

# **BEHAVIOUR OF CONCRETE MEMBERS UNDER CYCLIC LOADING**

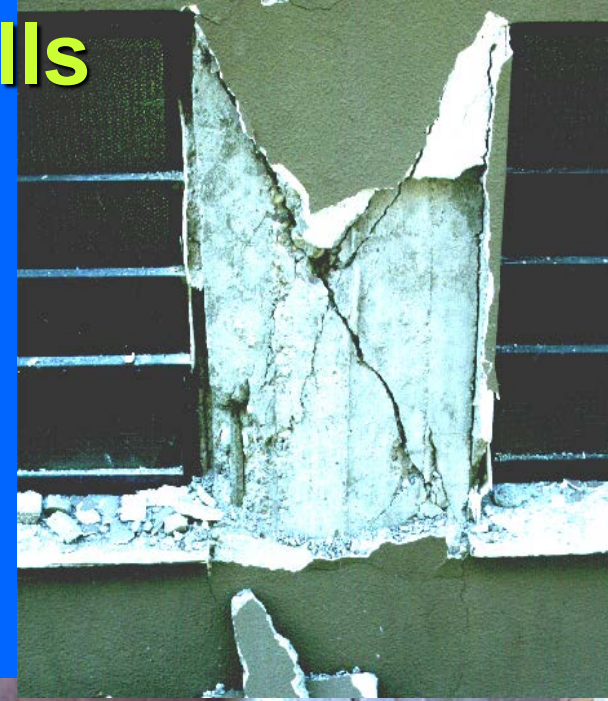


# II. CONCRETE MEMBERS

# Shear

**(w/ some effects of inelastic flexure)**

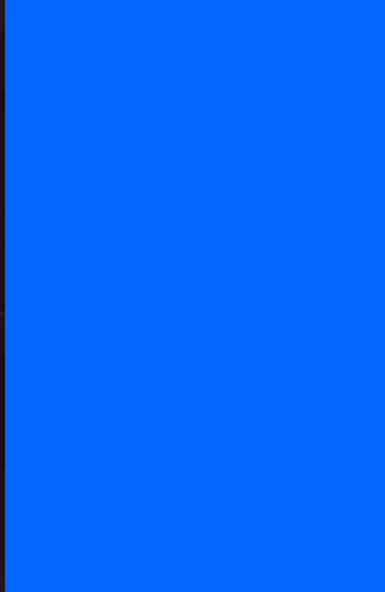
# Shear failures of columns or walls





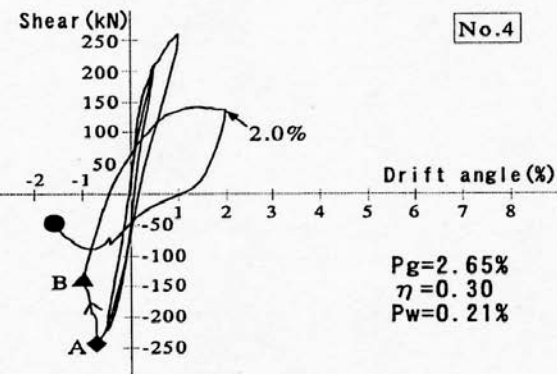
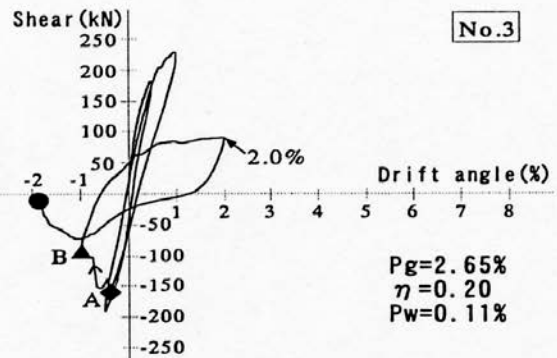
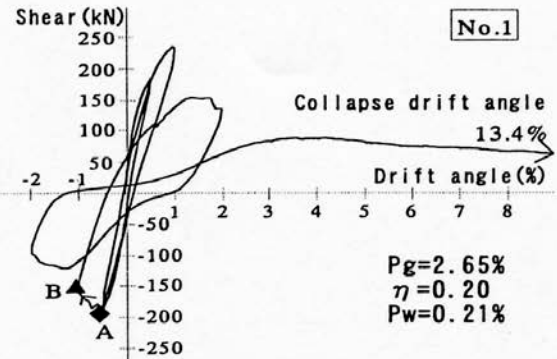


**Shear failures  
of columns or  
walls  
(top two, in  
plastic hinge  
region)**

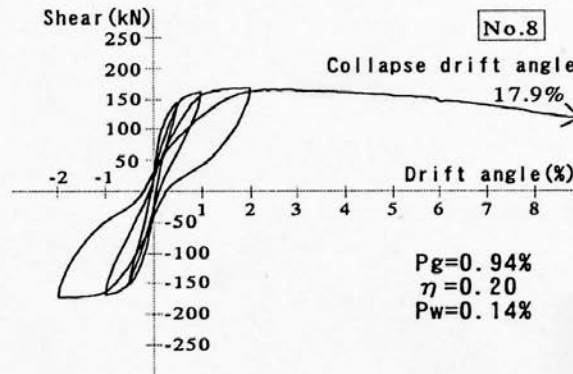
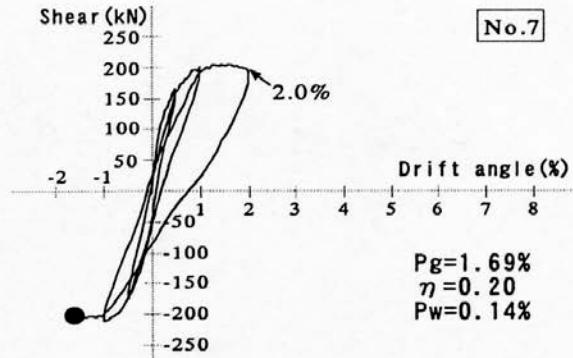
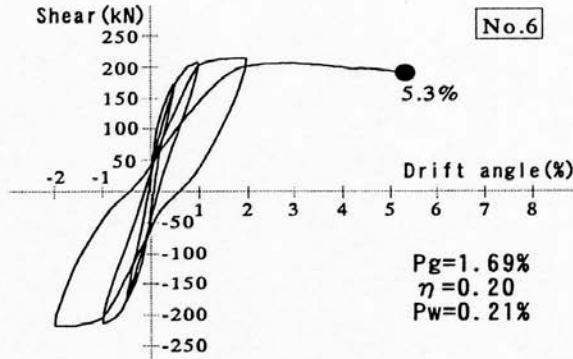


# Brittle vs ductile behaviour in cyclic shear

◆ Shear failure  
● Collapse



(a) Shear mode

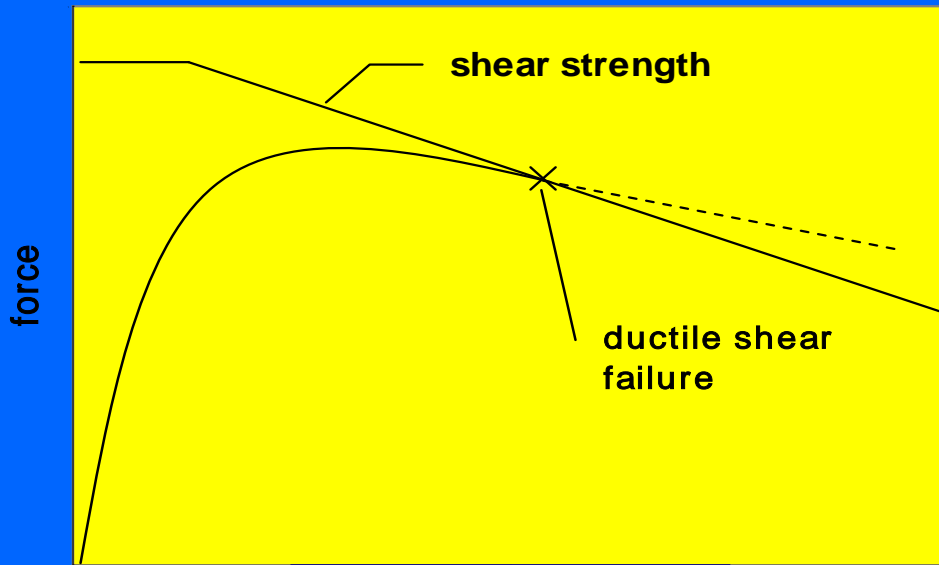
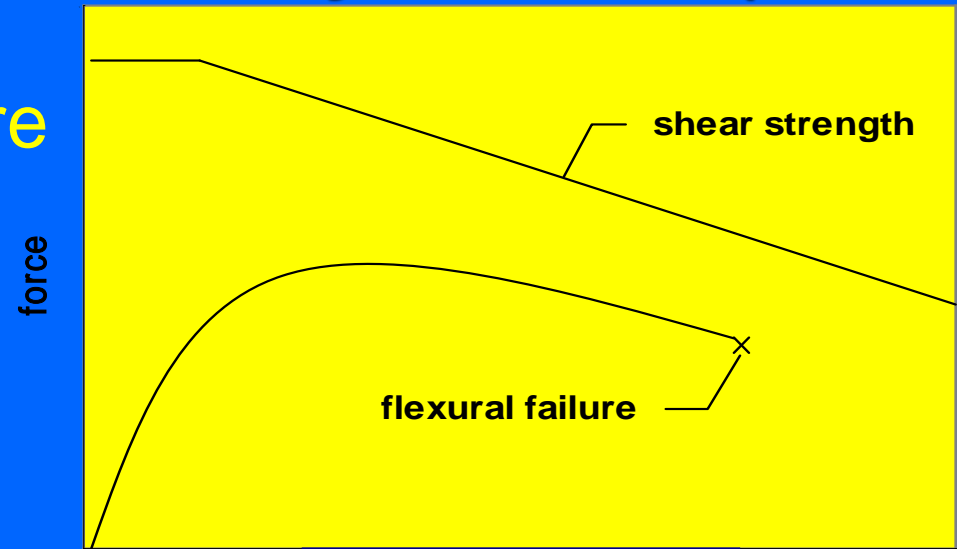


(b) Flexure-Shear mode  
(No.8: Flexure mode)

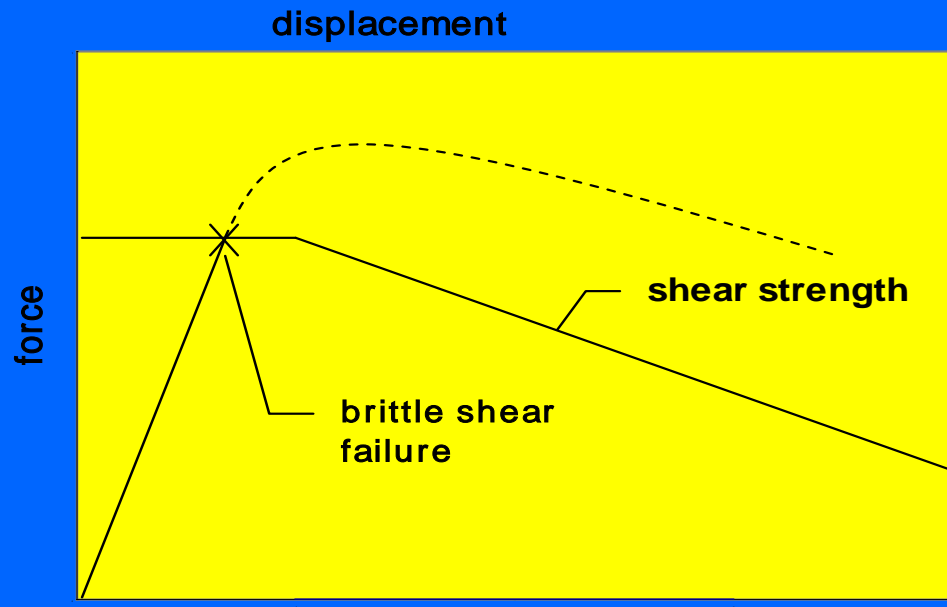
Shear force-chord rotation behaviour:  
(a) brittle shear;  
(b) “ductile shear” or flexural behaviour

# Brittle vs ductile behaviour in cyclic shear (cont'd)

## Flexural failure



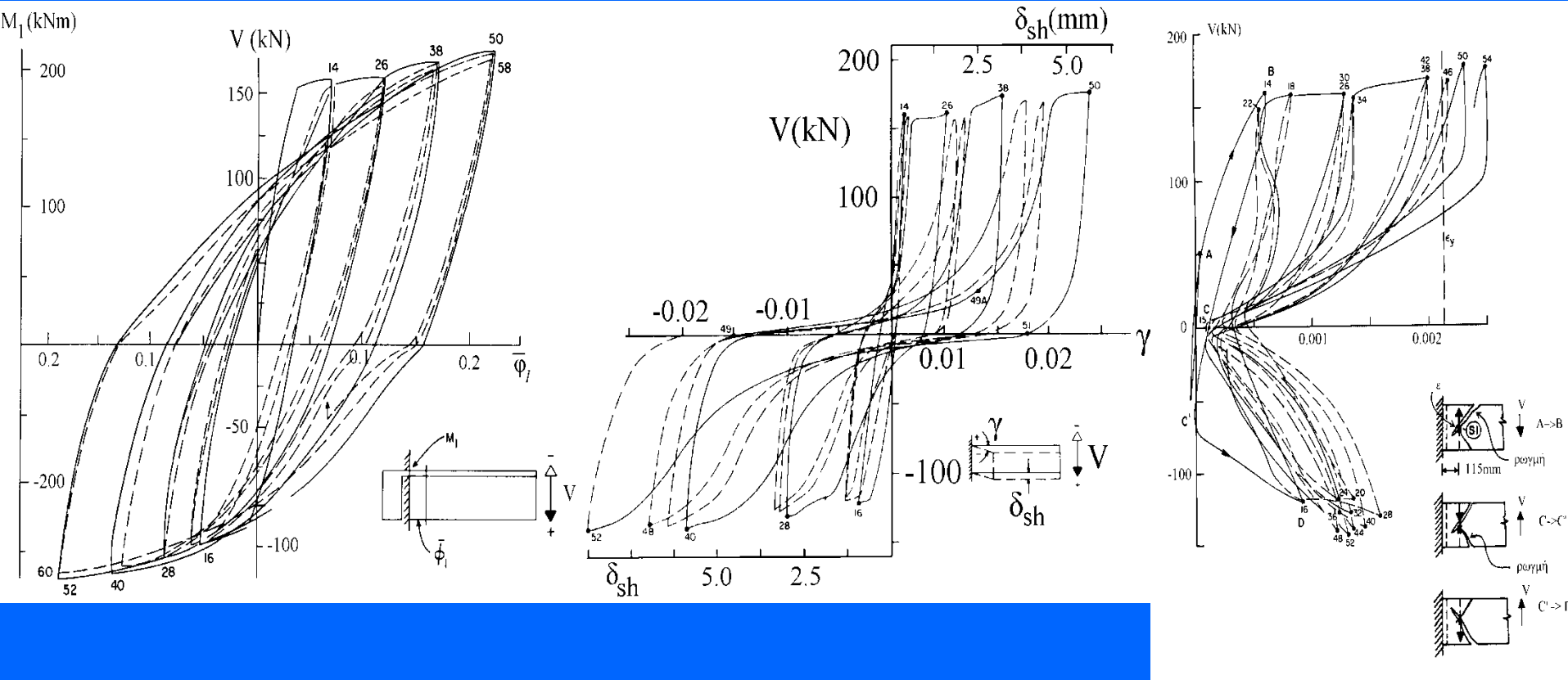
displacement  
**Ductile shear failure**



displacement  
**Brittle shear failure**



# Effect of cyclic inelastic deformations on shear behaviour after flexural yielding



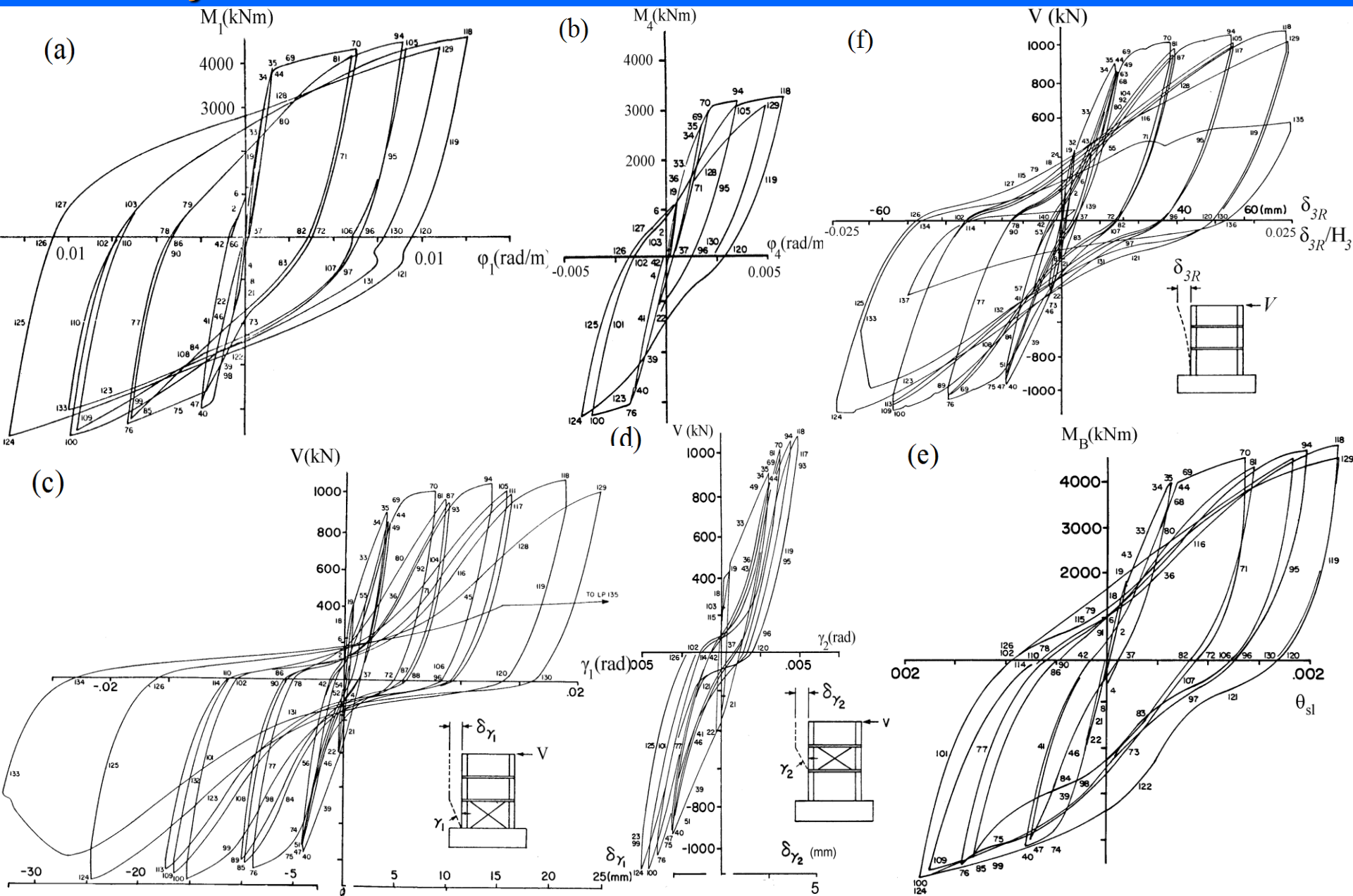
(a)

(b)

(c)

(a)  $M$ - $\phi$  loops next to end section; (b)  $V$ - $\gamma$  loops in plastic hinge region; (c) loops of shear force ( $V$ ) - stirrup strain

# Effect of cyclic inelastic flexural deformations on shear behaviour -cont'd



(a), (b):  $M-\phi$  loops next to base of 1<sup>st</sup> & 2<sup>nd</sup> storey; (c), (d):  $V-\gamma$  loops over 1<sup>st</sup> & 2<sup>nd</sup> storey; (f) base shear  $v$  top deflection (e) loops of base moment  $v$  fixed-end rotation due to bar pull-out from footing

# Fundamental models - monotonic shear resistance

## Truss model w/ variable strut inclination $\delta$ , CEB/FIP Model Code 90 & Eurocode 2

– Shear resistance in diagonal tension,

due to transverse reinforcement:  $V_R = \rho_w f_{yw} b_w z \cot \delta + 0.5N(h-x)/L_s$

- Eurocode 2:  $0.4 \leq \tan \delta \leq 1$ ,  $22^\circ \leq \delta \leq 45^\circ$ ,  
Model Code 90:  $1/3 \leq \tan \delta \leq 1$ ,  $18^\circ \leq \delta \leq 45^\circ$

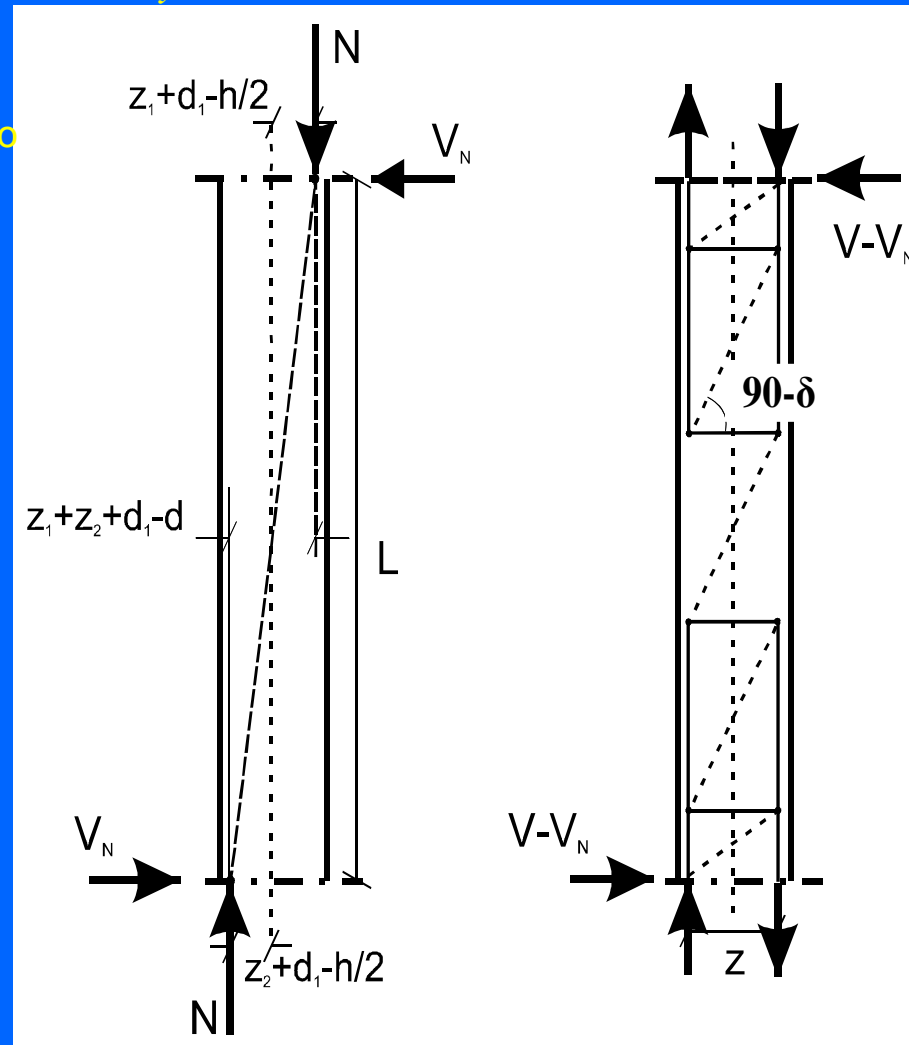
– Diagonal compression field at angle  $\delta$  to member axis

$$= \rho_w f_{yw} (1 + \cot^2 \delta) < n f_c$$

– may reach diagonal concrete strength,  $n f_c$

–  $n$ : reduction factor due to transverse tensile stresses/strains

- Eurocode 2 & Model Code 90:  
 $n = 0.6(1 - f_c(\text{MPa})/250)$



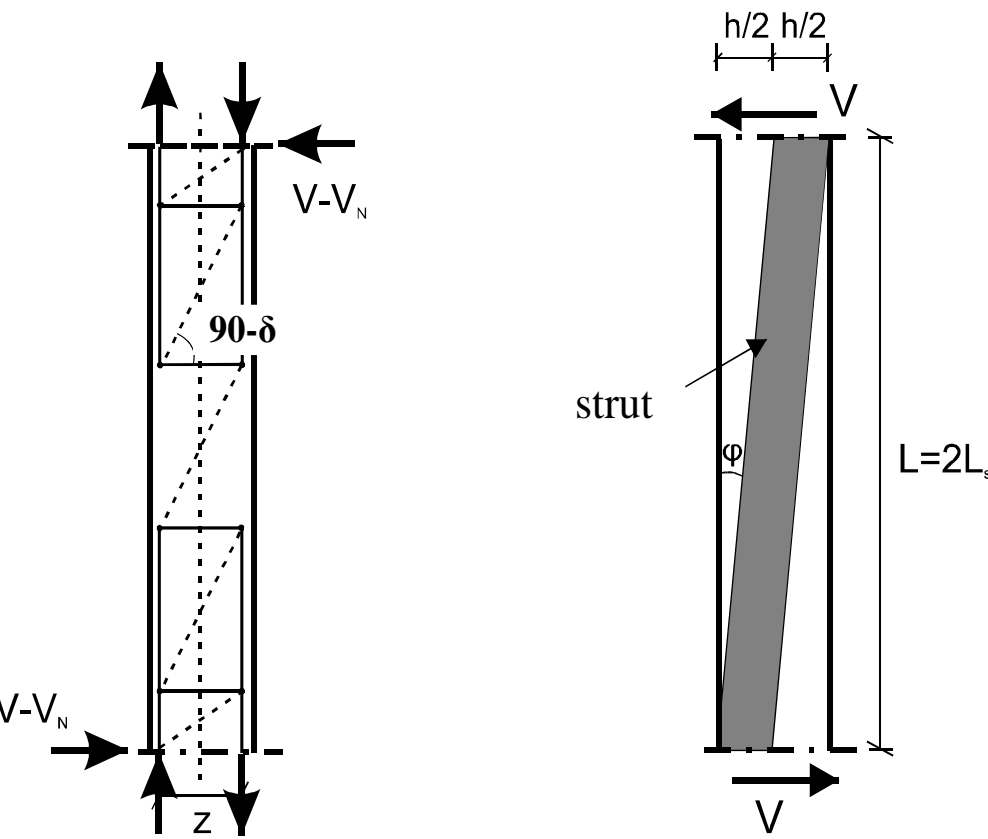
# Fundamental models for monotonic shear resistance (cont'd)

## AIJ Guidelines model

Concrete strut w/ width equal to 50% of section depth:

- contributes to  $V_R$  via transverse component of strut force;
- consumes part of the diagonal concrete strength,  $nf_c$
- rest of concrete strength is available for diagonal compression field in truss mechanism (angle:  $\delta$ )

$$V_R = \rho_w f_{yw} b_w z \cot \delta + 0.5 b_w h [n f_c - \rho_w f_{yw} (1 + \cot^2 \delta)] \tan \phi$$



$V_R$  for  $\cot \delta \leq \min[2; \sqrt{(n f_c / \rho_w f_{yw} - 1)}]$   
 unless:  $0.5 \tan \phi (\approx h/2 L_s = h/L) \geq 2z/h$   
 Then  $V_R$  reaches maximum value if:  
 $\cot \delta = z / (h \tan \phi) \approx 4 L_s z / h^2$   
 Maximum  $V_R$  equal to (with  $\zeta = z/h$ ):

$$V_R = 0.5 b_w h [n f_c \tan \phi + \rho_w f_{yw} (\zeta^2 - \tan^2 \phi)]$$

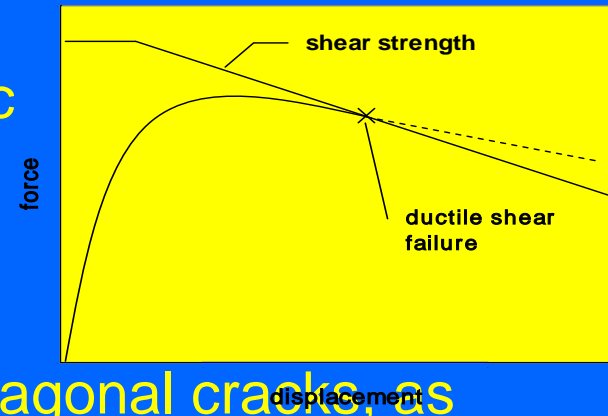
Inelastic cyclic deformation effect:

- $\cot \delta \leq \max(2 - 50 \theta_{pl}, 1)$ ;
  - $n$  on  $f_c$  multiplied x  $\max(0.25, 1 - 15 \theta_{pl})$
- with  $\theta_{pl} = (\mu_\theta - 1) \theta_y$



# Cyclic shear strength degradation

- Shear resistance degrades with cyclic loading: RC member that yields in flexure may ultimately fail in shear.
- Provisions of concrete design codes for shear strength apply to monotonic loading;
- Seismic codes (e.g. EC8) may reduce  $V_R$  if cyclic ductility demands are high.



## Degradation mechanisms :

- Gradual reduction of aggregate interlock along diagonal cracks, as interfaces become smoother with cyclic loading.
- Degradation of dowel action (also due to accumulation of inelastic strains in longitudinal reinforcement).
- Development of flexural cracks throughout the depth of the member → reduction of contribution of compression zone to shear resistance.
- Bond slippage & accumulation of inelastic strains in shear reinforcement → aggregate interlock reduced as diagonal cracks gradually open up.
- Softening of concrete in diagonal compression due to accumulation of transverse tensile strains.

# Models for diagonal tension cyclic shear resistance after flexural yielding

Biskinis et al 2004, Part 3 of EC8

(circular columns, rectangular beams/columns/walls, non-rectangular walls, hollow rectangular piers)

$$V_R = \frac{h-x}{2L_s} \min(N, 0.55A_c f_c) + 0.16 \cdot \left(1 - 0.095 \min\left(5, \mu_\theta^{pl}\right)\right) \max(0.5, 100 \rho_{tot}) \left(1 - 0.16 \min\left(5, \frac{L_s}{h}\right)\right) \sqrt{f_c} A_c + V_w$$

or:

$$V_R = \frac{h-x}{2L_s} \min(N, 0.55A_c f_c) + \left(1 - 0.05 \min\left(5, \mu_\theta^{pl}\right)\right) \left[0.16 \max(0.5, 100 \rho_{tot}) \left(1 - 0.16 \min\left(5, \frac{L_s}{h}\right)\right) \sqrt{f_c} A_c + V_w\right]$$

- $V_w$ ,  $V_N$ ,  $V_c$  terms;

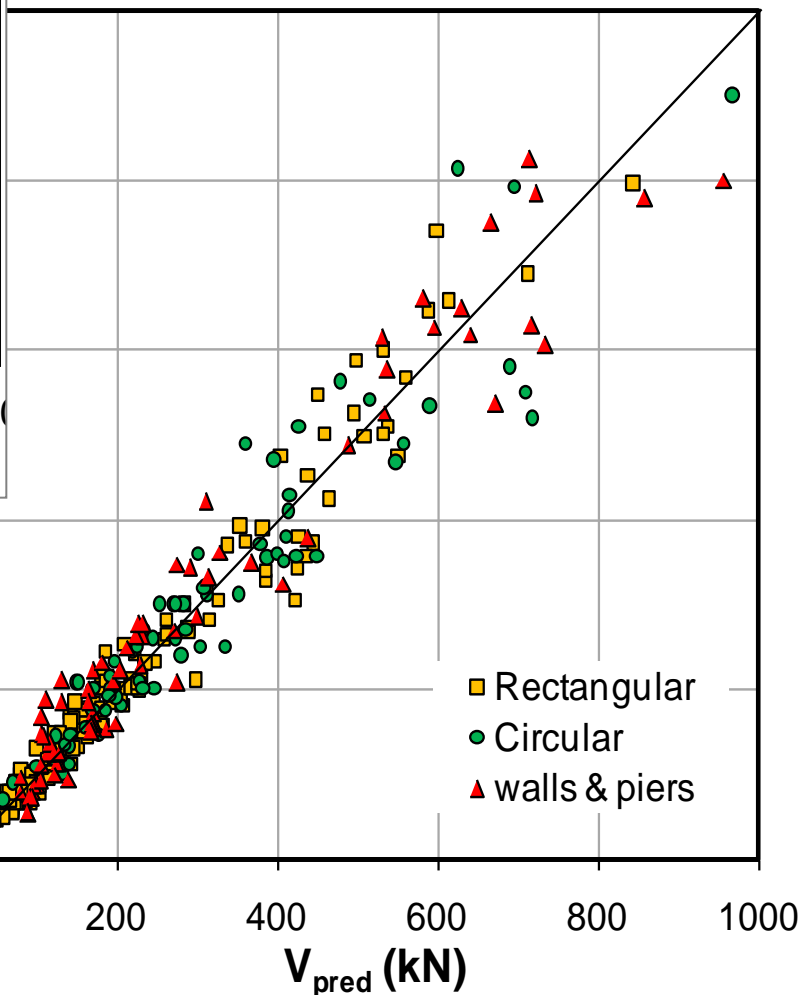
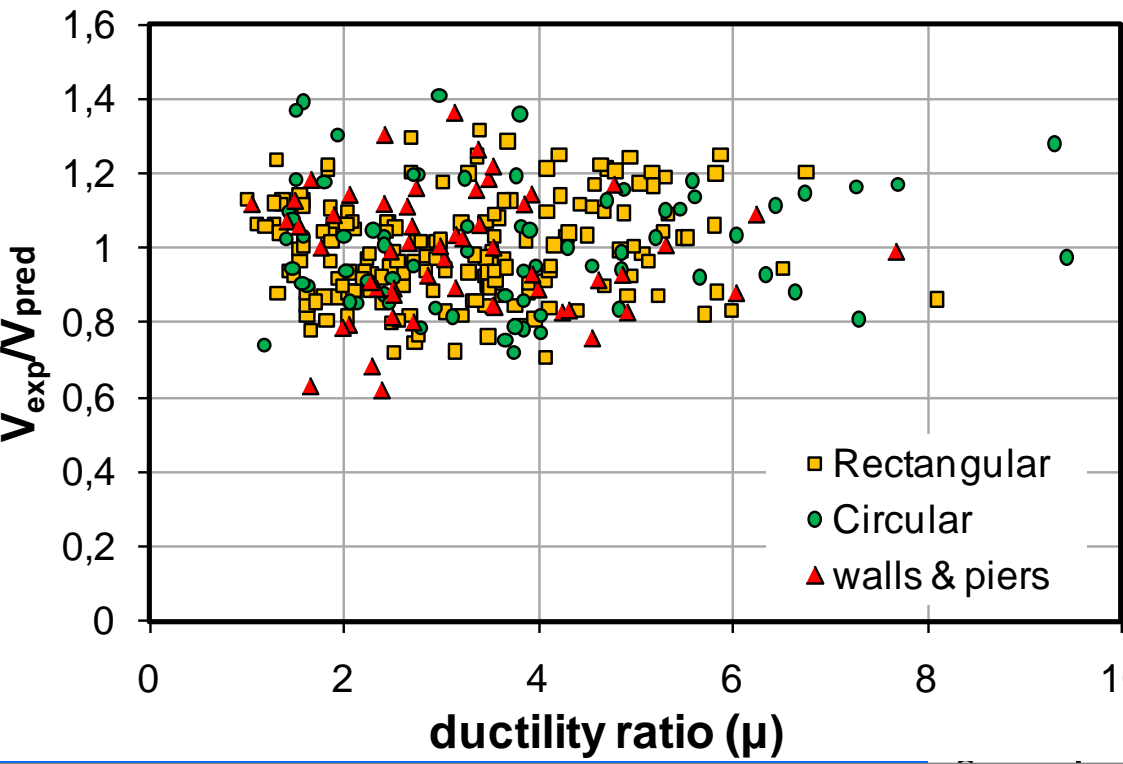
- Inclination of compression struts:  $\delta = 45^\circ$        $V_w = \rho_w b_w z f_{yw}$

- Linear degradation of  $V_c$  for ductility ratio demand from 1 to 6;

- In 1<sup>st</sup> model:  $V_c$  for  $\mu_\theta \geq 6$  is 52.5% of initial one

- In 2<sup>nd</sup> model:  $V_w + V_c$  for  $\mu_\theta \geq 6$  is 75% of initial one.

# Test v model: Diagonal tension cyclic shear resistance in plastic hinge (after flexural yielding)



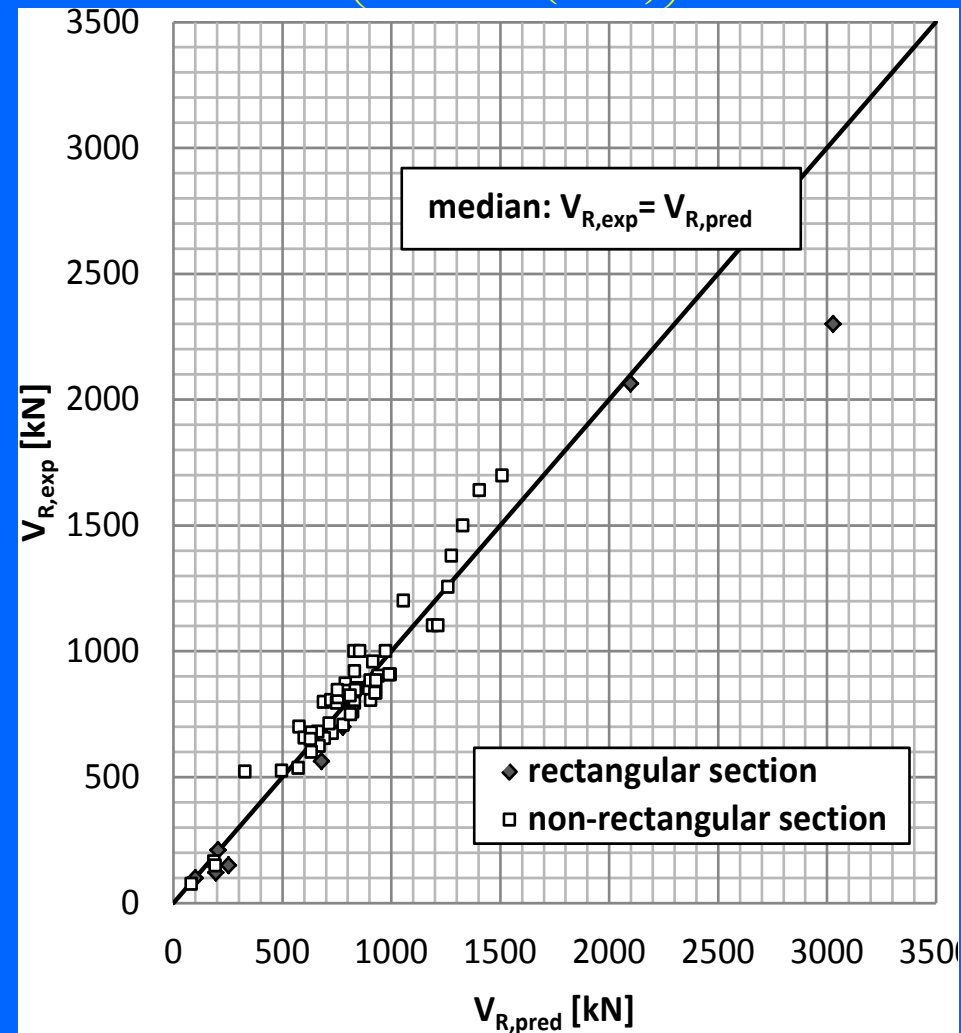
no. tests: 335  
median=1.00  
CoV=16.2%

# Cyclic shear resistance of squat walls in diagonal compression before or after flexural yielding

$$V_{R,max} = 0.85 \left( 1 - 0.06 \min \left( 5; \mu_{\theta}^{pl} \right) \right) \left( 1 + 1.8 \min \left( 0.15; \frac{N}{A_c f_c} \right) \right) \left( 1 + 0.25 \max \left( 1.75; 100 \rho_{tot} \right) \right) \left( 1 - 0.2 \min \left( 2; \frac{L_s}{h} \right) \right) \sqrt{\min \left( f_c; 100 \right) b_w z}$$

no. tests: 62, median=1.00,  
CoV=14.5%

Experimental cyclic shear  
resistance for shear  
compression failure of squat  
walls v predictions





# **Flexure-shear interaction in squat members**

# Monotonic lateral force resistance of squat members w/ flexure-shear interaction

## Generalization of AIJ Guidelines model

Concrete strut over depth  $x$  of compression zone:

- takes also the axial load,  $N$ ;
- contributes to  $V_R$  via transverse component of strut force;
- consumes part of diagonal concrete strength,  $nf_c$
- rest of concrete strength is available for diagonal compression field in truss mechanism, at angle  $\delta$ , w/  $\cot\delta \leq \sqrt{(nf_c/\rho_w f_{yw} - 1)}$ ;  $\cot\delta \sim L_s/h$ .

# Monotonic lateral force resistance of squat members w/ flexure-shear interaction (cont'd)

1. In axial force range:  $N_1 = 0.5bhnf_c - A_{s,tot}f_y + \rho_w b_w f_{yw} [\cot\delta(2L_s + (z-0.5h)\cot\delta) - 0.5h] \leq N \leq N_2 = 0.5bhnf_c + A_{s,tot}f_y - \rho_w b_w f_{yw} [\cot\delta(2L_s - (z-0.5h)\cot\delta) + 0.5h]$

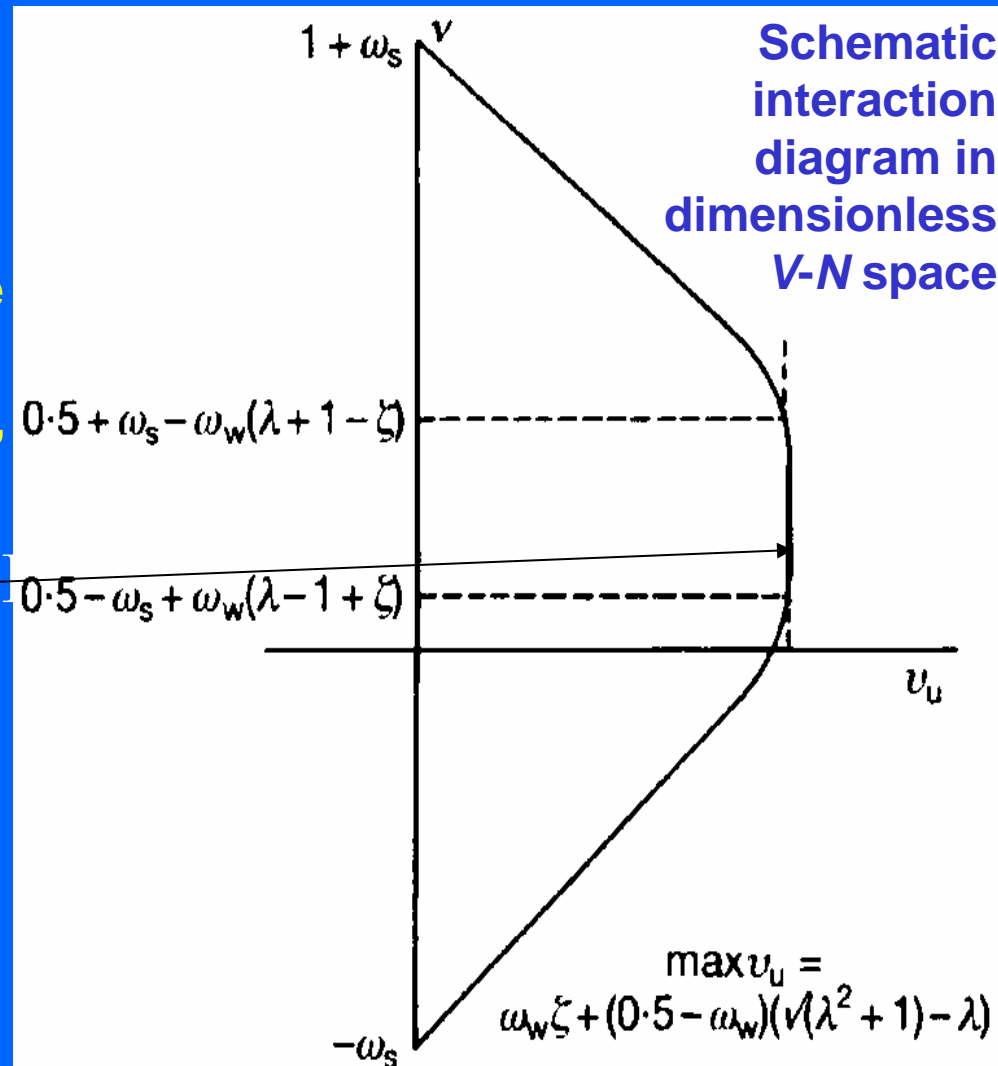
Strut inclination is: 
$$\tan \phi = \sqrt{\left(\frac{2L_s}{h}\right)^2 + 1} - \frac{2L_s}{h}$$

Very brittle failure:

Concrete fails in diagonal compression, w/ yielding of transverse reinforcement, but no yielding of tension or compression reinforcement, at an ultimate shear force of:

$$V_R = 0.5b_w h [nf_c \tan \phi + \rho_w f_{yw} (\zeta^2 - \tan^2 \phi)]$$

N-range exists if: 
$$\cot \delta \leq \frac{\omega_{tot}}{\omega_w} \frac{h}{2L_s}$$



# Monotonic lateral force resistance of squat members w/ flexure-shear interaction (cont'd)

2. In axial force range:  $N_1 \geq \mathbf{N} \geq -A_{s,tot} f_y$

Strut inclination is: 
$$\tan \phi = \min \left( \sqrt{\left(\frac{L_s}{\eta h}\right)^2 + \frac{1-\eta}{\eta}} - \frac{L_s}{\eta h}, \frac{h}{2L_s} \right)$$

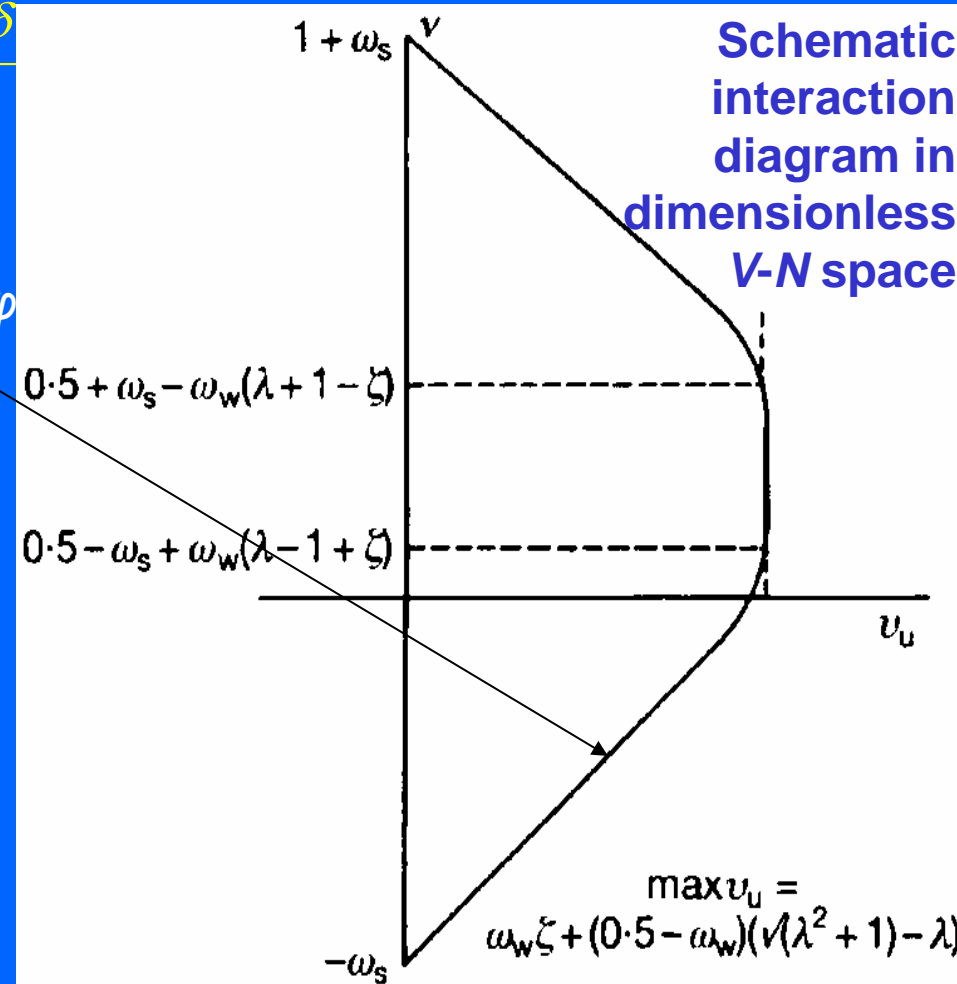
where:

$$\eta = \frac{N + A_{s,tot} f_y - \rho_w f_{yw} b_w (2L_s + z \cot \delta) \cot \delta}{b_w h (n f_c - \rho_w f_{yw} (1 + \cot^2 \delta))}$$

& ultimate shear is:

$$V_R = (N + A_{s,tot} f_y) \tan \phi + \rho_w f_{yw} b_w \cot \delta [z - (2L_s + z \cot \delta) \tan \phi]$$

Moderately brittle failure:  
Concrete fails by diagonal compression, w/ yielding of transverse reinforcement & of tension reinforcement.





# Monotonic lateral force resistance of squat members w/ flexure-shear interaction (cont'd)

3. In axial force range:  $Nf_c b_w h + A_{s,tot} f_y \geq N \geq N_2$

Strut inclination is: 
$$\tan \phi = \min \left( \sqrt{\left(\frac{L_s}{\eta h}\right)^2 + \frac{1-\eta}{\eta}} - \frac{L_s}{\eta h}, \frac{h}{2L_s} \right)$$

where:

$$\eta = \frac{(N - A_{s,tot} f_y) + \rho_w f_{yw} b_w (2L_s - z \cot \delta) \cot \delta}{b_w h (n f_c - \rho_w f_{yw} (1 + \cot^2 \delta))}$$

& ultimate shear is:

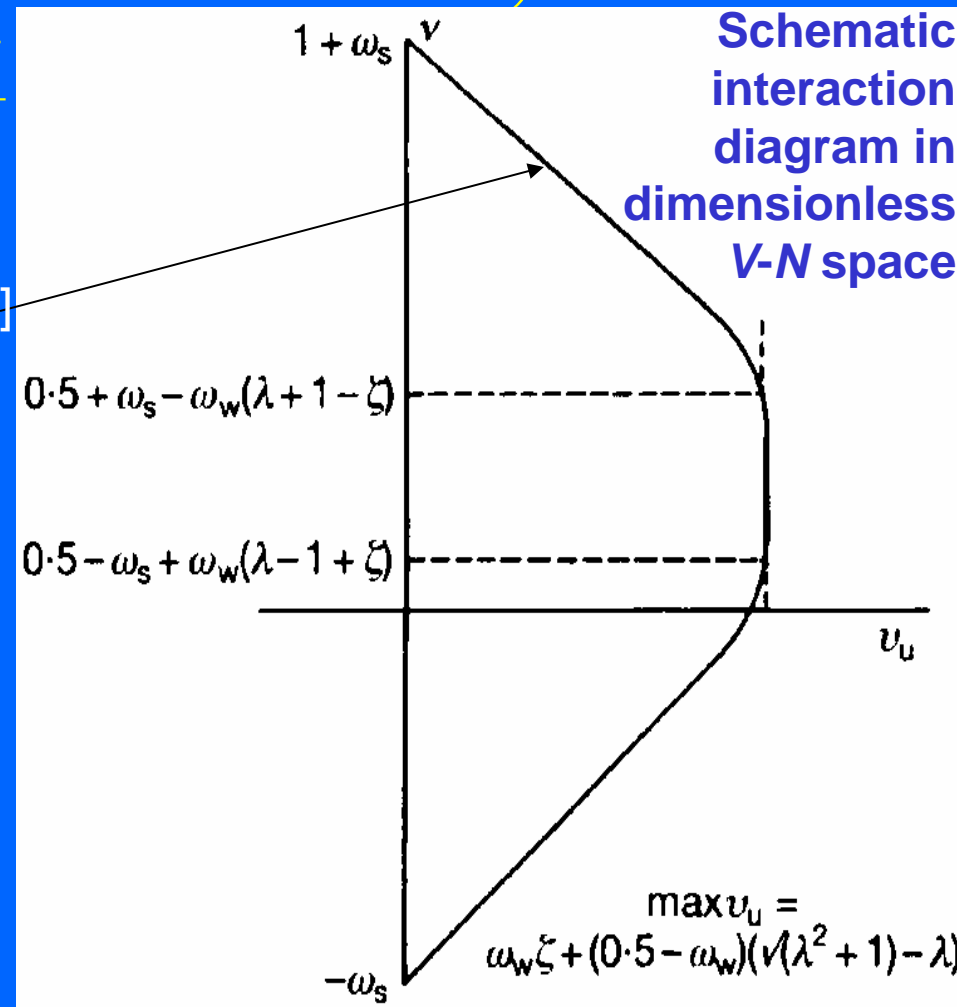
$$V_R = (N - A_{s,tot} f_y) \tan \phi + \rho_w f_{yw} b_w \cot \delta [z + (2L_s - z \cot \delta) \tan \phi]$$

Moderately brittle failure:

Concrete fails by diagonal compression, w/ yielding of transverse reinforcement & of compression reinforcement.

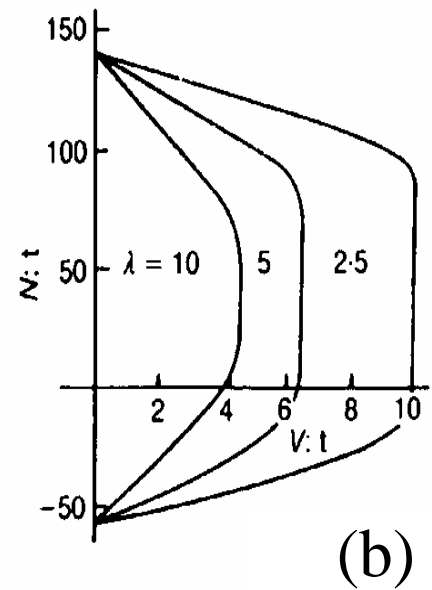
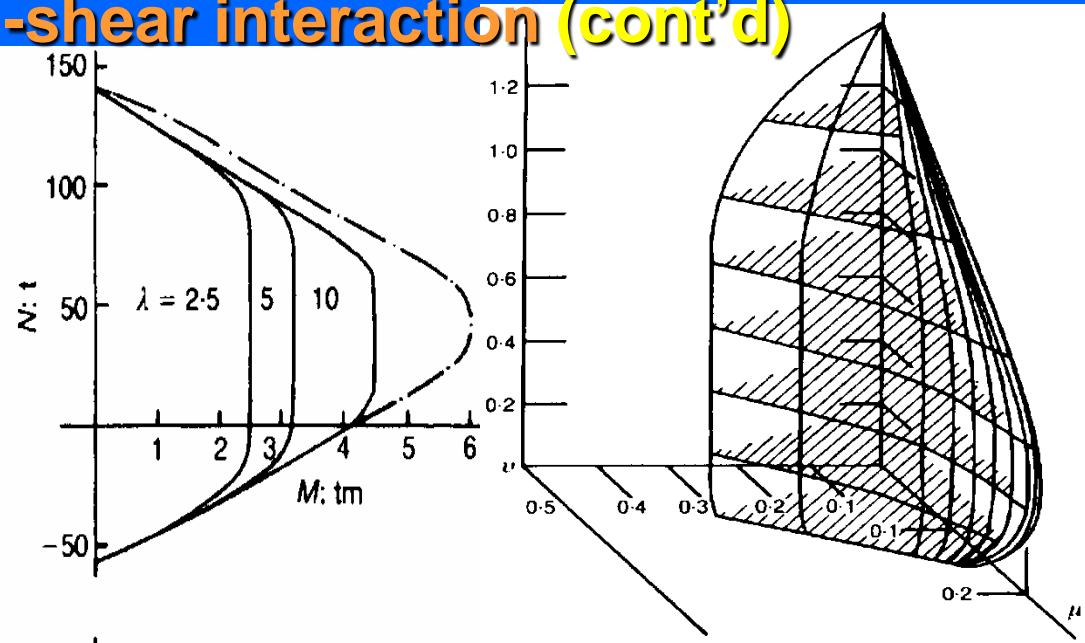
If shear effects unimportant ( $L_s/h \gg 2.5$ ), interaction diagram degenerates into simple  $\mu$ - $v$  diagram:

$$\begin{aligned} \mu &= 0.5 \zeta (v + \omega_{tot}) & \text{for } 0.5n > v \geq -\omega_{tot} \\ \mu &= 0.5 \zeta (n + \omega_{tot} - v) & \text{for } n + \omega_{tot} \geq v \geq 0.5n \end{aligned}$$

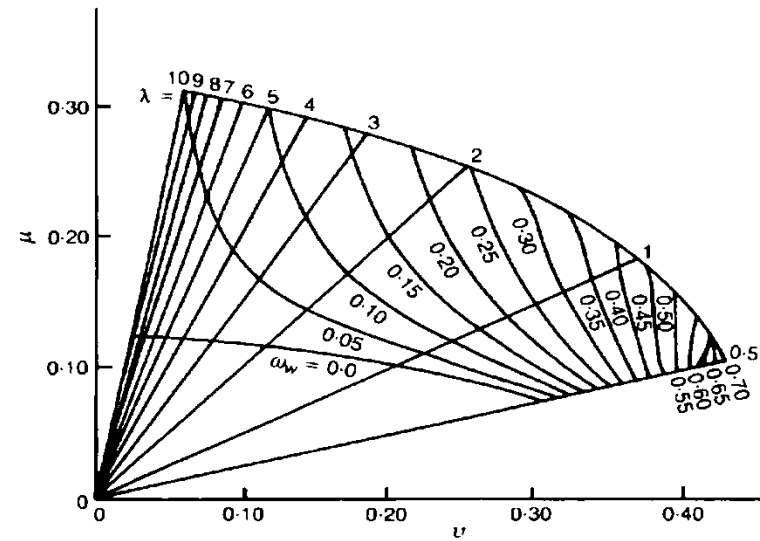
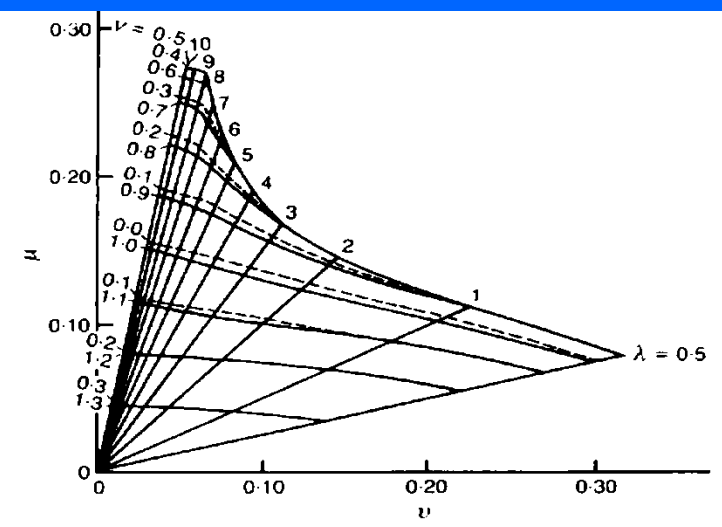


# Monotonic lateral force resistance of squat members w/ flexure-shear interaction (cont'd)

## -shear interaction (cont'd)



(c)



(b) Dimensional interaction  $M-N$  and  $V-N$  diagrams of 200mm square column with four 16mm bars; (c) example dimensionless  $M-V-N$  diagrams

# Monotonic lateral force resistance of squat members w/ flexure-shear interaction (cont'd)

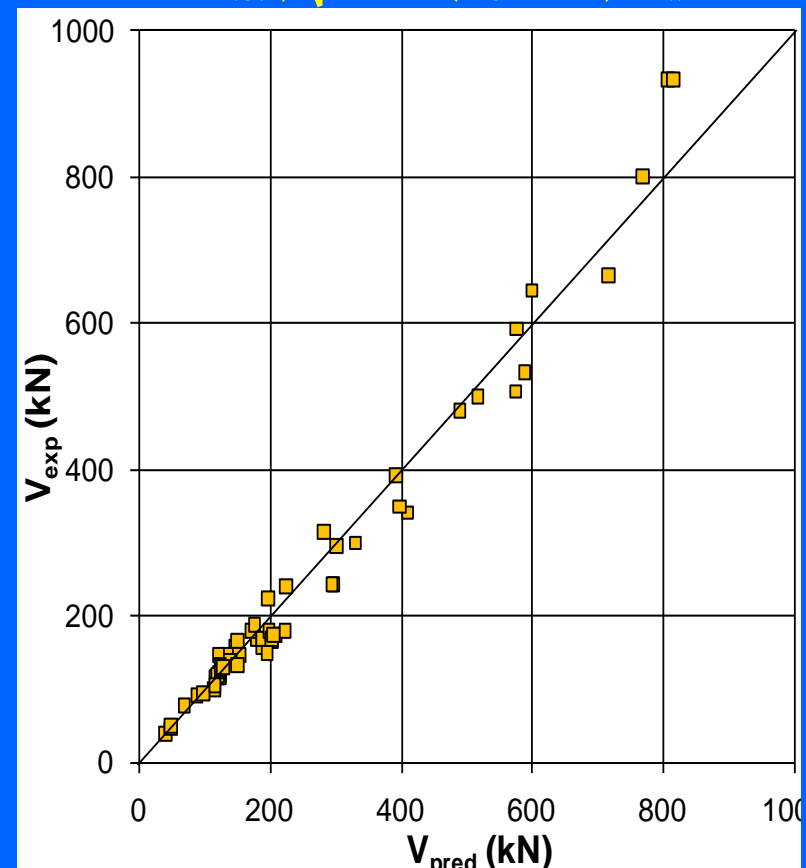
## Cyclic shear resistance of squat columns in diagonal compression after flexural yielding

$\delta$  = angle of column  
diagonal to member  
axis:  $\tan \delta = h/2L_s$

$$V_{R,max} = \frac{4}{7} \left( 1 - 0.02 \min(5; \mu_{\theta}^{pl}) \right) \left( 1 + 1.35 \frac{N}{A_c f_c} \right) \left( 1 + 0.45 \cdot 100 \rho_{tot} \right) \sqrt{\min(f_c; 40)} b_w z \sin 2\delta$$

Experimental cyclic shear  
resistance for shear compression  
failure of squat columns after  
flexural yielding v predictions

no. tests: 64, median=1.00,  
CoV=10.4%



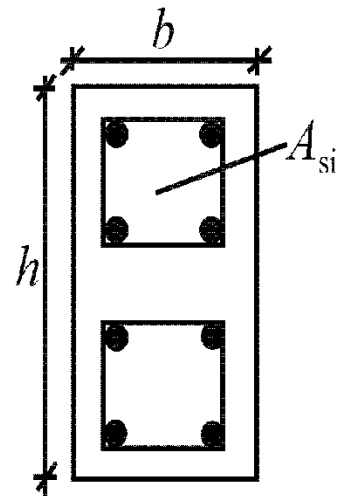
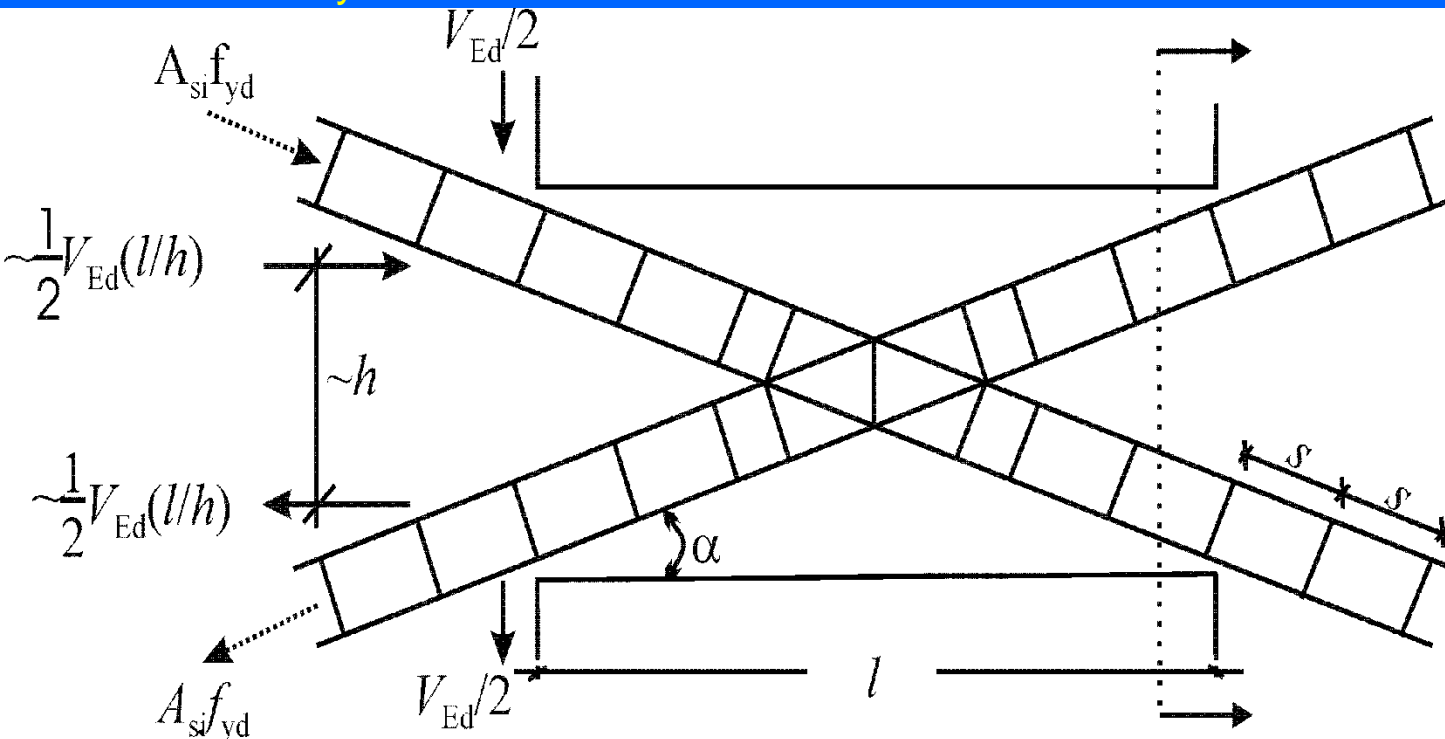
# Monotonic lateral force resistance of squat members w/ flexure-shear interaction (cont'd)

## Diagonal reinforcement in squat columns or deep beams

$$V_{Ed} = 2A_{sd}f_{yd}\sin\delta$$

$$(\tan\delta = z/L = z/2L_s)$$

$$M_d = zA_{sd}f_{yd}\cos\delta$$



Coupling beams w/ diagonal reinforcement in Eurocode 8