{\tau} e^{-t/\tau}$ | $\frac{1}{\tau s + 1}$ |
| 7. $\frac{t^{n-1} e^{-bt}}{(n-1)!}$ ($n > 0$) | $\frac{1}{(s+b)^n}$ |
| 8. $\frac{1}{\tau^n (n-1)!} t^{n-1} e^{-t/\tau}$ | $\frac{1}{(\tau s + 1)^n}$ |
| 9. $\frac{1}{b_1 - b_2} (e^{-b_2 t} - e^{-b_1 t})$ | $\frac{1}{(s+b_1)(s+b_2)}$ |
| 10. $\frac{1}{\tau_1 - \tau_2} (e^{-t/\tau_1} - e^{-t/\tau_2})$ | $\frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)}$ |
| 11. $\frac{b_3 - b_1}{b_2 - b_1} e^{-b_1 t} + \frac{b_3 - b_2}{b_1 - b_2} e^{-b_2 t}$ | $\frac{s + b_3}{(s+b_1)(s+b_2)}$ |
| 12. $\frac{1}{\tau_1} \frac{\tau_1 - \tau_3}{\tau_1 - \tau_2} e^{-t/\tau_1} + \frac{1}{\tau_2} \frac{\tau_2 - \tau_3}{\tau_2 - \tau_1} e^{-t/\tau_2}$ | $\frac{\tau_3 s + 1}{(\tau_1 s + 1)(\tau_2 s + 1)}$ |
| 13. $1 - e^{-t/\tau}$ | $\frac{1}{s(\tau s + 1)}$ |
| 14. $\sin \omega t$ | $\frac{\omega}{s^2 + \omega^2}$ |
| 15. $\cos \omega t$ | $\frac{s}{s^2 + \omega^2}$ |
| 16. $\sin(\omega t + \phi)$ | $\frac{\omega \cos \phi + s \sin \phi}{s^2 + \omega^2}$ |
| 17. $e^{-bt} \sin \omega t$ | $\left\{ \begin{array}{l} \frac{\omega}{(s+b)^2 + \omega^2} \\ \frac{s+b}{(s+b)^2 + \omega^2} \end{array} \right.$ |
| 18. $e^{-bt} \cos \omega t$ | |
| 19. $\frac{1}{\tau \sqrt{1-\zeta^2}} e^{-\zeta t/\tau} \sin(\sqrt{1-\zeta^2} t/\tau)$ ($0 \leq \zeta < 1$) | $\frac{1}{\tau^2 s^2 + 2\zeta \tau s + 1}$ |
| 20. $1 + \frac{1}{\tau_2 - \tau_1} (\tau_1 e^{-t/\tau_1} - \tau_2 e^{-t/\tau_2})$ ($\tau_1 \neq \tau_2$) | $\frac{1}{s(\tau_1 s + 1)(\tau_2 s + 1)}$ |
| 21. $1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta t/\tau} \sin[\sqrt{1-\zeta^2} t/\tau + \psi]$ $\psi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$, ($0 \leq \zeta < 1$) | $\frac{1}{s(\tau^2 s^2 + 2\zeta \tau s + 1)}$ |
| 22. $1 - e^{-\zeta t/\tau} [\cos(\sqrt{1-\zeta^2} t/\tau) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\sqrt{1-\zeta^2} t/\tau)]$ ($0 \leq \zeta < 1$) | $\frac{1}{s(\tau^2 s^2 + 2\zeta \tau s + 1)}$ |