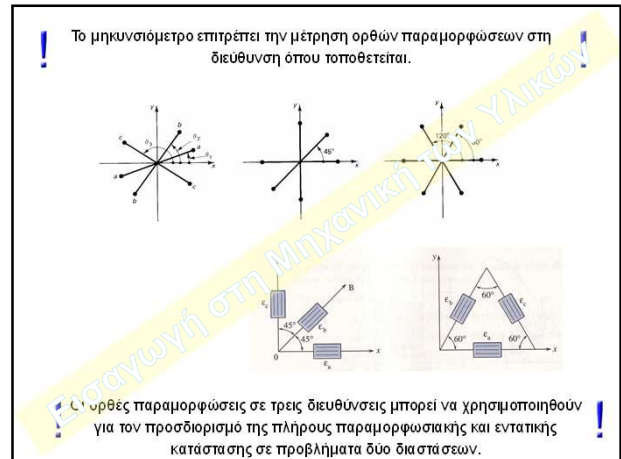
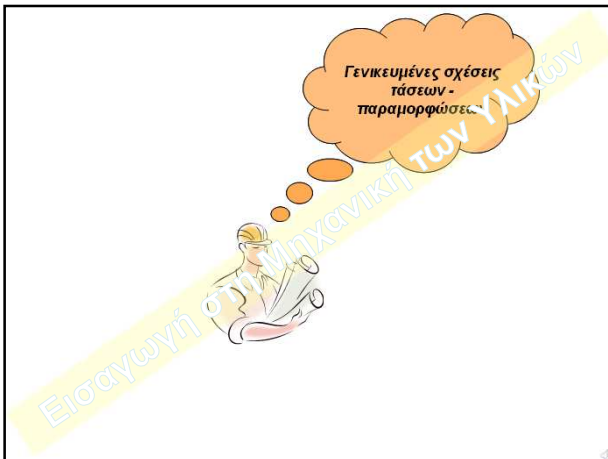




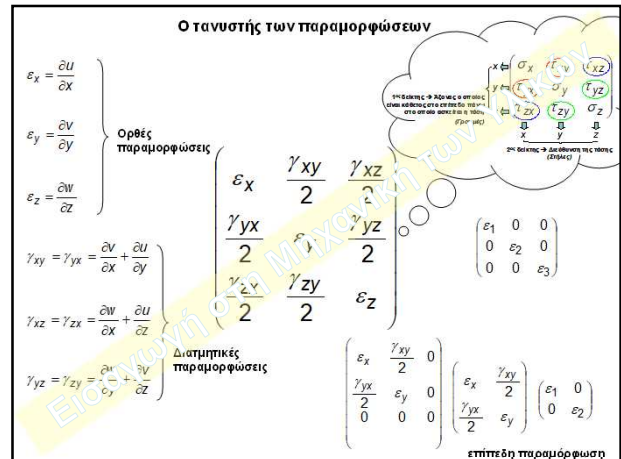
1



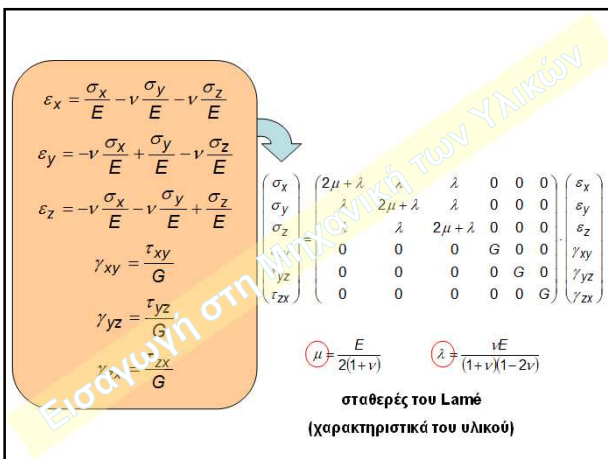
2



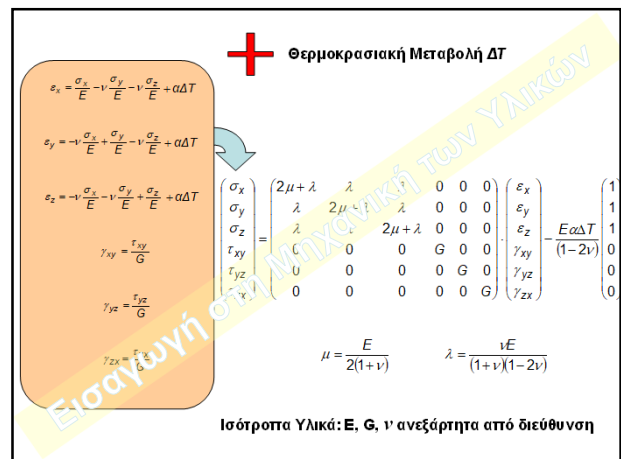
3



4



5



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Επίπεδη εντατική κατάσταση

$$\begin{pmatrix} \sigma_x & \tau & 0 \\ \tau & \sigma_y & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} & 0 \\ \frac{\gamma_{yx}}{2} & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}$$

Επίπεδη παραμορφωση

$$\begin{pmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} \\ \frac{\gamma_{yx}}{2} & \varepsilon_y \end{pmatrix} \neq \begin{pmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} \\ \frac{\gamma_{yx}}{2} & \varepsilon_y \end{pmatrix}$$

Επίπεδη εντατική κατάσταση κατάστασι

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E}$$

$$\varepsilon_y = -\nu \frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E}$$

$$\varepsilon_z = -\nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G}$$

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Επίπεδη παραμορφωση:

$$\begin{pmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} \\ \frac{\gamma_{yx}}{2} & \varepsilon_y \end{pmatrix} = \begin{pmatrix} 2\mu + \lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2\mu + \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2\mu + \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{pmatrix}$$

Μη μηδενικές τάσεις

$$\mu = \frac{E}{2(1+\nu)} \quad \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

Επίπεδη εντατική κατάσταση κατάστασι

$$\varepsilon_z = -\nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} + \frac{\sigma_z}{E} = 0 \Rightarrow \sigma_z = \nu(\sigma_x + \sigma_y)$$

αναλλοίωτη ποσότητα, ανεξάρτητη του προσανατολισμού των επιπέδων

$\sigma_x + \sigma_y = \sigma_x + \sigma_y$

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Επίπεδη εντατική κατάσταση - Κύρια επίπεδα

$$\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}$$

Επίπεδη εντατική κατάσταση κατάστασι

$$\varepsilon_1 = \frac{\sigma_1}{E} - \nu \frac{\sigma_2}{E} - \nu \frac{\sigma_3}{E} = \frac{1}{E}(\sigma_1 - \nu\sigma_2)$$

$$\varepsilon_2 = -\nu \frac{\sigma_1}{E} + \frac{\sigma_2}{E} - \nu \frac{\sigma_3}{E} = \frac{1}{E}(-\nu\sigma_1 + \sigma_2)$$

$$\varepsilon_3 = -\nu \frac{\sigma_1}{E} - \nu \frac{\sigma_2}{E} + \frac{\sigma_3}{E} = -\frac{\nu}{E}(\sigma_1 + \sigma_2)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G}$$

Κύρια επίπεδα

$$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2)$$

$$\sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

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Επίπεδη εντατική κατάσταση κατάστασι

$$\varepsilon_x' = \varepsilon_x \cos^2 \theta + \varepsilon_y \sin^2 \theta + \gamma_{xy} \sin \theta \cos \theta$$

$$\varepsilon_y' = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

αγνωστές παραμορφώσεις

$$\varepsilon_{\theta_1} = \varepsilon_x \cos^2 \theta_1 + \varepsilon_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1$$

$$\varepsilon_{\theta_2} = \varepsilon_x \cos^2 \theta_2 + \varepsilon_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2$$

$$\varepsilon_{\theta_3} = \varepsilon_x \cos^2 \theta_3 + \varepsilon_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3$$

γνωστές παραμορφώσεις

Εάν: $\theta_1 = 0^\circ, \theta_2 = 45^\circ, \theta_3 = 90^\circ$

Τότε: $\varepsilon_x = \varepsilon_{0^\circ}, \varepsilon_y = \varepsilon_{90^\circ}$

$$\gamma_{xy} = 2\varepsilon_{45^\circ} - (\varepsilon_{0^\circ} + \varepsilon_{90^\circ})$$

$$\varepsilon_x + \varepsilon_y + \gamma_{xy} = 2\varepsilon_{45^\circ}$$

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Επίπεδη εντατική κατάσταση κατάστασι

Εάν: $\theta_1 = 0^\circ, \theta_2 = 60^\circ, \theta_3 = 120^\circ$

Τότε: $\varepsilon_x = \varepsilon_{0^\circ}, \varepsilon_y = (2\varepsilon_{60^\circ} + 2\varepsilon_{120^\circ} - \varepsilon_{0^\circ})/2$

$$\gamma_{xy} = 2(\varepsilon_{60^\circ} - \varepsilon_{120^\circ})/(\sqrt{3})$$

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Επίπεδη εντατική κατάσταση κατάστασι

$$\varepsilon_{0^\circ} = -500 \cdot 10^{-6} \Rightarrow \varepsilon_x = -500 \cdot 10^{-6}$$

$$\varepsilon_{45^\circ} = 200 \cdot 10^{-6}$$

$$\varepsilon_{90^\circ} = 300 \cdot 10^{-6} \Rightarrow \varepsilon_y = 300 \cdot 10^{-6}$$

$$\gamma_{xy} = 2\varepsilon_{45^\circ} - (\varepsilon_{0^\circ} + \varepsilon_{90^\circ}) = 600 \cdot 10^{-6}$$

$$\varepsilon_{12} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \begin{cases} \varepsilon_1 = 100 \cdot 10^{-6} \\ \varepsilon_2 = -600 \cdot 10^{-6} \end{cases}$$

$$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) \quad \sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

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