CHAPTER 5

SHEAR STRENGTHENING OF REINFORCED CONCRETE

5.1 General

Shear strengthening of RC members using FRP may be provided by bonding the external reinforcement (typically in the form of sheets) with the principal fiber direction as parallel as practically possible to that of maximum principal tensile stresses, so that the effectiveness of FRP is maximized (see Fig. 5.1 for the dependence of the FRP elastic modulus on the fiber orientation). For the most common case of structural members subjected to lateral loads, that is loads perpendicular to the member axis (e.g. beams under gravity loads or columns under seismic forces), the maximum principal stress trajectories in the shear-critical zones form an angle with the member axis which may be taken roughly equal to 45°. However, it is normally more practical to attach the external FRP reinforcement with the principal fiber direction perpendicular to the member axis (Fig. 5.2). Photographs of typical applications are shown in Fig. 5.3.

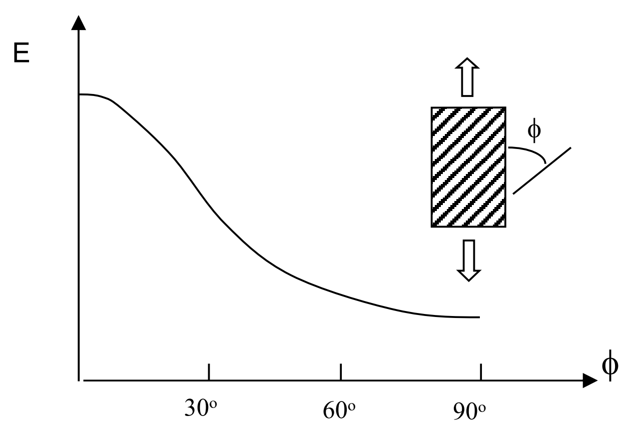


Fig. 5.1 Dependence of FRP elastic modulus on fiber orientation.

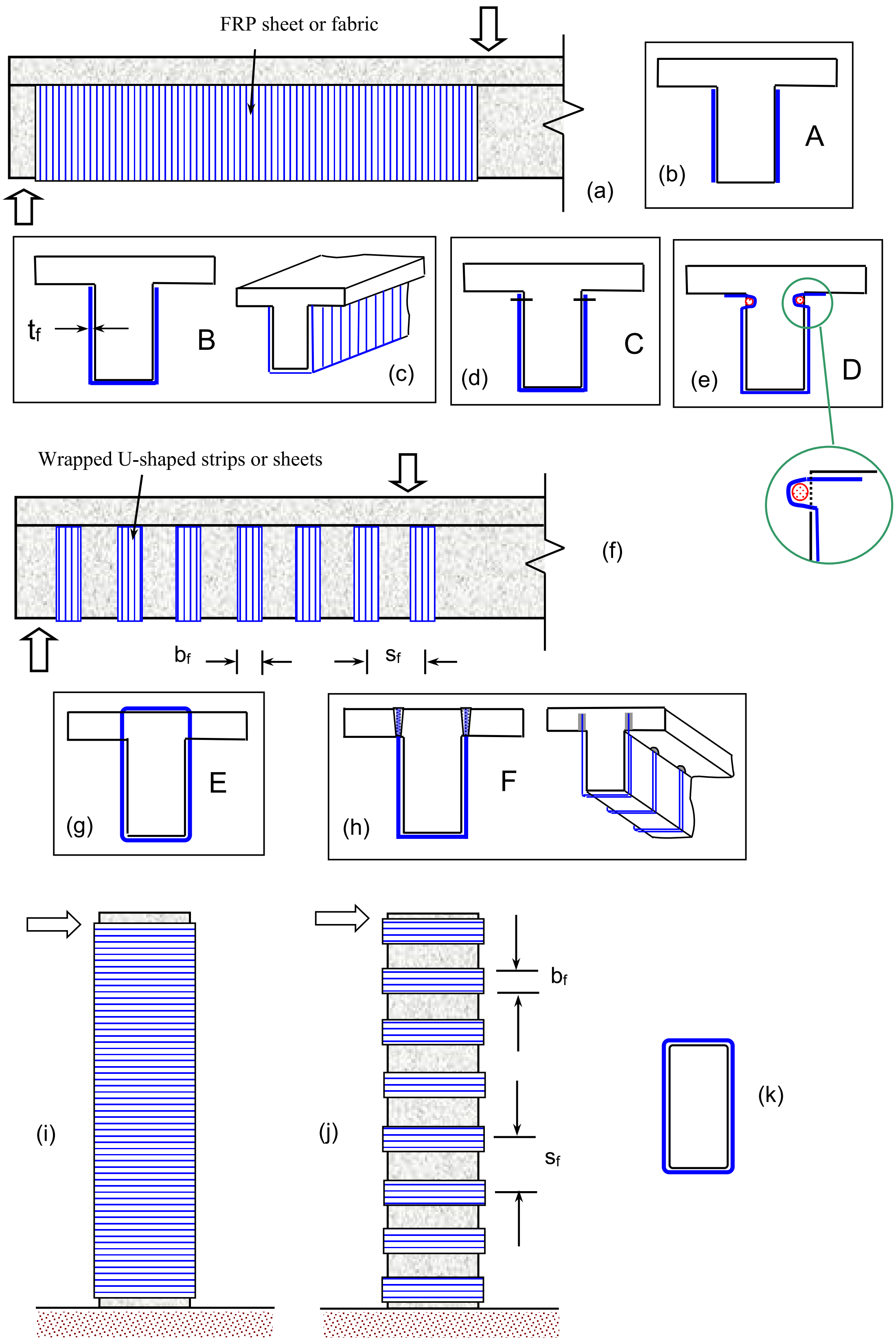


Fig. 5.2 Shear strengthening of: (a)-(h) beams, (i)-(k) columns and shear walls.

(a)

(b)

Fig. 5.3 Shear strengthening (a) of beam end with CFRP, (b) of column with GFRP.

5.2 Shear carried by FRP

At the ultimate limit state in shear, the fibers crossing a diagonal crack are activated and carry tension **in analogy to internal stirrups**, Fig. 5.4.

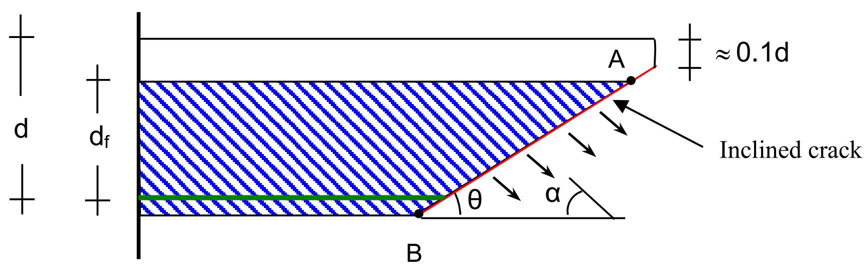


Fig. 5.4 Contribution of FRP to shear resistance (Triantafillou 1998).

If shear strengthening is achieved with strips of thickness t_f and width b_f (measured perpendicular to the axis of the strips), at a spacing s_f (parallel to the member axis), the design shear carried by the FRP, $V_{Rd,f}$, may be calculated from the following expression:

$$V_{Rd,f} = \frac{2t_f b_f}{s_f} d_f \sigma_{fed} (\cot\theta + \cot\alpha)\sin\alpha \tag{5.1}$$

where d_f = height of FRP crossed by the shear crack, measured from the longitudinal steel reinforcement (equals $0.9d$ in the case of fully wrapped members, e.g. Fig. 5.2g-k), θ = angle of diagonal crack with respect to the member axis (may be taken equal to 45^o, based on the classical Mörsch-Ritter truss analogy), α = angle between principal fiber orientation and longitudinal axis of member, σ_{fed} = design value of mean stress in the FRP crossing the shear crack, in the principal fiber direction ("effective" stress).

Note that the differences between eq. (5.1) and that for the contribution of internal stirrups to shear resistance ($V_{Rd,s}$) are: use of $2t_f b_f$ instead of A_{sw} (cross section area of stirrups), s_f instead of s_h (spacing of stirrups) and σ_{fed} instead of f_{ywd} (yield stress of stirrups).

For the most common case of continuous sheets or fabrics (instead of equally spaced strips) $b_f = s_f \sin\alpha$ and eq. (5.1) gives:

$$V_{Rd,f} = 2t_f d_f \sigma_{fed}(\cot\theta + \cot\alpha)\sin^2\alpha \tag{5.2}$$

Furthermore, for the typical case where the FRP is applied with the fibers perpendicular to the member axis ($\alpha = 90^o$), we obtain:

$$V_{Rd,f} = 2t_f d_f \sigma_{fed} \cot\theta \tag{5.3}$$

The exact calculation of the effective stress σ_{fed} is not a straightforward task. This stress depends on the type of FRP jacket (closed, three-sided, two-sided) and on the bond-slip model. Based on the model of Monti et al. (2004), Eurocode 8 gives the following equations for σ_{fed}:

Closed jackets or properly anchored in the compression zone

$$\sigma_{fed} = f_{fbd}\left[1-\left(1-\frac{2}{\pi}\right)\frac{\ell_{b,max}\sin\alpha}{2d_f}\right] + \frac{1}{2}\left[f_{fu,W}(R) - f_{fbd}\right]\left[1-\frac{\ell_{b,max}\sin\alpha}{d_f}\right] \tag{5.4a}$$

where $f_{fu,W}(R)$ = tensile strength of closed (wrapped) jacket which has been applied to a member of width b_w and radius at the corners of the cross section R:

$$f_{fu,W}(R) = f_{fbd} + \langle \eta_R f_{fd} - f_{fbd} \rangle \tag{5.5}$$

If the expression inside the $\langle\ \rangle$ brackets is negative in eq. (5.5), then the second term is taken equal to zero. The coefficient η_R equals (Campione and Miraglia 2003):

$$\eta_R = 0.2 + 1.6\frac{R}{b_w} \qquad 0 \le \frac{R}{b_w} \le 0.5 \tag{5.6}$$

U-jackets (three-sided)

$$\sigma_{fed} = f_{fbd}\left[1-\left(1-\frac{2}{\pi}\right)\frac{\ell_{b,max}\sin\alpha}{d_f}\right] \tag{5.4b}$$

//- jackets (two-sided)

$$\sigma_{fed} = f_{fbd}\frac{d_f-\ell_{b,max}\sin\alpha+\dfrac{k_bE_f}{3f_{fbd}}\sin\alpha}{d_f}\left[1-\sqrt{\frac{\left(1-\dfrac{2}{\pi}\right)k_bE_f\sin\alpha}{3f_{fbd}\left(d_f-\ell_{b,max}\sin\alpha+\dfrac{k_bE_f}{3f_{fbd}}\sin\alpha\right)}}\right]^2 \tag{5.4c}$$

Note that $\ell_{b,max}$, k_b and f_{fbd} in the above expressions are given by eq. (3.4), (3.5) and (3.6a), respectively, with $b = s_f\sin\alpha$. Moreover, if the FRP jacket covers the member's full height, d_f is taken equal to $0.9d$.

It is interesting to note that in most cases closed (fully wrapped) jackets (e.g. Fig. 5.2g, h, k) fail due to fracture of the fibers, whereas three-sided or two-sided jackets (Fig. 5.2b-e) failure is due to debonding (Fig. 5.5).

Fig. 5.5 Debonding of FRP strips used for shear strengthening.

Also note that in the case of U-shaped (three-sided) jackets the best-anchored part of the FRP is that at the maximum crack opening (Fig. 5.6a), where in the case of two-sided jackets the best-anchored part of the FRP is at the middle of the shear crack (Fig. 5.6b). This explains the reduced effectiveness of two-sided jackets, which should be avoided.

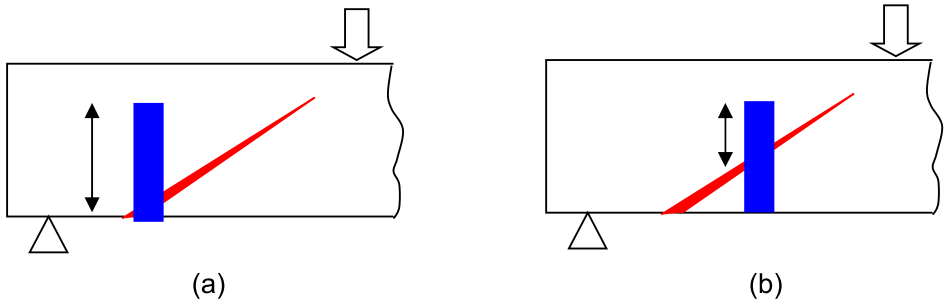


Fig. 5.6 Bond length of (a) U-shaped FRP jacket, (b) two-sided FRP.

The improved anchorage shown in Fig. 5.2e, where the FRP end is rolled around a rod and inserted into grooves is an interesting solution, which may be considered of effectiveness in between that for open (Fig. 5.2c) and closed (Fig. 5.2g-k) jackets. This case could be treated using the expressions for U-shaped jackets, with σ_{fed} [given by eq. (5.4b)] increased by approximately 30%.

Some researchers have proposed that the effective strain in the FRP be limited to a maximum value, in the order of 0.004, to maintain the integrity of concrete and secure activation of the aggregate interlock mechanism. With this limitation, σ_{fed} should not be taken higher than $0.004E_f$.

5.3 Summary of design procedure

The contribution of FRP to shear resistance is provided through the term $V_{Rd,f}$ in the well-known equation for the design shear resistance:

$$V_{Rd} = \frac{1}{\gamma_{Rd}} \min\left(V_{Rd,c} + V_{Rd,s} + V_{Rd,f}, V_{Rd,max}\right) \tag{5.7}$$

where $V_{Rd,c}$ = shear resistance of member without stirrups (“concrete contribution”), $V_{Rd,max}$ = maximum shear resistance determined from crushing of the diagonal concrete struts and γ_{Rd} = safety factor (>1) for the determination of the shear resistance in existing members (γ_{Rd}=1.20).. The FRP contribution $V_{Rd,f}$ in eq. (5.7) is given by eq. (5.1) or (5.2) for FRP in the form of strips at equal spacing or continuous jackets, respectively, with σ_{fed} calculated from eq. (5.4).

At this point we should mention that the shear resistance of RC members under cyclic (e.g. seismic) loading depends on the target ductility factor: high values of ductility result in reduced shear resistance (e.g. Moehle et al. 2001), which affects (reduces) the terms of eq. (5.7), but not the one regarding the FRP contribution ($V_{Rd,f}$). Hence the reduced shear capacity due to cycling does not affect the equations presented above.

Finally it should be noted that if shear strengthening is provided by means of equally spaced strips, the spacing should be such that the shear crack intersects at least two strips, that is $s_f \le s_{f,max} = 0.5\min(d_f, 0.9d)$ (for $\theta = 45^o$ and $\alpha = 90^o$).

Example 5.1

Consider the T-beam of Fig. 5.7, with $b = 250$ mm, height $h = 500$ mm and static depth $d = 460$ mm. The mean tensile strength of concrete is assumed $f_{ctm} = 2$ N/mm². Determine the required CFRP thickness for an additional shear resistance $V_{Rd,f} = 75$ kN. Assume the fibers perpendicular to the axis of the beam ($\alpha = 90^o$) and take $\theta = 45^o$.

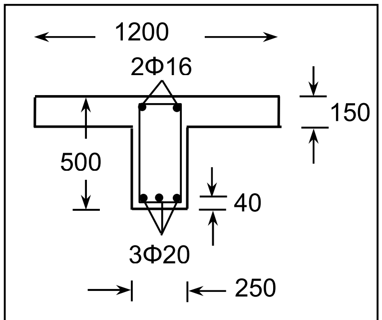


Fig. 5.7

We assume the following properties for the CFRP: thickness of one layer = 0.12 mm, elastic modulus $E_f = 230$ kN/mm², design tensile strength f_{fd} =3200 N/mm². The jacket will be applied according to the configuration shown in Fig. 5.2c (U-shaped). For a continuous jacket k_b =1.0. Moreover, d_f = 310 mm.

The problem will be solved trying different numbers of layers, in order to illustrate their relative effectiveness in carrying shear.

Jacket with <u>one layer</u>: $$\ell_{b,max} = 0.6\sqrt{\frac{230000 \times 0.12}{\sqrt{2.0 \times 1.0}}} = 84 \text{ mm}$$

$$f_{fbd} = \frac{1}{1.5}\sqrt{\frac{0.6 \times 230000 \times 2.0 \times 1.0}{0.12}} = 1010 \text{ N/mm}^2$$

$$\sigma_{fed} = 1010\left[1 - \left(1 - \frac{2}{\pi}\right)\frac{84}{310}\right] = 910 \text{ N/mm}^2$$

$$(0.004\,E_f = 0.004 \times 230000 = 920 \text{ N/mm}^2 > 910 \text{ N/mm}^2)$$

$$V_{Rd,f} = 2 \times 0.12 \times 310 \times 910 \times 10^{-3} = 67.7 \text{ kN}$$

Jacket with <u>two layers</u>: $$\ell_{b,max} = 0.6\sqrt{\frac{230000 \times 2 \times 0.12}{\sqrt{2.0 \times 1.0}}} = 100 \text{ mm}$$

$$f_{fbd} = \frac{1}{1.5}\sqrt{\frac{0.6 \times 230000 \times 2.0 \times 1.0}{2 \times 0.12}} = 715 \text{ N/mm}^2$$

$$\sigma_{fed} = 715\left[1 - \left(1 - \frac{2}{\pi}\right)\frac{100}{310}\right] = 630 \text{ N/mm}^2$$

$$(0.004\,E_f = 0.004 \times 230000 = 920 \text{ N/mm}^2 > 630 \text{ N/mm}^2)$$

$$V_{Rd,f} = 2 \times (2 \times 0.12) \times 310 \times 630 \times 10^{-3} = \underline{93.7 \text{ kN} > 75 \text{ kN}} \text{ OK}$$

Example 5.2

Consider the T-beam of Example 5.1. Design an appropriate shear strengthening system for an additional shear $V_{Rd,f}$ = 80 kN, based on carbon fiber strips at constant spacing. The strips may be assumed fully anchored in the compression zone (Fig. 5.2h).

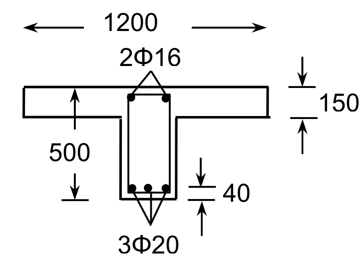


Fig. 5.8

We assume that the strips have a width b_f = 40 mm, thickness t_f = 1.4 mm, elastic modulus E_f =120 kN/mm^2 and design strength f_{fde} =1700 N/mm^2. The strips will be placed perpendicular to the beam axis at a maximum spacing of $0.5 \times 0.9 \times 460 = 207$ mm. The radius at the (bottom) corners of the cross section is taken equal to $R = 15$ mm.

$$\eta_R = 0.2 + 1.6\frac{15}{250} = 0.3 \qquad \text{and} \qquad 0 < \frac{15}{250} < 0.5$$

Assumed distance between strips: $s_f = 150$ mm.

$$k_b = \sqrt{\frac{1.5\left(2 - \frac{40}{150}\right)}{1 + \frac{40}{100}}} = 1.36 \qquad \ell_{b,max} = 0.6\sqrt{\frac{120000 \times 1.4}{\sqrt{2.0 \times 1.36}}} = 191 \text{ mm}$$

$$f_{fbd} = \frac{1}{1.5}\sqrt{\frac{0.6 \times 120000 \times 2.0 \times 1.36}{1.4}} = 250 \text{ N/mm}^2$$

$$f_{fu,W}(R) = 250 + \langle 0.3 \times 1700 - 250 \rangle = 510 \text{ N/mm}^2$$

$$\sigma_{fed} = 250\left[1 - \left(1 - \frac{2}{\pi}\right)\frac{191 \times 1.0}{2 \times 0.9 \times 460}\right] + \frac{1}{2}[510 - 250]\left[1 - \frac{191 \times 1.0}{0.9 \times 460}\right] = 299 \text{ N/mm}^2$$

$$V_{Rd,f} = \frac{2 \times 1.4 \times 40}{150} 0.9 \times 460 \times 299 \times 10^{-3} = \underline{92.4 \text{ kN} > 80 \text{ kN}} \;\; \text{OK}$$

Example 5.3

Consider a column of rectangular cross section, 250x400 mm, with a static width of 365 mm. The mean tensile strength of concrete is assumed f_{ctm} = 2 N/mm². Design a CFRP jacket with fibers perpendicular to the column axis (α =90°) for an additional shear $V_{Rd,f}$ = 100 kN corresponding to strong axis bending. Take $\theta = 45°$.

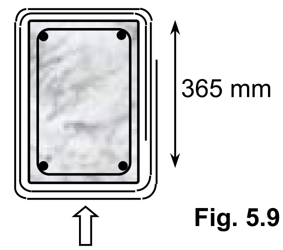


Fig. 5.9

We assume the following properties for the CFRP: thickness of one layer = 0.12 mm, elastic modulus E_f = 230 kN/mm², design tensile strength f_{fd} = 3200 N/mm². The jacket will be applied according to the configuration shown in Fig. 5.2i (full wrapping). The radius at the corners of the cross section is taken equal to $R = 15$ mm. For a continuous jacket k_b =1.0.

$$\eta_R = 0.2 + 1.6\frac{15}{250} = 0.3 \qquad \text{και} \qquad 0 < \frac{15}{250} < 0.5$$

This problem also will be solved trying different numbers of layers, in order to illustrate their relative effectiveness in carrying shear.

Jacket with <u>one layer</u>: $\ell_{b,max} = 0.6\sqrt{\dfrac{230000 \times 0.12}{\sqrt{2.0 \times 1.0}}} = 84$ mm

$$f_{fbd} = \frac{1}{1.5}\sqrt{\frac{0.6 \times 230000 \times 2.0 \times 1.0}{0.12}} = 1010 \text{ N/mm}^2$$

$$f_{fu,W}(R) = 1010 + \langle 0.3 \times 3200 - 1010 \rangle = 1010 \text{ N/mm}^2$$

$$\sigma_{fed} = 1010\left[1 - \left(1 - \frac{2}{\pi}\right)\frac{84 \times 1.0}{2 \times 0.9 \times 365}\right] + \frac{1}{2}\left[1010 - 1010\right]\left[1 - \frac{84 \times 1.0}{0.9 \times 365}\right] = 963 \text{ N/mm}^2$$

$$(0.004 E_f = 0.004 \times 230000 = 920 \text{ N/mm}^2 < 963 \text{ N/mm}^2)$$

$$V_{Rd,f} = 2 \times 0.12 \times 0.9 \times 365 \times 920 \times 10^{-3} = 72.5 \text{ kN}$$

Jacket with <u>two layers</u>: $\ell_{b,max} = 0.6\sqrt{\dfrac{230000 \times 2 \times 0.12}{\sqrt{2.0 \times 1.0}}} = 118.5$ mm

$$f_{fbd} = \frac{1}{1.5}\sqrt{\frac{0.6 \times 230000 \times 2.0 \times 1.0}{2 \times 0.12}} = 715 \text{ N/mm}^2$$

$$f_{fu,W}(R) = 715 + \langle 0.3 \times 3200 - 715 \rangle = 960 \text{ N/mm}^2$$

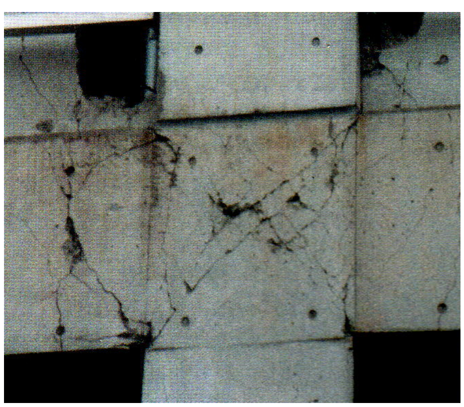
$$\sigma_{fed} = 715\left[1 - \left(1 - \frac{2}{\pi}\right)\frac{118.5 \times 1.0}{2 \times 0.9 \times 365}\right] + \frac{1}{2}[960 - 715]\left[1 - \frac{118.5 \times 1.0}{0.9 \times 365}\right] = 746.5 \text{ N/mm}^2$$

$$(0.004\,E_f = 0.004 \times 230000 = 920 \text{ N/mm}^2 > 746.5 \text{ N/mm}^2)$$

$V_{Rd,f}$ = 2 x (2 x 0.12) x 0.9 x 365 x 746.5 x 10^{-3} = <u>118 kN > 100 kN</u> OK

5.4 Beam-column joints

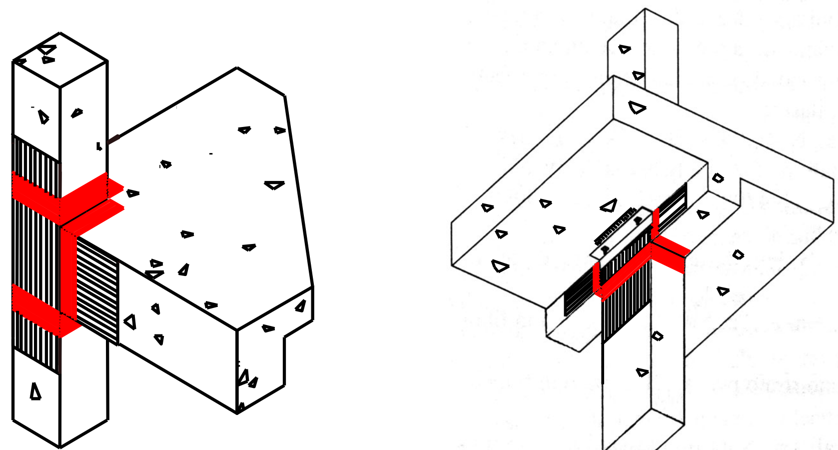
Typical shear failures of (exterior) beam-column joints are shown in Fig. 5.10. Studies on joints strengthened with FRP in shear demonstrated that even very thin FRP jackets (e.g. 2-3 layers of carbon fiber sheets with layer thickness in the order of 0.12 mm) properly anchored outside the joints can provide an increase in shear capacity by well above 80-100% (Antonopoulos 2001, Antonopoulos and Triantafillou 2002, Antonopoulos and Triantafillou 2003). This is feasible provided that the sheets will be made of fibers primarily in the beam direction, but if possible, also in the column (Fig. 5.11).



(a)

(b)

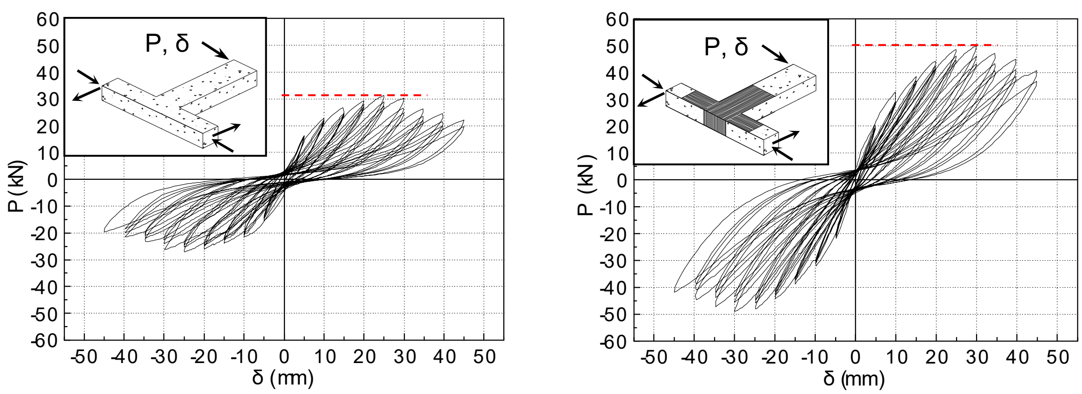
Fig. 5.10 Shear failure of exterior joints: (a) Hyogo-ken Nanbu earthquake, Japan, 1995. (b) Kalamata earthquake, Greece, 1986 (*fib* 2003).



(a) (b)

Fig. 5.11 Typical configurations for shear strengthening of beam-column joints and anchorage outside the joint. (a) Exterior joint, (b) Interior joint.

The substantial increase in shear capacity of beam-column joints is demonstrated schematically in Fig. 5.12, which gives load-displacement loops for non-strengthened as well as strengthened (with two layers of 0.12 mm thick carbon fiber sheets) joints under cyclic loading (Antonopoulos and Triantafillou 2003).



(a) (b)

Fig. 5.12 Load-displacement loops for poorly detailed (lack of stirrups) beam-column joints. (a) Non-strengthened specimen, (b) Strengthened specimen, which shows a 70% increase in shear strength.

An approximate and simple method to account for the contribution of FRP to the shear resistance of joints is to assume that the fibers in the beam direction are activated up to a strain equal to 0.004.