

**CHAPTER 6**  
**CONFINEMENT OF CONCRETE**

**6.1 General**

Confinement is generally applied to members in compression (Fig. 6.1), with the aim of enhancing their load carrying capacity or, in cases of seismic upgrading, to increase their ductility. FRP, as opposed to steel that applies a constant confining pressure after yielding, has an elastic behavior up to failure and therefore exerts a continuously increasing confining action. The confining stresses applied by the FRP result in one or more of the following:

1. Increase of concrete compressive strength and deformability (ultimate strain).
2. Increase of chord rotation after flexural yielding of columns (that is, increase of ductility).
3. Increase of bond strength at lap-splices, hence prevention of lap-splice failures.
4. Delay of rebar buckling in compression zones with poor detailing (inadequate spacing of stirrups).

Each one of the above is briefly described in the following sections.



(a)

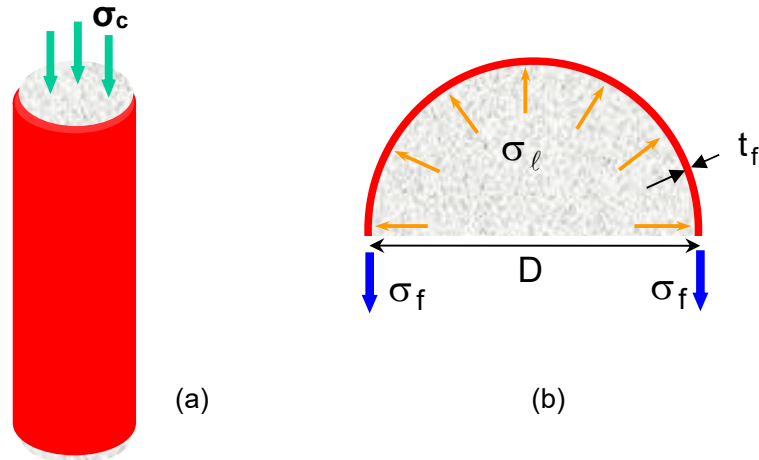
(b)

**Fig. 6.1** Confinement of columns with FRP jackets: (a) CFRP, fibers in the horizontal direction, (b) helically applied GFRP.

## 6.2 Behavior and constitutive modeling of FRP-confined concrete

### 6.2.1 Behavior

Consider a concrete cylinder (Fig. 6.2a) with diameter  $D$ , fully wrapped with an FRP jacket with thickness  $t_f$  and elastic modulus  $E_f$  (in the direction of the fibers, that is circumferentially).



**Fig. 6.2** (a) Axially loaded column. (b) Lateral stresses due to confinement.

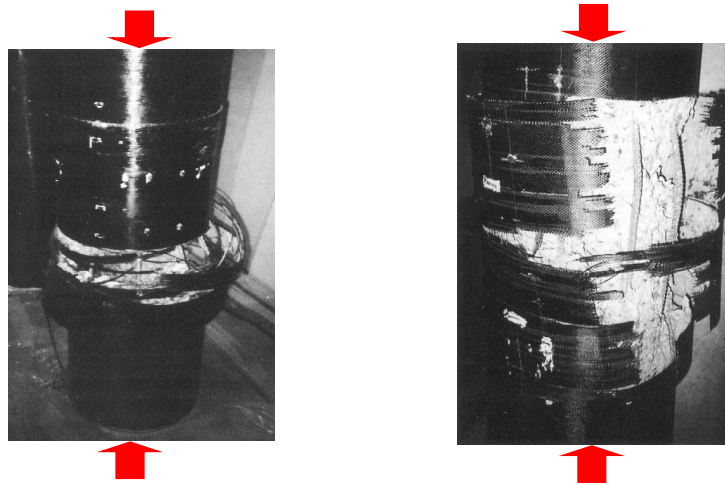
The lateral stresses  $\sigma_\ell$  (in the radial direction, due to dilation of the concrete) exerted in the jacket (equal but of opposite sign act on the concrete) are calculated as follows:

$$\sigma_\ell = \frac{2t_f}{D} \sigma_f = \frac{2t_f}{D} E_f \varepsilon_f = \frac{1}{2} \rho_f E_f \varepsilon_f \quad (6.1)$$

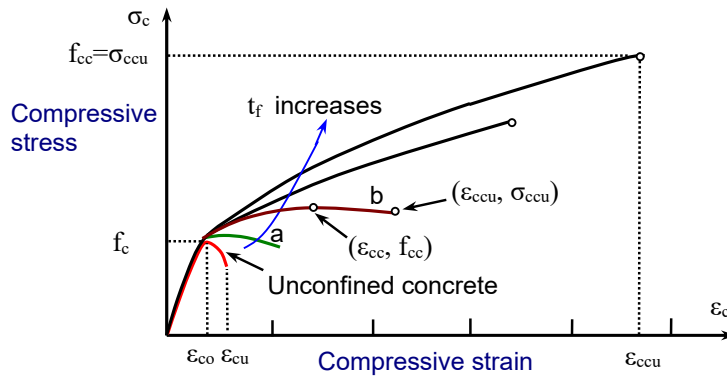
where  $\sigma_f$  and  $\varepsilon_f$  = FRP tensile stress and strain, respectively, and  $\rho_f$  = volumetric ratio of FRP. The result of confining stresses  $\sigma_\ell$  is control of lateral expansion and hence increase of deformability, until the tensile stress  $\sigma_f$  (corresponding strain  $\varepsilon_f$ ) in the FRP reaches its tensile strength  $f_{fde}$  (corresponding strain  $\varepsilon_{fue}$ ); at this point the jacket fractures (Fig. 6.3) and the member fails. Of course the mechanism described above is possible only provided that premature debonding of the FRP (at its ends) will not occur.

Let us remind here that the circumferential tensile strength of the jacket is, in general, lower than the tensile strength of FRP measured in a uniaxial tension test. This is attributed to the multiaxial state of stress in the FRP, stress concentrations, the use of many layers, the quality of application etc., and may be taken into account through the reduction factor  $\eta_e$ , with values in the of 0.6-0.9:

$$f_{fde} = \eta_e f_{fd} \tag{6.2}$$



**Fig. 6.3** Tensile fracture of FRP jacket in the circumferential direction when the tensile stress  $\sigma_f$  reaches the design FRP strength  $f_{fde}$ .



**Fig. 6.4** Compressive stress-strain curves for concrete confined with FRP.

The stress-strain relationship for concrete confined with FRP is given schematically in Fig. 6.4. On the basis of experimental support, one may draw the following conclusions:

- The stress-strain curve is approximately bilinear, with change of slope at a strain ( $\epsilon_{co} \approx 0.002$ ) corresponding to the peak stress for unconfined concrete ( $f_c$ ).
- Jackets of very low thickness increase only the ultimate strain  $\epsilon_{ccu}$  (curve a in Fig. 6.4).
- Jackets of low thickness result in confined concrete strength  $f_{cc}$  which corresponds to strain  $\epsilon_{cc}$  lower than that at ultimate ( $\epsilon_{ccu}$ ) (curve b in Fig. 6.4).

- For a given type of FRP, the strength  $f_{cc}$  and ultimate strain  $\varepsilon_{ccu}$  of confined concrete increase with the thickness of the jacket.
- For jackets of equal thickness but with different types of fibers (e.g. carbon versus glass) the confined strength  $f_{cc}$  increases with the jacket strength  $f_{fde}$  (carbon is better than glass in this case), whereas the ultimate strain  $\varepsilon_{ccu}$  increases with the jacket strength  $f_{fde}$  but also, mainly, with its ultimate strain  $\varepsilon_{fue}$  (glass is better than carbon in this case).
- For jackets of equal stiffness (expressed by the product  $E_f t_f$ ), the confined strength  $f_{cc}$  increases with the ultimate strain of FRP  $\varepsilon_{fue}$ .

### 6.2.2 Design model

As far as the design of FRP jackets for confinement is concerned, typically we aim at calculating the required thickness  $t_f$  (for a given type of FRP) for a target confined strength  $f_{ccd}$  (design value) and/or for a target ultimate strain  $\varepsilon_{ccu}$ . The international literature on FRP-concrete confinement models is vast. One of these models is described next (*fib* 2001). The model applies to columns with rectangular cross section (dimensions  $b$  and  $d$ ,  $b \geq d$ ), rounded at the corners with a radius  $R$  (*fib* 2001).

$$f_{ccd} = E_{sec,ud} \varepsilon_{ccu} \geq f_{cd} \quad (6.3)$$

$$\varepsilon_{ccu} = 0.002 \left[ 1 + 5(\alpha_{1d} \alpha_{2d} - 1) \right] \left[ \frac{E_{sec,Md} (E_c - E_{sec,ud})}{E_{sec,ud} (E_c - E_{sec,Md})} \right]^{1 - \frac{E_{sec,Md}}{E_c}} \quad (6.4)$$

$$E_{sec,ud} = \frac{E_c}{1 + 2 \left( \frac{E_c}{f_{cd}} - \frac{1}{0.002} \right) \frac{f_{fde}}{E_f}} \quad (6.5)$$

$$E_{sec,Md} = \frac{\alpha_{1d} \alpha_{2d} f_{cd}}{0.002 \left[ 1 + 5(\alpha_{1d} \alpha_{2d} - 1) \right]} \quad (6.6)$$

$$\alpha_{1d} = 2.254 \sqrt{1 + 7.94 \frac{\sigma_{lud,b}}{f_{cd}} - 2 \frac{\sigma_{lud,b}}{f_{cd}} - 1.254} \quad (6.7)$$

$$\alpha_{2d} = 1 - \left[ 0.6 \left( \frac{d}{b} \right)^2 - 1.4 \frac{d}{b} + 0.8 \right] \sqrt{\frac{\sigma_{lud,b}}{f_{cd}}} \quad (6.8)$$

$$\sigma_{lud,b} = \alpha_f \frac{2t_f}{d} f_{fde} \quad (6.9)$$

In the above expressions  $E_c$  = initial modulus of elasticity for concrete [  $E_c = 1.05 \times 9500 \times (f_{ck} + 8)^{1/3}$  ] and  $\alpha_f$  = confinement **effectiveness coefficient** for the specific jacket used, depending on: (a) the cross section geometry (aspect ratio, radius at corners, Fig. 6.5), (b) the degree of concrete coverage (Fig. 6.6a) and (c) the fiber orientation with respect to the member axis (Fig. 6.6b). Specifically:

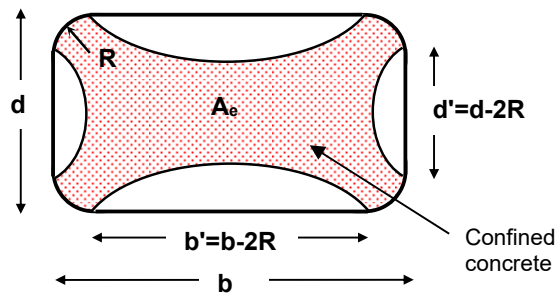
$$\alpha_f = \alpha_n \times \alpha_s \times \alpha_a \leq 1 \quad (6.10)$$

Shape coefficient: 
$$\alpha_n = \frac{A_e}{A_g} = 1 - \frac{b'^2 + d'^2}{3A_g \left(1 - \frac{A_s}{A_g}\right)} \approx 1 - \frac{(b - 2R)^2 + (d - 2R)^2}{3bd} \quad (6.11)$$

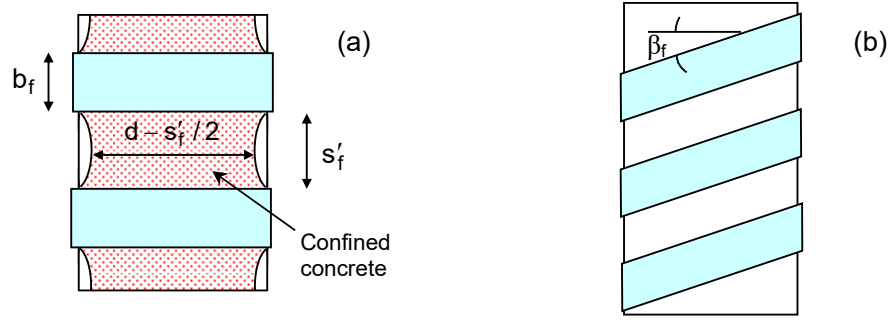
Coverage coefficient: 
$$\alpha_s = \frac{\left(1 - \frac{s'_f}{2d}\right)^2}{1 - \frac{A_s}{A_g}} \approx \left(1 - \frac{s'_f}{2d}\right)^2 \quad (6.12)$$

Fiber orientation coefficient: 
$$\alpha_a = \frac{1}{1 + (\tan \beta_f)^2} \quad (6.13)$$

where  $A_g$  = area of cross section,  $A_s$  = cross section area of longitudinal steel,  $s'_f$  = clear space between strips, for the case of partial coverage (Fig. 6.6a),  $d$  = smallest dimension of the cross section (or diameter, in the case of circular columns) and  $\beta_f$  = fiber orientation with respect to member axis (Fig. 6.6b). For circular cross sections  $\alpha_n=1$ , for fully covered members  $\alpha_s=1$  and for fibers in the direction perpendicular to the member axis  $\alpha_a=1$ .



**Fig. 6.5** Confinement of rectangular cross sections is achieved by rounding the corners.



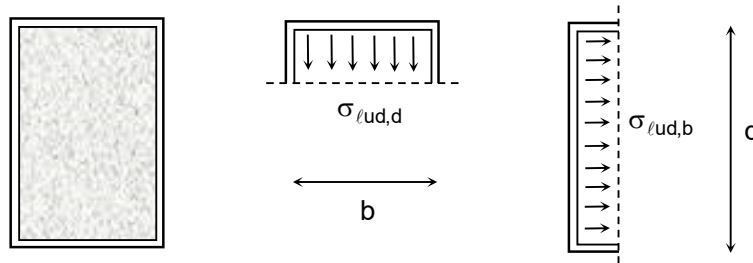
**Fig. 6.6** Confinement (a) with equally spaced strips, (b) with helically applied fibers.

Other confinement models found in the international literature are much simpler, typically in the form:

$$\frac{f_{ccd}}{f_{cd}} = 1 + k_1 \left( \frac{\sigma_{lud}}{f_{cd}} \right)^m \quad (6.14)$$

$$\varepsilon_{ccu} = \varepsilon_{cu} + k_2 \left( \frac{\sigma_{lud}}{f_{cd}} \right)^n \quad (6.15)$$

In eqs. (6.14)-(6.15)  $\sigma_{lud}$  is the mean confining stress (at failure of the jacket), approximately equal to (Fig. 6.7):



**Fig. 6.7** Mean confining stress in each direction of rectangular cross section.

$$\begin{aligned} \sigma_{lud} &= \frac{\sigma_{lud,b} + \sigma_{lud,d}}{2} = \frac{1}{2} \left( \alpha_f \frac{2t_f}{d} f_{fde} + \alpha_f \frac{2t_f}{b} f_{fde} \right) \\ &= \frac{1}{2} \alpha_f (\rho_{f,b} + \rho_{f,d}) f_{fde} = \alpha_f \frac{(b+d)}{bd} t_f f_{fde} \end{aligned} \quad (6.16)$$

where  $\sigma_{\ell_{ud,b}}$  and  $\sigma_{\ell_{ud,d}}$  are the mean confining stresses in the direction of sides b and d, respectively. In eq. (6.16)  $\rho_{f,b}$  and  $\rho_{f,d}$  is the volumetric ratio of FRP in each direction:  $\rho_{f,b} = 2t_f / d$  and  $\rho_{f,d} = 2t_f / b$ .

Typical values found in the international literature for the empirical constants in eqs. (6.14) – (6.15) are as follows:  $k_1 = 2.15$ ,  $m = 1$ ,  $k_2 = 0.02$  or  $0.04$  for carbon or glass fibers, respectively, and  $n = 1$ . Alternatively,  $k_1 = 2.6$ ,  $m = 2/3$ ,  $k_2 = 0.015$  (regardless of the type of fibers) and  $n = 0.5$ . The ultimate strain of unconfined concrete is may be taken equal to  $\epsilon_{cu} = 0.0035$ .

If the full **constitutive law** in uniaxial compression is of interest (e.g for column analysis under the combination of axial load and bending moment), the model of Lam and Teng (2003), described in Fig. 6.8, may be adopted.

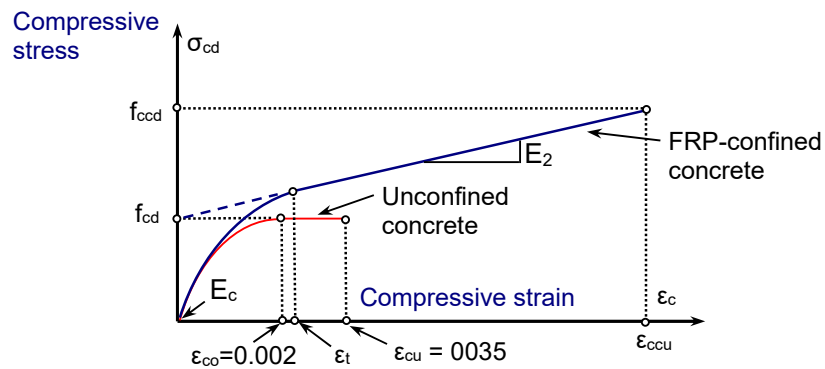
$$\sigma_{cd} = E_c \epsilon_c - \frac{(E_c - E_2)^2}{4f_{cd}} \epsilon_c^2 \quad \text{if } 0 \leq \epsilon_c \leq \epsilon_t \quad (6.17a)$$

$$\sigma_{cd} = f_{cd} + E_2 \epsilon_c \quad \text{if } \epsilon_t \leq \epsilon_c \leq \epsilon_{ccu} \quad (6.17b)$$

where

$$\epsilon_t = \frac{2f_{cd}}{(E_c - E_2)} \quad (6.18)$$

$$E_2 = \frac{f_{ccd} - f_{cd}}{\epsilon_{ccu}} \quad (6.19)$$



**Fig. 6.8** Stress-strain model for unconfined and FRP-confined concrete.

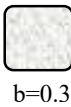
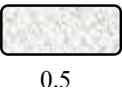

Finally, one may rely on the simpler, but not so accurate for the case of FRP-confined concrete, models described in Eurocodes 2 or 8.

**Example 6.1**

Consider a concrete column of rectangular cross section, with unconfined strength  $f_{cd} = 20 \text{ MPa}$  and elastic modulus  $E_c = 33.5 \text{ kN/mm}^2$ . The column is to be jacketed with either CFRP or GFRP, aiming at increasing the compressive strength to  $f_{ccd} = 35 \text{ N/mm}^2$  and the ultimate strain to  $\varepsilon_{ccu} = 0.025$ : (a) For CFRP we assume  $E_f = 230 \text{ kN/mm}^2$ ,  $f_{fd} = 2895 \text{ N/mm}^2$ , thickness of one layer = 0.12 mm. (b) For GFRP we take  $E_f = 70 \text{ kN/mm}^2$ ,  $f_{fd} = 1565 \text{ N/mm}^2$  and thickness of one layer 0.17 mm. Finally we assume that the tensile strength of the jacket is reduced by 15% with respect to tension testing specimens (that is  $\eta_e = 0.85$ ).

For CFRP  $f_{fde} = \eta_e f_{fd} = 0.85 \times 2895 = 2460 \text{ N/mm}^2$  and for GFRP  $f_{fde} = 0.85 \times 1565 = 1330 \text{ N/mm}^2$ . The results for the required fiber sheet thickness and the corresponding number of layers are calculated in Table 6.1, based on the analytical model of eq. (6.3) – (6.9), for three different cross sections. The results given in this table verify if the aim of confinement is to increase strength then the required CFRP is much less than GFRP, whereas the opposite is the case if the aim is to increase deformability.

**Table 6.1** Required fiber sheet thickness for various types of cross sections.

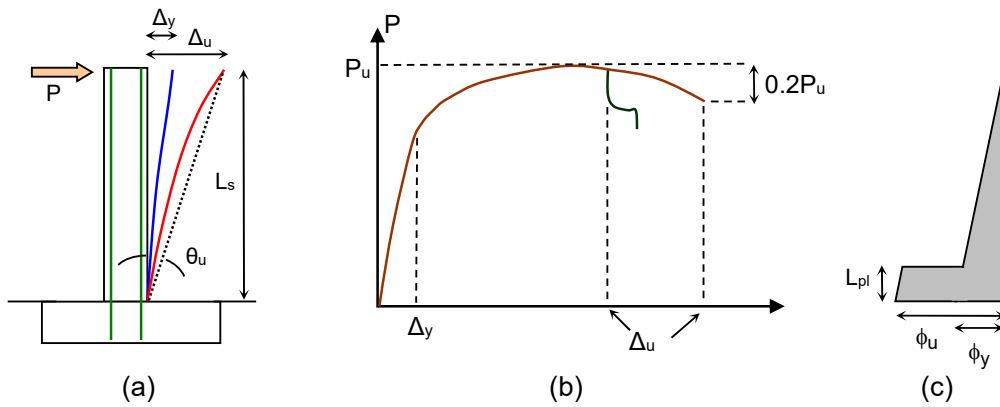
Cross section (b, d σε m)	R (cm)	$A_g$ (cm <sup>2</sup> )	$\alpha_f$ (effectiveness)	Required thickness of fiber sheet $t_f$ (mm) [in ( ) the corresponding number of layers]			
				Carbon fibers		Glass fibers	
				for $f_{ccd} = 35 \text{ N/mm}^2$	for $\varepsilon_{ccu} = 0.025$	for $f_{ccd} = 35 \text{ N/mm}^2$	for $\varepsilon_{ccu} = 0.025$
 d=0.3 b=0.3	2	896.5	0.50	<b>0.39 (4)</b>	<b>0.31 (3)</b>	<b>0.82 (7)</b>	<b>0.12 (1)</b>
 0.25 0.5	2	1246.5	0.32	<b>0.74 (7)</b>	<b>0.56 (5)</b>	<b>1.56 (13)</b>	<b>0.22 (2)</b>
 0.3 0.3	4	886.2	0.64	<b>0.31 (3)</b>	<b>0.24 (2)</b>	<b>0.64 (6)</b>	<b>0.10 (1)</b>

**6.3 Chord rotation and ductility**

According to the philosophy of the upcoming version of Eurocode 8, of utmost importance in seismic retrofitting is the increase of a member's (column) **chord rotation**



at failure  $\theta_u$  (Fig. 6.9a), which is more or less equivalent to increasing the **ductility**. The ductility may be quantified through the member chord rotation ductility factor,  $\mu_\theta = \theta_u / \theta_y$ , or through the curvature ductility factor,  $\mu_\phi = \phi_u / \phi_y$ , where:  $\theta_y$  = chord rotation at yielding,  $\phi_u$  = curvature at failure and  $\phi_y$  = curvature at yielding. Note that, essentially, the chord rotation ductility factor  $\mu_\theta$  is equal to the member (relative end) displacement ductility factor,  $\mu_\Delta = \Delta_u / \Delta_y$ , where  $\Delta_u$  and  $\Delta_y$  the relative displacement of member ends at ultimate and yielding, respectively (Fig. 6.9). In the above definitions “failure” is considered when either there is an abrupt fall in the member’s response (e.g. load – displacement curve) or the response parameter (e.g. force) has been reduced by 20% with respect to its peak (Fig. 6.9b).



**Fig. 6.9** (a) Lateral loading of RC member. (b) Load-displacement diagram. (c) Curvature.

$\theta_u$  can be calculated from the simple expression:

$$\theta_u = \theta_y + (\phi_u - \phi_y)L_{pl} \left( 1 - 0.5 \frac{L_{pl}}{L_s} \right) \quad (6.20)$$

where  $L_s$  = shear span (distance from base of column to the point where the bending moment is zero, equal to the ratio of moment to shear at the column end) and  $L_{pl}$  = plastic hinge length. The chord rotation at yielding,  $\theta_y$ , is not affected by FRP jacketing and equals:

For beams or columns:

$$\theta_y = \phi_y \frac{L_s + a_v z}{3} + 0.0013 \left( 1 + 1.5 \frac{h}{L_s} \right) + 0.13 \phi_y \frac{f_y}{\sqrt{f_c}} d_b \quad (6.21)$$

For shear walls:

$$\theta_y = \phi_y \frac{L_s + a_v z}{3} + 0.002 \left( 1 - 1.125 \frac{L_s}{h} \right) + 0.13 \phi_y \frac{f_y}{\sqrt{f_c}} d_b \quad (6.22)$$

where  $d_b$  = mean diameter of tension steel rebars,  $h$  = height of cross section,  $f_y$  = yield stress of longitudinal steel (N/mm<sup>2</sup>) and  $f_c$  = concrete strength (MPa). The above material data are taken as mean values of in-situ assessed properties, divided by a data reliability factor (1.0, 1.2, 1.35), as per Eurocode 8. The term  $a_v z$  is the tension shift of the bending moment diagram  $a_\ell$  for shear cracking at 45° and expresses the effect of tension forces shifted by  $a_\ell$  to the member's flexural deformations. The coefficient  $a_v$ , which multiplies the internal force lever arm  $z$  at the end cross section, equals 0 if the shear force at flexural yielding,  $V_{My} = M_y / L_s$ , is less than the shear cracking force  $V_{cr}$ , or 1 otherwise. Note that the shear cracking force may be taken as the shear resistance of the member without shear reinforcement,  $V_{R,c}$ , as calculated by Eurocode 2 with a safety factor  $\gamma_c = 1$ .

The plastic hinge length  $L_{pl}$  may be estimated from the following expression:

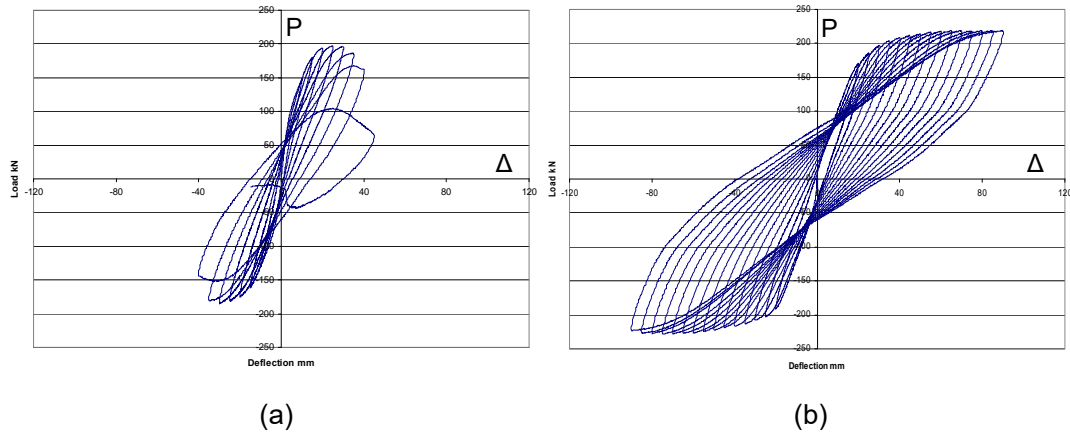
$$L_{pl} = 0.1L_s + 0.17h + \frac{0.24f_y}{\sqrt{f_c}} d_b \quad (6.23)$$

where  $f_y$  and  $f_c$  are in MPa. The curvatures  $\phi_y$  και  $\phi_u$  are calculated based on section analysis at yielding and failure.  $\phi_u$  is calculated as  $\phi_u = \epsilon_{ccu} / x_u$ , where  $x_u$  = depth of compression zone at failure and  $\epsilon_{ccu}$  = ultimate strain of concrete, as provided by the confinement model, e.g. eq. (6.15) (it is this term that is mainly affected by the properties of the FRP jacket!).

The chord rotation  $\theta_u$  (or the curvature at failure  $\phi_u$ ) can increase by jacketing the RC member at its critical regions (member ends), Fig. 6.10, where strains in concrete and steel are expected to be high. In these regions the confinement exerted by the FRP increases the ultimate strain of concrete (in addition to delaying rebar buckling and bond failure at lap-splices) and hence the ductility (Fig. 6.11).



**Fig. 6.10** FRP wrapping at member ends aiming at increased ductility.



**Fig. 6.11** Load-displacement loops for RC column of 0.25x0.50 m cross section under cyclic loading. (a) Unretrofitted member. (b) Member retrofitted with two layers of carbon sheet (thickness of each layer = 0.12 mm) at 0.60 m of the column base.

In summary, the design of FRP jackets for a given chord rotation at failure  $\theta_u$  (which is introduced in the compliance criteria for the performance levels specified in Eurocode 8) requires the expression of  $\theta_u$  in terms of the jacket properties. This is achieved through the following steps:

- Determine the plastic hinge length  $L_{pl}$  from eq. (6.23).
- Calculate the yield curvature  $\phi_y$ , based on cross section analysis.
- Calculate the chord rotation at yielding from eq. (6.21) or (6.22).
- Solve eq. (6.20) for the required jacket characteristics.

An alternative approach for relating the FRP jacket characteristics to the ultimate chord rotation (mean value) at flexural failure of *beams or columns* designed according to old provisions for seismic design is based on the use of the following empirical relationship (Eurocode 8 and KANEPE 2005):

$$\theta_{um} = 0.016(0.3^v) \left[ \frac{\max(0.01, \omega')}{\max(0.01, \omega)} f_c \right]^{0.225} \left( \frac{L_s}{h} \right)^{0.35} 25^{\left( \alpha_{psx} \frac{f_{yw}}{f_c} + \alpha_{fpx} \frac{f_{fe}}{f_c} \right)} (1.25^{100\rho_d}) \quad (6.24)$$

where:

$\omega$  = mechanical reinforcement ratio of tension longitudinal reinforcement (including any longitudinal reinforcement between the tension and compression flanges),

$\omega'$  = mechanical reinforcement ratio of compression longitudinal reinforcement,

$v = N/bhf_c$  = normalized axial force (compression taken as positive,  $b$  = width of compression zone,  $h$  = cross section side parallel to the loading direction),

$\rho_{sx} = A_{sw} / b_w s_h$  = transverse steel ratio parallel to the direction  $x$  of loading,

$\rho_{fx}$  = geometric ratio of FRP parallel to the direction  $x$  of loading,  
 $s_h$  = spacing of stirrups,  
 $f_{yw}$  = yield stress of stirrups,  
 $\rho_d$  = geometric ratio of diagonal reinforcement, if any,  
 $\alpha_f$  = effectiveness coefficient for confinement with FRP, and  
 $\alpha$  = effectiveness coefficient for confinement with stirrups, equal to

$$\alpha = \left(1 - \frac{s_h}{2b_o}\right) \left(1 - \frac{s_h}{2h_o}\right) \left(1 - \frac{\sum b_i^2}{6b_o h_o}\right) \quad (6.25)$$

In eq. (6.25)  $b_o$  and  $h_o$  are the dimensions of confined concrete core to the centerline of the stirrups and  $b_i$  is the centerline spacing of longitudinal rebars supported by stirrups. It is strongly recommended that if the stirrup ends are not bent towards the concrete core ( $\geq 135^\circ$  at corners,  $\geq 90^\circ$  on the sides), the confinement provided by stirrups should be neglected ( $\alpha = 0$ ).

The corresponding to eq. (6.24) formula for the mean value of the plastic part of the ultimate chord rotation ( $\theta_u^{pl} = \theta_u - \theta_y$ ) is:

$$\theta_{um}^{pl} = 0.0145 (0.25^v) \left[ \frac{\max(0.01, \omega')}{\max(0.01, \omega)} \right]^{0.3} (f_c)^{0.2} \left( \frac{L_s}{h} \right)^{0.35} 25^{\left( \alpha \rho_{sx} \frac{f_{yw}}{f_c} + \alpha_f \rho_{fx} \frac{f_{fe}}{f_c} \right)} \left( 1.275^{100\rho_d} \right) \quad (6.26)$$

For *shear walls* designed according to old seismic design code provisions the right part of eq. (6.24) και (6.26) should be multiplied by 0.625 and 0.6, respectively (0.016 and 0.0145 are replaced by 0.01 and 0.0087).

A careful examination of eq. (6.24) and (6.26) reveals that the contribution of FRP lies only in the exponent of 25. According to Eurocode 8, the jacket effective strength  $f_{fe}$  is determined from the following empirical formula:

$$f_{fe} = \min(f_f, \varepsilon_{fu} E_f) \left[ 1 - 0.7 \min(f_f, \varepsilon_{fu} E_f) \frac{\rho_{fx}}{f_c} \right] \quad (6.27)$$

where  $f_f$ ,  $\varepsilon_{fu}$  and  $E_f$  is the tensile strength, ultimate strain and elastic modulus of the FRP. The recommended value for  $\varepsilon_{fu}$  is 0.015 for CFRP or AFRP (however this value is too low for AFRP) and 0.020 for GFRP.

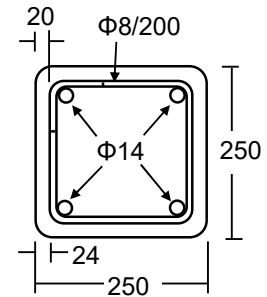
Another alternative approach to deal with the design of FRP jackets for a target ductility is to use the following simple but highly conservative equation proposed by Tastani and Pantazopoulou (2002):

$$\mu_{\Delta} = \mu_{\theta} = 1.3 + 12.4 \left( \frac{\sigma_{\ell u}}{f_c} - 0.1 \right) \geq 1.3 \quad (6.28)$$

$\sigma_{\ell u}$  in eq. (6.28) is the confining stress at the ultimate limit state, given e.g. by eq. (6.9), which neglects the contribution of stirrups. Note that the use of eq. (6.9) in rectangular columns applies with  $d$  taken as the cross section dimension perpendicular to the plane of bending. The application of this approach is illustrated in the next example.

**Example 6.2**

Consider a fixed-end column with cross section properties as shown in Fig. 6.12. The column has a height of 3.2 m and is subjected to top displacement due to seismic loading, combined with an axial load  $N = 300$  kN. Material properties:  $f_c = 28$  N/mm<sup>2</sup>, deformed steel with  $f_y = 450$  N/mm<sup>2</sup> and  $f_{yw} = 460$  N/mm<sup>2</sup>. The column ends are confined with a CFRP jacket, which has the following properties:  $E_f = 225$  kN/mm<sup>2</sup>, tensile strength  $f_f = 3500$  N/mm<sup>2</sup>, application of two layers, thickness of each layer = 0.12 mm. Calculate the (mean) ultimate chord rotation [eq. (6.24)].



**Fig. 6.12**

Based on the above data  $\rho_{fx} = 0.00192$ , hence from eq. (6.27)  $f_{fe} = 2828$  N/mm<sup>2</sup>. With  $b_o = h_o = 202$  mm,  $b_i = 180$  mm and  $s_n = 200$  mm we calculate  $\alpha = 0.12$ . Moreover,  $\alpha_f = 0.57$ ,  $\rho_{sx} = 0.002$ ,  $\omega = \omega' = 0.092$ ,  $\nu = 0.17$ ,  $L_s = 1600$  mm and  $h = 250$  mm, hence eq. (6.24) gives  $\theta_{um} = 4.3\%$ .

**Example 6.3**

Consider a column with cross section 0.30x0.40 m, subjected to strong axis bending (Fig. 6.13). The column edges are rounded at a radius  $R = 25$  mm; the concrete strength is 11 MPa; and the carbon fiber sheets to be used have an elastic modulus 230 kN/mm<sup>2</sup>, tensile strength 3500 N/mm<sup>2</sup> and thickness 0.12 mm (one layer). We assume that the FRP strength reduction coefficient is  $\eta_e = 0.90$ . The objective is to design the jacket (that is to calculate the required number of layers) for a target displacement (or chord rotation) ductility factor  $\mu_{\Delta} (= \mu_{\theta}) = 4$ , using the conservative eq. (6.28).

Tensile strength of the jacket:  $0.90 \times 3500 = 3150$  N/mm<sup>2</sup>.

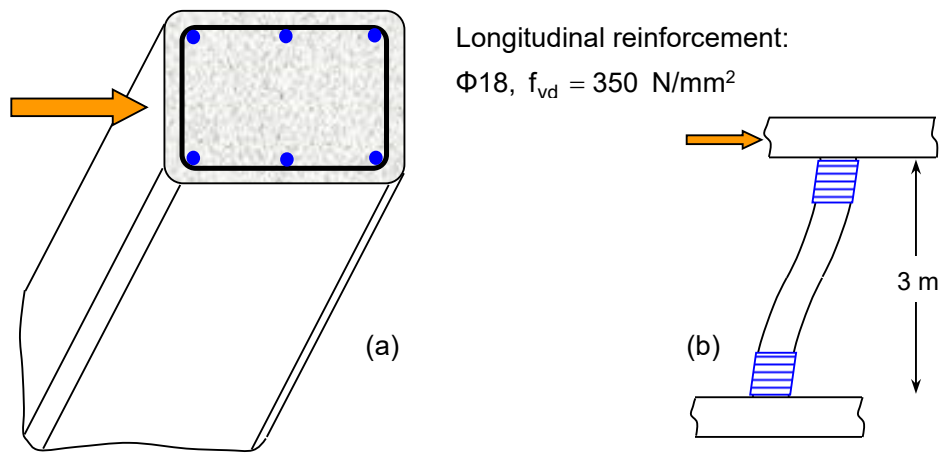
Confinement effectiveness coefficient, eq. (6.11):  $A_g = 1195$  cm<sup>2</sup>,  $A_s = 15.25$  cm<sup>2</sup>.

$$\alpha_n = 1 - \frac{35^2 + 25^2}{3 \times 1195 \times \left(1 - \frac{15.25}{1195}\right)} = 0.48$$

From eq. (6.28):

$$4 = 1.3 + 12.4 \left( \frac{0.48 \times \frac{2t_f}{300} \times 3150}{11} - 0.1 \right) \text{ hence } t_f = 0.35 \text{ mm}$$

that is  $0.35/0.12 = 2.9 \rightarrow$  3 layers (if repeated with  $t_f = 3 \times 0.12$  mm, the calculations give  $\mu_\theta = 4.15$ ).

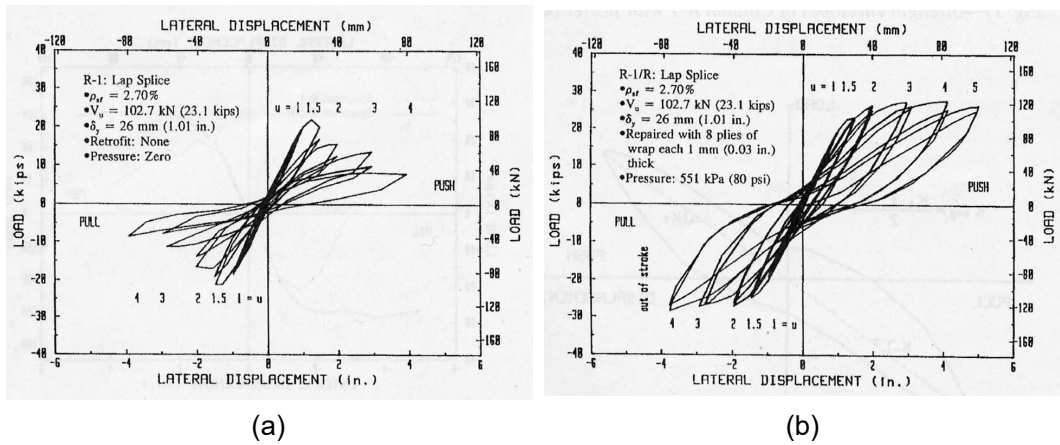


**Fig. 6.13** (a) Loading of column and (b) retrofitting for ductility.

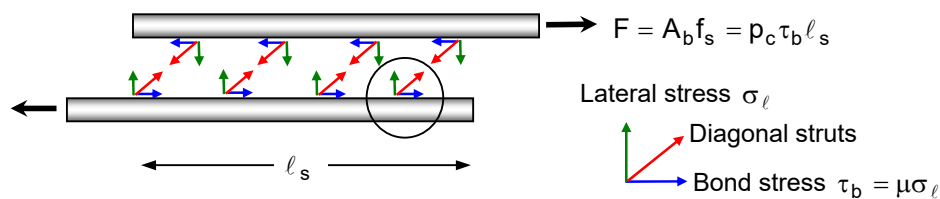
## 6.4 Lap-splices

### 6.4.1 Behavior and design

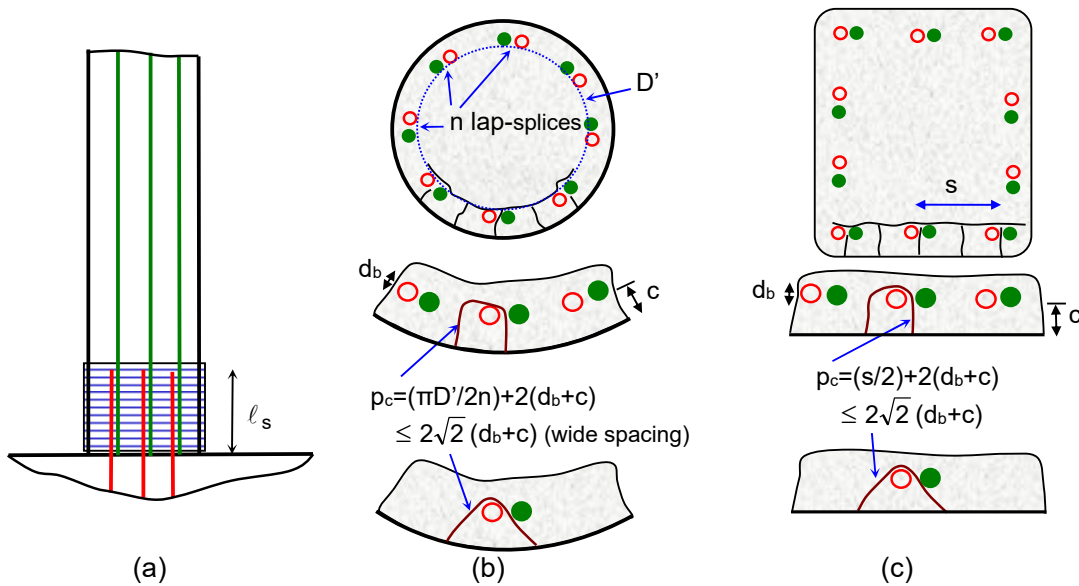
FRP jackets in regions with straight lap-spliced rebars provide confinement which increases the friction between lap-splices and prevents slippage (typically this is not of concern in lap-splices with 180° hooks, in which case slippage is not activated). The improved behavior in FRP-confined lap-spliced regions has been demonstrated in many studies, including those of Ma and Xiao (1997), Saadatmanesh et al. (1997), Seible et al. (1997), Restrepo et al. (1998), Osada et al. (1999), Chang et al. (2001), Haroun et al. (2001) etc. Typical results are shown in Fig. 6.14.



**Fig. 6.14** Cyclic loading response of column with rectangular cross section: (a) unretrofitted member, (b) member retrofitted at lap-splices (Saadatmanesh et al. 1997).



**Fig. 6.15** State of stress at lap-splice (friction mechanism).



**Fig. 6.16** (a) Column confinement at lap-splice region. (b) Cracking of circular section in the tension zone due to bond failure and definition of critical crack path. (c) Similarly for rectangular columns.

According to the friction model of Fig. 6.15, lap splice failure will be prevented provided that steel yielding will precede bond failure of the rebar. This condition is satisfied when

$$A_b f_y = p_c \mu \sigma_\ell \ell_s \quad (6.29)$$

where  $\sigma_\ell$  is the confining stress provided by the jacket at the ultimate limit state (ignoring, for simplicity, the contribution from stirrups), that is  $\sigma_\ell = \sigma_{\ell U}$ . By combining eq. (6.29) and (6.16), assuming that the required jacket thickness increases linearly as the ratio of the available lap-splice length to that required to prevent slippage decreases and introducing the safety factor to account for model uncertainties, the jacket thickness required to prevent lap-splice failure is given as follows:

$$t_f = \gamma_{Rd} \frac{bd \left( 1 - \frac{\ell_s}{\ell_{s,min}} \right) A_b f_y}{\alpha_f (b+d) p_c \mu f_{fe} \ell_s} \quad (6.30)$$

where  $A_b$  = cross section area and diameter of one spliced rebar,  $\ell_s$  = available lap-splice length,  $\ell_{s,min}$  = lap-splice length required to prevent slippage,  $p_c$  = perimeter of crack at lap-splice failure (Fig. 6.16b,c),  $f_y$  = yield stress of longitudinal rebars,  $b$  and  $d$  = dimensions of rectangular cross section,  $\mu$  = friction coefficient,  $f_{fe}$  = effective FRP jacket strength in circumferential direction and  $\gamma_{Rd}$  = safety factor. An additional condition to meet in order to prevent lap-splice failure according to Seible et al. (1997) is that the radial concrete strain should be kept below a critical value, in the order of 0.001. Hence,  $f_{fe}$  in eq. (6.28) should exceed the value

$$f_{fe} \leq 0.001 \times E_f \quad (6.31)$$

Closing this section we should point out that the effect of FRP confinement at lap-spliced rebars is favorable only for the corner rebars (in rectangular cross sections), where confining stresses are substantial due to rounding of the corners.

#### Example 6.4

Consider the column of Fig. 6.13a (0.30x0.40 m cross section) with  $\Phi 16$  rebars and  $f_y = 230$  MPa, under lateral loading which causes bending with respect to either the strong or the weak axis. We assume that the radius at column edges is  $R = 25$  mm and that the concrete cover is  $c = 30$  mm. The concrete strength is  $f_c = 11$  MPa, the friction coefficient is taken  $\mu = 1.4$ , the lap-splice length is  $\ell_s = 0.25$  m and  $\ell_{s,min} = 0.35$  m.



Assuming that confinement at the lap-splice region is provided with carbon fiber sheets with elastic modulus  $E_f = 240 \text{ kN/mm}^2$ , tensile strength  $f_{fe} = 2600 \text{ MPa}$  and thickness of one layer  $0.13 \text{ mm}$ , determine the required number of layers to prevent lap-splice failure. Take  $\gamma_{Rd} = 1.5$ .

(a) Strong axis bending

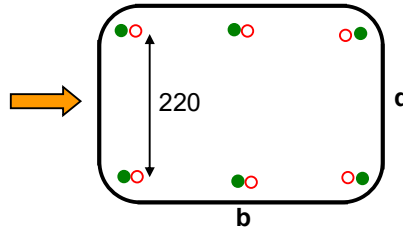


Fig. 6.17a

Critical crack path:  $p_c = \min\left\{\left[\left(\frac{220}{2}\right) + 2(16 + 30)\right], 2\sqrt{2}(16 + 30)\right\} = 130 \text{ mm}$ .

Rebar cross section area:  $A_b = (\pi \times 16^2) / 4 = 200 \text{ mm}^2$ .

$f_{fe} = \min(2600, 0.001 \times 240000) = \min(2600, 240) = 240 \text{ N/mm}^2$ .

From Example 6.3,  $\alpha_f = 0.48 \text{ mm}$ .

Required jacket thickness:  $t_f = \frac{1.5 \times 300 \times 400 \times \left(1 - \frac{0.25}{0.35}\right) \times 200 \times 230}{0.48 \times (300 + 400) \times 130 \times 1.4 \times 240 \times 250} = 0.64 \text{ mm}$ .

Required number of layers:  $0.64 / 0.13 = 4.92 \rightarrow$  5 layers.

(b) Weak axis bending

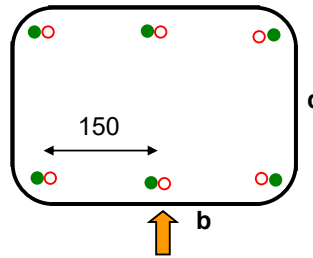


Fig. 6.17b

The calculations are as above, but note that FRP jacketing will prevent lap-splice failure only at the corner rebars.

### 6.4.2 Effect of lap-splices on chord rotation

The effect of lap-splices on chord rotation is taken into account by computing the yield chord rotation  $\theta_y$  and the plastic part of the ultimate chord rotation  $\theta_u^{pl}$  with  $\omega'$  twice as high compared to that outside the lap-splice region. The same applies for  $\phi_y$  and  $M_y$ . Moreover, if  $l_s < l_{s,min}$ , then  $\theta_u^{pl}$ ,  $\theta_u$ ,  $M_y$  and  $\phi_y$  should be computed by multiplying the yield stress of longitudinal rebars by  $l_s / l_{s,min}$ . Also, the 2<sup>nd</sup> term in eq.

(6.21) – (6.22) should be multiplied by the ratio of the reduced yield moment to that outside the lap-splice region. Finally, the right part of eq. (6.26) should be multiplied by  $\ell_s / \ell_{su,min}$ .

For lap-splices without FRP jacketing:

$$\ell_{s,min} = \frac{0.3f_y}{\sqrt{f_c}} d_b \quad (6.32)$$

$$\ell_{su,min} = \frac{f_y}{\left(1.05 + 14.5\alpha_{\ell} \rho_{sx} \frac{f_{yw}}{f_c}\right) \sqrt{f_c}} d_b \quad (6.33)$$

where

$$\alpha_{\ell} = \left(1 - \frac{s_h}{2b_o}\right) \left(1 - \frac{s_h}{2h_o}\right) \frac{n_{restr}}{n} \quad (6.34)$$

$n$  = total number of longitudinal rebars in the column perimeter and  $n_{restr}$  = number of rebars supported at corners of stirrups or by cross ties.

For lap-splices with FRP jacketing at a height at least equal to  $2\ell_s/3$ :

$$\ell_{s,min} = \frac{0.2f_y}{\sqrt{f_c}} d_b \quad (6.35)$$

$$\ell_{su,min} = \frac{f_y}{\left(1.05 + 14.5\alpha_{\ell,f} \rho_{fx} \frac{f_{fde}}{f_c}\right) \sqrt{f_c}} d_b \quad (6.36)$$

where  $\alpha_{\ell,f} = 4/n$  (because confinement is effective only in the vicinity of the four corner rebars). Note here that in order to avoid accounting for the FRP contribution twice in the correction for  $\theta_u^p$ ,  $\alpha_f$  in the power of 25 in eq. (6.26) should be taken as zero. Finally, all strength parameters in the above equations are given in N/mm<sup>2</sup>.

## 6.5 Rebar buckling

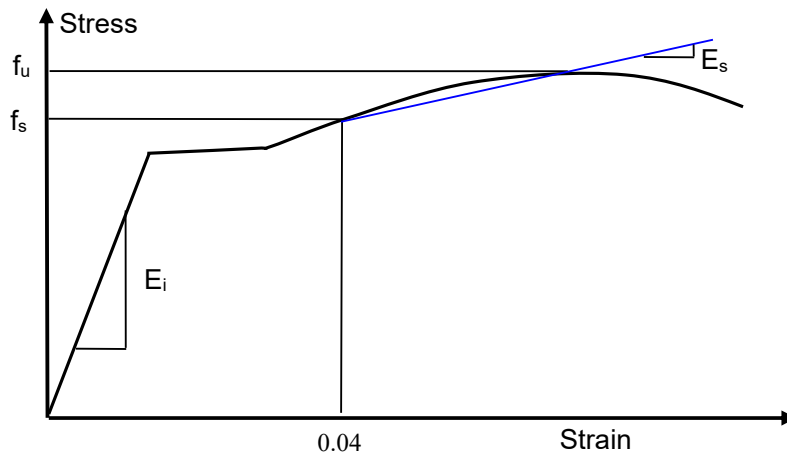
According to Priestley et al. (1996), in columns with  $M/Vd > 4$  ( $M$  and  $V$  is the maximum acting bending moment and shear force, respectively, and  $d$  is the cross section dimension parallel to the plane of bending) and the ratio of stirrup spacing to rebar diameter  $s_h/d_b$  exceeds a critical value, buckling of the longitudinal rebars is likely

to occur due to high axial strains. Such buckling may be delayed when the FRP confining jacket has a thickness equal to:

$$t_f = \frac{0.45nf_s^2d}{4E_{ds}E_f\alpha_f} \tag{6.37}$$

where  $n$  = total number of longitudinal rebars in the cross section,  $f_s$  = stress in the rebars at a strain equal to 0.04 and  $E_{ds}$  = “double” modulus of rebars, defined as follows (Fig. 6.18):

$$E_{ds} = \frac{4E_sE_i}{(\sqrt{E_s} + \sqrt{E_i})^2} \tag{6.38}$$



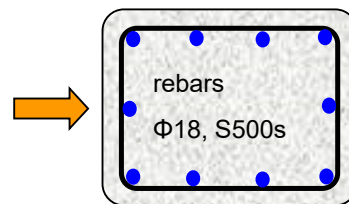
**Fig. 6.18** Definition of steel moduli.

In eq. (6.38)  $E_s$  = secant modulus from stress  $f_s$  to  $f_u$  (strength of steel) and  $E_i$  = initial modulus of rebars. Finally, in eq. (6.37) the quantity  $0.45f_s^2/E_{ds}$  may be taken approximately (and conservatively) equal to  $40 \text{ N/mm}^2$ . Hence, with the introduction of the safety factor we have:

$$t_f = \gamma_{Rd} \frac{10nd}{E_f\alpha_f} \quad (E_f \text{ in N/mm}^2) \tag{6.39}$$

**Example 6.5**

Consider a column with  $0.30 \times 0.40 \text{ m}$  cross section (Fig. 6.19) and 10 longitudinal rebars



**Fig. 6.19**

Φ18. The radius at the rounded edges is assumed  $R = 25$  mm and  $\gamma_{Rd} = 1.5$ .

For carbon fiber sheets with  $E_f = 230$  kN/mm<sup>2</sup> and thickness of one layer equal to 0.12 mm, the required sheet thickness to delay rebar buckling is:

$$t_f = \frac{1.5 \times 10 \times 10 \times 400}{230000 \times 0.48} = 0.54 \text{ mm}$$

which implies  $0.54/0.12 = 4.5 \rightarrow$  5 layers.

## 6.6 General comments on FRP-jacketed columns

It must be made clear that FRP jacketing in RC columns: (a) increases the axial load resistance, if the predominant loading is axial and (b) increases substantially the deformability (ductility, chord rotation) and/or the shear resistance, if the predominant loading is lateral (seismic forces). Contrary to the case of steel jacketing, the **stiffness is not affected** by FRP jacketing, implying that very flexible structures (e.g. buildings with pilotis) may remain vulnerable and may require stiffening in addition to strengthening, as per the structural analysis results.

Under the condition that the intervention does not aim to increase the stiffness (or the flexural resistance!), any given seismic excitation will provide (through the structural analysis) (a) the target chord rotation (or ductility) and (b) the design shear (accounting for capacity design, that is flexural yielding before shear cracking). The required thickness of FRP jackets should be determined as the maximum given by the calculations for chord rotation, shear resistance, delay or rebar buckling and prevention of lap-splice failures.