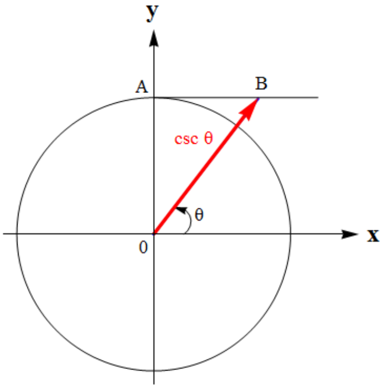
Λύση 1ης άσκησης

Να υπολογισθεί γεωμετρικά η συντέμνουσα πάνω στον μοναδιαίο τριγωνομετρικό κύκλο.

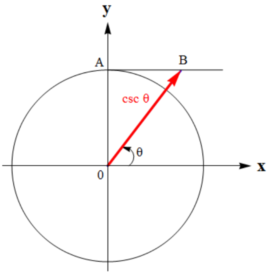


$$\theta = \widehat{OBA} \quad \text{άρα} \quad \sin\theta = \frac{OA}{OB} = \frac{1}{OB} \Longrightarrow OB = \frac{1}{\sin\theta} = \csc\theta.$$

Λύση 2ης άσκησης

Να αποδειχθεί η τριγωνομετρική ταυτότητα $\csc^2\theta = 1 + \cot^2\theta$.

1ος τρόπος:



$$OB^2 = OA^2 + AB^2 \Longrightarrow \csc^2\theta = 1 + \cot^2\theta.$$

2ος τρόπος:

$$\csc^2\theta = \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + cos^2\theta}{\sin^2\theta} = 1 + \cot^2\theta.$$

Λύση 3ης άσκησης - (α)

$$y = \sin^{1/3} x \, \csc^2 x \Rightarrow y = \frac{\sin^{1/3} x}{\sin^2 x} = \frac{1}{\sin^{5/3} x} \Rightarrow$$

$$y' = -\frac{\frac{5}{3} \sin^{2/3} x \, \cos x}{\sin^{10/3} x} = -\frac{5 \cos x}{3 \sin^{8/3} x}$$

Λύση 3ης άσκησης - (β)

$$y = \cos x \sec^3 x^2 \Rightarrow y' = (-\sin x)\sec^3 x^2 + \cos x\, 3\sec^2 x^2 (\sec x^2)' =$$
$$= -\sin x \sec^3 x^2 + 3\cos x \sec^2 x^2 \sec x^2 \tan x^2 (x^2)' =$$
$$= -\sin x \sec^3 x^2 + 6x\cos x \sec^3 x^2 \tan x^2 =$$
$$= -\sec^3 x^2 (\sin x - 6x\cos x \tan x^2)$$

Λύση 3ης άσκησης - (γ)

$$
\begin{aligned}
y &= \frac{\tan x + 1}{\sec^2 x} = \cos^2 x(1+\tan x) \Rightarrow \\
y' &= 2\cos x(-\sin x)(1+\tan x) + \cos^2 x\ \sec^2 x = \\
&= -2\sin x\ \cos x - 2\sin x \cos x\ \tan x + \cos^2 x \frac{1}{\cos^2 x} = \\
&= -2\sin x\ \cos x - 2\sin^2 x + 1 = -\sin 2x + \cos 2x
\end{aligned}
$$

Λύση 4ης άσκησης

$$\begin{aligned} y &= e^{\sec x} \Rightarrow y' = e^{\sec x}\sec x\ \tan x \quad (\text{αφού } (\sec x)' = \sec x\ \tan x) \quad \text{άρα} \\ y'' &= (e^{\sec x})' \sec x\ \tan x + e^{\sec x}(\sec x)' \tan x + e^{\sec x} \sec x (\tan x)' = \\ &= e^{\sec x}\sec^2 x\ \tan^2 x + e^{\sec x}\sec x\ \tan^2 x + e^{\sec x}\sec x\ \sec^2 x = \\ &= e^{\sec x}\sec x(\sec x\ \tan^2 x + \tan^2 x + \sec^2 x) = \\ &= e^{\sec x}\sec x(\sec x\ \tan^2 x + 2\tan^2 x + 1) \quad (\text{αφού } \sec^2 x = 1 + \tan^2 x) \end{aligned}$$

Λύση 5ης άσκησης: $y^2 \sin x + y = \arctan x$

Έμμεση παραγώγιση:

$$2yy' \sin x + y^2 \cos x + y' = \frac{1}{1+x^2} \Rightarrow$$

$$y'(2y \sin x + 1) = \frac{1}{1+x^2} - y^2 \cos x \Rightarrow$$

$$y' = \frac{1-(1+x^2)y^2 \cos x}{(1+x^2)(2y \sin x + 1)}$$

Λύση 6ης άσκησης

- $\frac{d}{dx}\left[\arcsin\sqrt{x}\right] = \frac{\frac{1}{2}x^{-1/2}}{\sqrt{1-x}} = \frac{1}{2\sqrt{x}\sqrt{1-x}} = \frac{1}{2\sqrt{x-x^2}}$.
- $\frac{d}{dx}\left[\operatorname{arcsec} e^{2x}\right] = \frac{2e^{2x}}{e^{2x}\sqrt{(e^{2x})^2-1}} = \frac{2e^{2x}}{e^{2x}\sqrt{e^{4x}-1}} = \frac{2}{\sqrt{e^{4x}-1}}$.
- $\frac{d}{dx}\left[\arcsin x + x\sqrt{1-x^2}\right] =$

$$= \frac{1}{\sqrt{1-x^2}} + \sqrt{1-x^2} + x\left(\frac{1}{2}\right)(1-x^2)^{-1/2}(-2x) =$$

$$= \frac{1}{\sqrt{1-x^2}} + \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}\left(1-x^2\right) + \sqrt{1-x^2} =$$

$$= \frac{\left(1-x^2\right)\sqrt{1-x^2}}{\sqrt{1-x^2}\sqrt{1-x^2}} + \sqrt{1-x^2} = \frac{\left(1-x^2\right)\sqrt{1-x^2}}{\left(1-x^2\right)} + \sqrt{1-x^2} =$$

$$= \sqrt{1-x^2} + \sqrt{1-x^2} = 2\sqrt{1-x^2}.$$

Λύση 7-8ης άσκησης

Λύση 7ης

$$\arctan(2x-3) = \frac{\pi}{4} \Rightarrow \tan[\arctan(2x-3)] = \tan\frac{\pi}{4} \Rightarrow$$
$$2x-3 = 1 \Rightarrow x = 2\,.$$

Λύση 8ης

$y = \arcsin x \Rightarrow x = \sin y$

Άρα

$$\cos y = \sqrt{1-x^2}$$