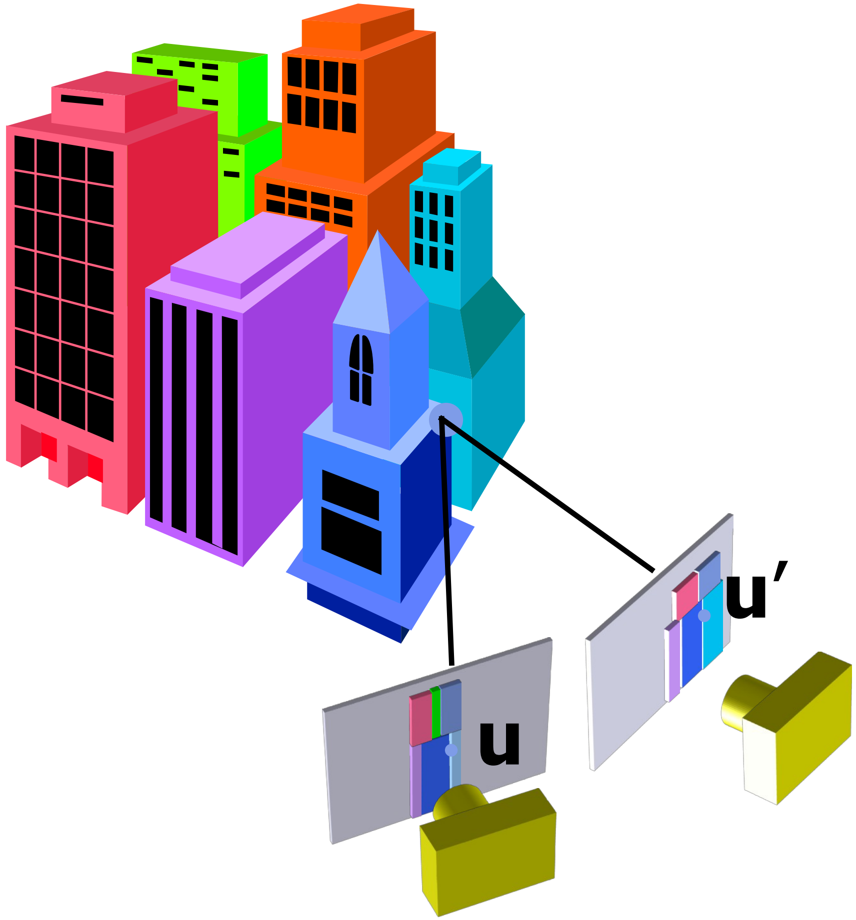
Τρισδιάστατη αναπαράσταση του χώρου απο δισδιάστατες εικόνες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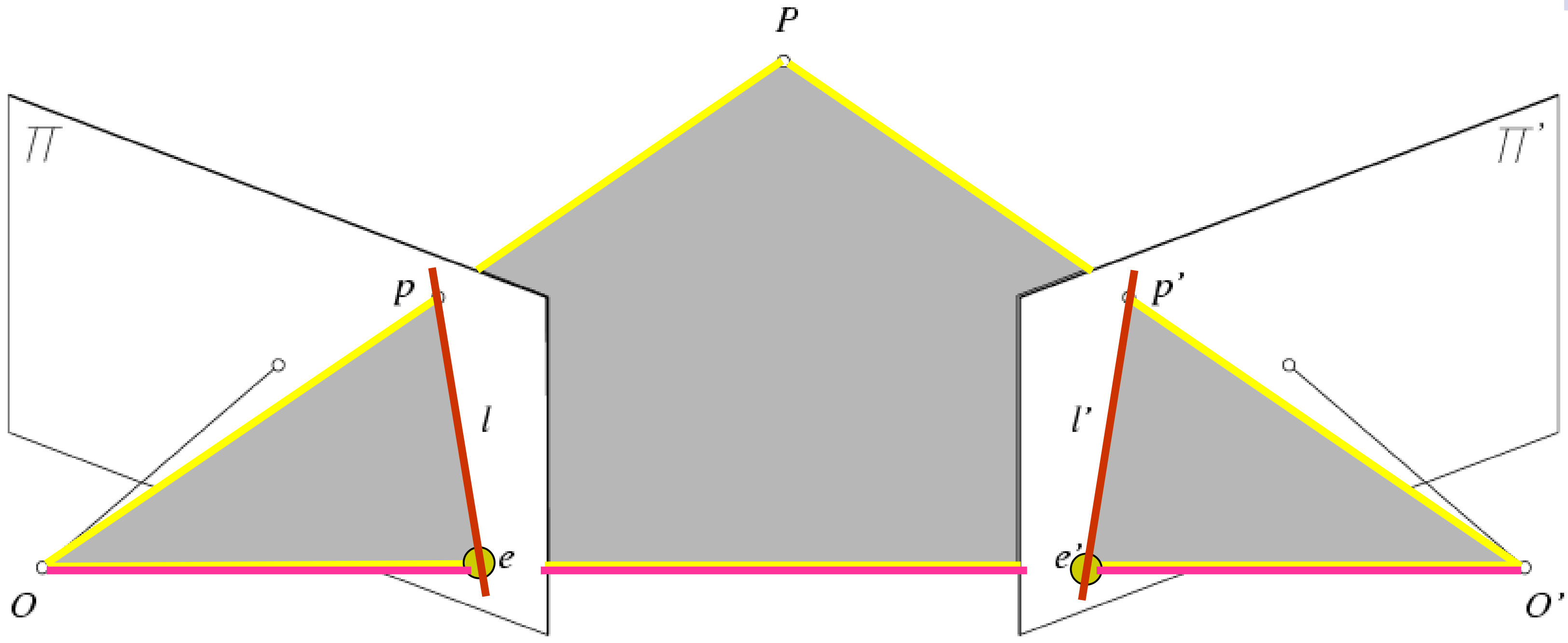
Προβολική γεωμετρία

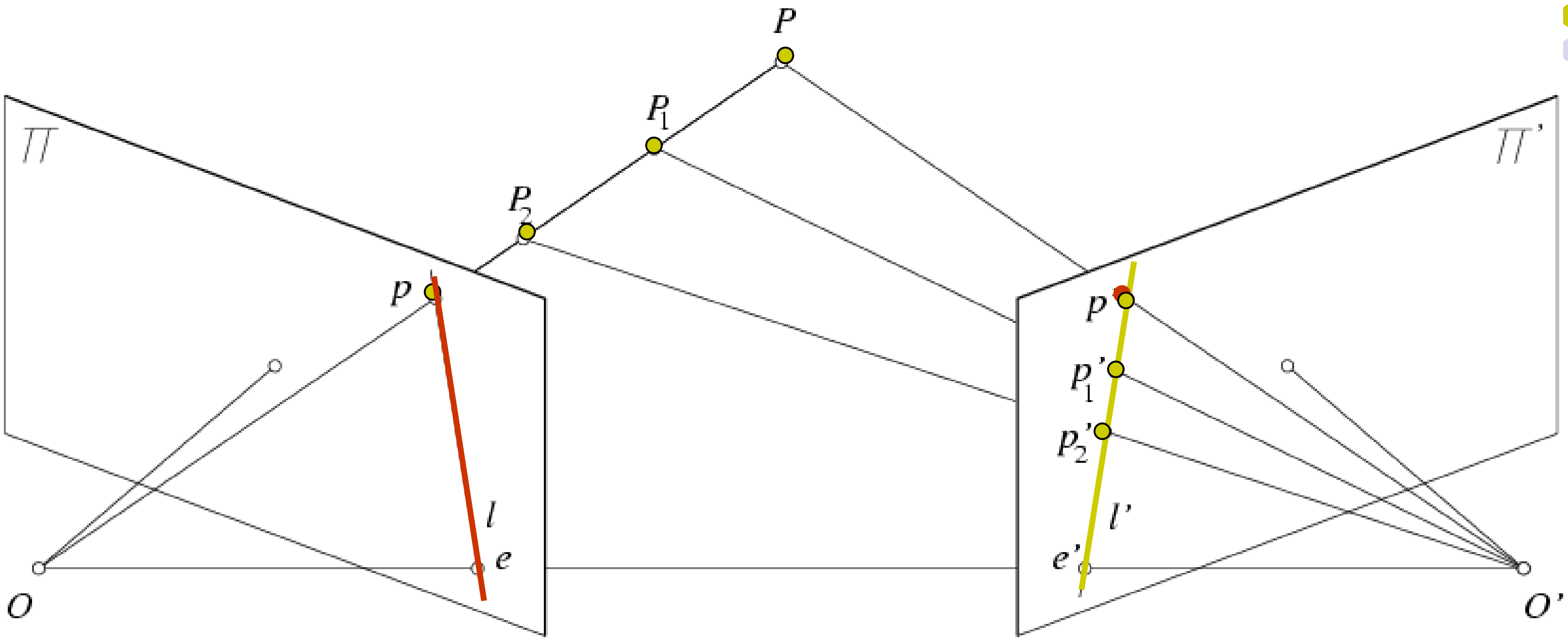
- Epipolar Geometry
  - The Essential Matrix
  - The Fundamental Matrix
  - Εκτίμηση θέσης

Επιπολική γεωμετρία



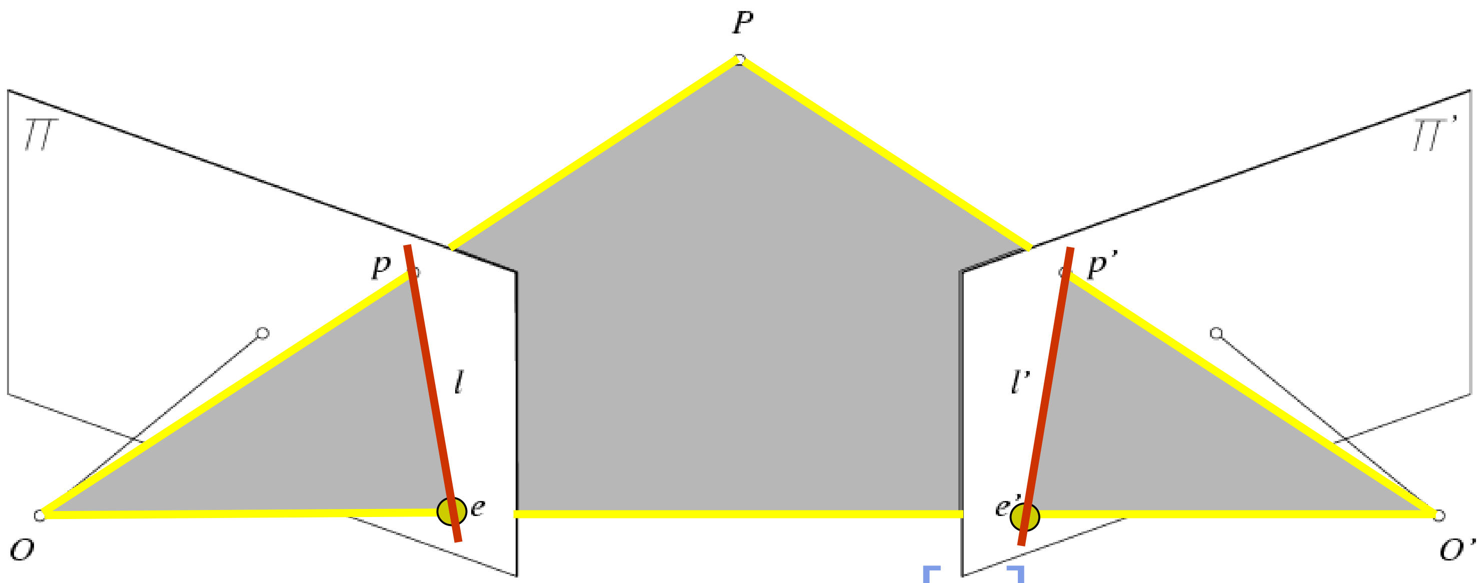
- Epipolar Plane - OPO'
- Baseline OO'
- Epipoles – e, e'
- Epipolar Lines – pe & p'e'

Επιπολικές συνθήκες



- Τα σημεία P,…P_1 που προβάλλονται στο P (Π), στην (Π') ανήκουν στην επιπολική γραμμή.

Epipolar Constraint: Calibrated Case



$$t \times p = [t_\times] p$$

$$\overrightarrow{Op} \cdot [\overrightarrow{OO'} \times \overrightarrow{O'p'}] = 0 \Rightarrow p^T (t \times R p') = 0$$

$$p^T (t \times R p') = p^T [t_\times] R p' = 0$$

$$\boldsymbol{p}^T \mathcal{E} \boldsymbol{p}' = 0 \quad \text{with} \quad \mathcal{E} = [\boldsymbol{t}_\times] \mathcal{R}$$

Essential Matrix
(Longuet-Higgins, 1981)

Properties of the Essential Matrix

$$\boldsymbol{p}^T \mathcal{E} \boldsymbol{p}' = 0 \quad \text{with} \quad \mathcal{E} = [\boldsymbol{t}_\times]\mathcal{R}$$

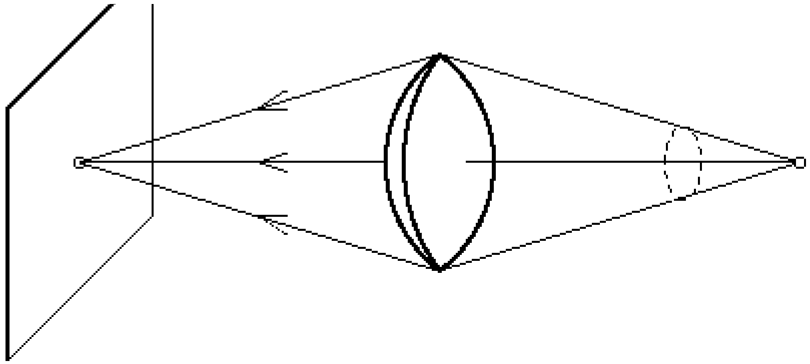
- $\mathcal{E}$ is singular.

•In fact, there are only 5 degrees of freedom in E,
    3 for rotation
    2 for translation (up to scale), determined by epipole

$$t \times p = [t]_x\, p$$

$$\begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} t_y z - t_z y \\ t_z x - t_x z \\ t_x y - t_y x \end{pmatrix} = \begin{pmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Camera Internal Parameters or Calibration matrix



- Background

The lens optical axis does not coincide with the sensor

We model this using a 3x3 matrix the ***Calibration matrix***

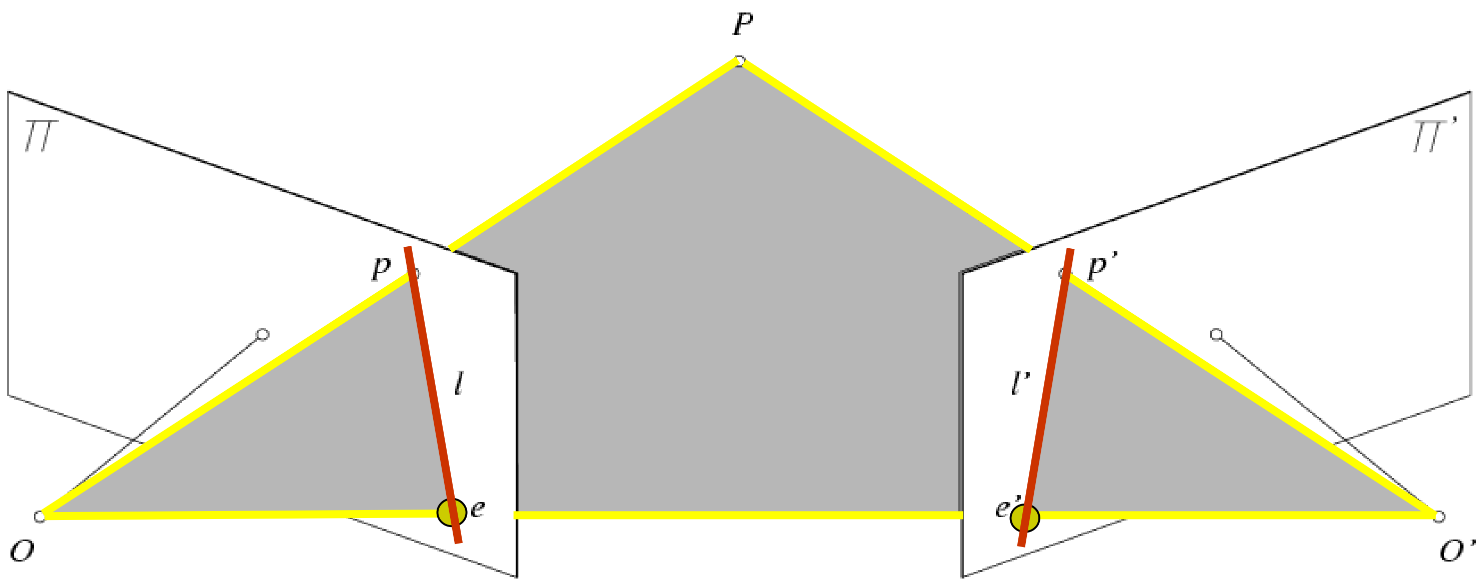
Camera Calibration matrix

- The difference between ideal sensor ant the real one is modeled by a 3x3 matrix K:

$$K = \begin{pmatrix} a_x & b & c_x \\ 0 & a_y & c_y \\ 0 & 0 & 1 \end{pmatrix}$$

- (c_x, c_y) camera center, (a_x, a_y) pixel dimensions, b skew
- We end with $q = K\bar{p}$

Epipolar Constraint: Uncalibrated Case



$$\hat{\boldsymbol{p}}^T \mathcal{E} \hat{\boldsymbol{p}}' = 0$$

$$\boldsymbol{p} = \mathcal{K} \hat{\boldsymbol{p}}$$

$$\boldsymbol{p}' = \mathcal{K}' \hat{\boldsymbol{p}}'$$

$$\boldsymbol{p}^T \mathcal{F} \boldsymbol{p}' = 0 \quad \text{with} \quad \mathcal{F} = \mathcal{K}^{-T} \mathcal{E} \mathcal{K}'^{-1}$$

Fundamental Matrix
(Faugeras and Luong, 1992)

The Eight-Point Algorithm (Longuet-Higgins, 1981)

$$(u, v, 1)\begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix}\begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0$$

$$(uu', uv', u, vu', vv', v, u', v', 1)\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

$$\begin{pmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' \\ u_2u_2' & u_2v_2' & u_2 & v_2u_2' & v_2v_2' & v_2 & u_2' & v_2' \\ u_3u_3' & u_3v_3' & u_3 & v_3u_3' & v_3v_3' & v_3 & u_3' & v_3' \\ u_4u_4' & u_4v_4' & u_4 & v_4u_4' & v_4v_4' & v_4 & u_4' & v_4' \\ u_5u_5' & u_5v_5' & u_5 & v_5u_5' & v_5v_5' & v_5 & u_5' & v_5' \\ u_6u_6' & u_6v_6' & u_6 & v_6u_6' & v_6v_6' & v_6 & u_6' & v_6' \\ u_7u_7' & u_7v_7' & u_7 & v_7u_7' & v_7v_7' & v_7 & u_7' & v_7' \\ u_8u_8' & u_8v_8' & u_8 & v_8u_8' & v_8v_8' & v_8 & u_8' & v_8' \end{pmatrix}\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = -\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\sum_{i=1}^{n} (\boldsymbol{p}_i^T \mathcal{F} \boldsymbol{p}_i')^2$$

under the constraint

$|\mathcal{F}|^2 = 1.$

Non-Linear Least-Squares Approach (Luong et al., 1993)

Minimize

$$\sum_{i=1}^{n} [\mathrm{d}^2(\boldsymbol{p}_i, \mathcal{F}\boldsymbol{p}'_i) + \mathrm{d}^2(\boldsymbol{p}'_i, \mathcal{F}^T\boldsymbol{p}_i)]$$

with respect to the coefficients of $\mathcal{F}$, using an appropriate rank-2 parameterization.

Εκτίμηση του R και t από τον E

An SVD of E gives $E = U\Sigma V^T$

$$\boldsymbol{\Sigma} = \begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

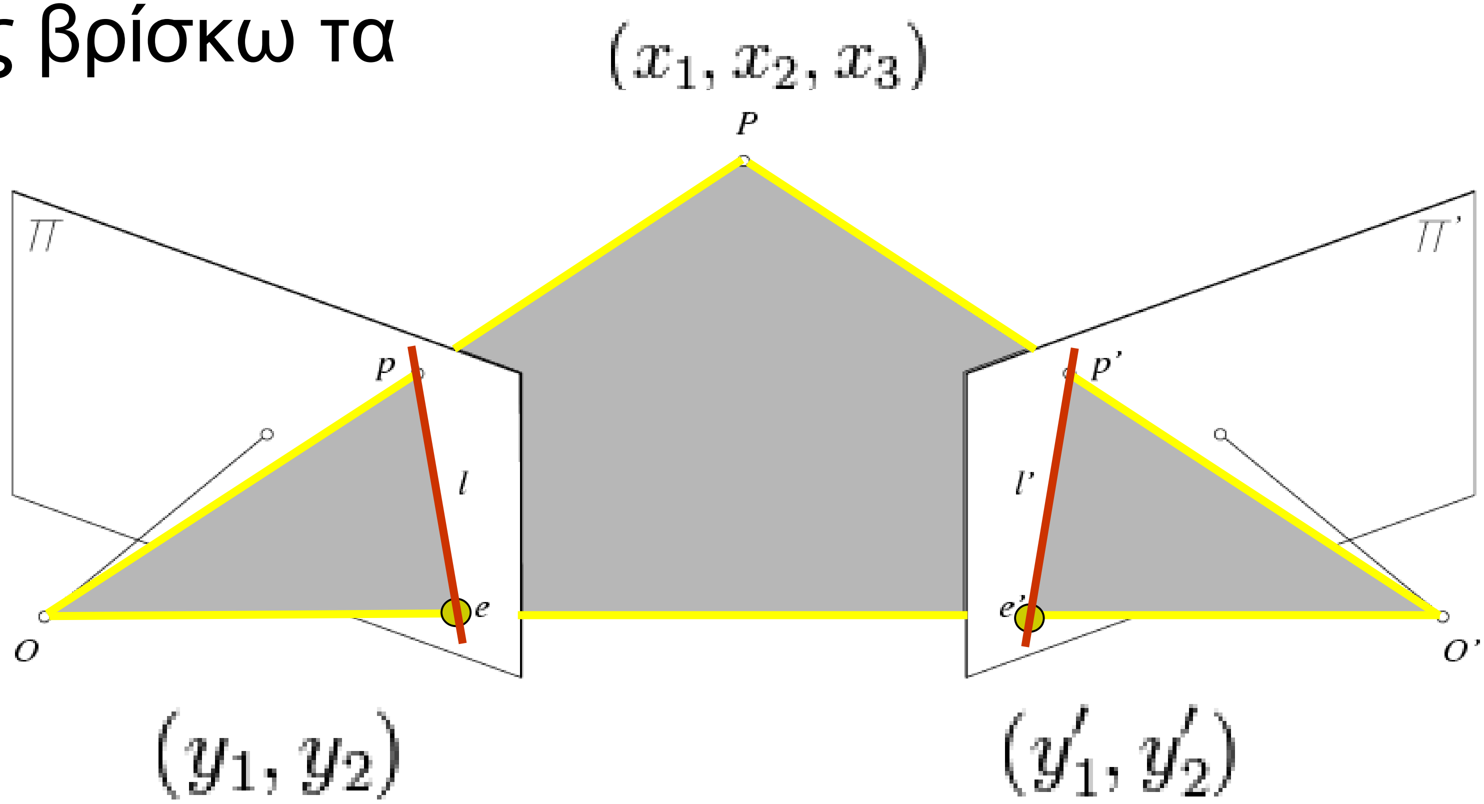
$$\mathbf{W} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$E = U\ \Sigma\ V^T = U\ W^{-1}\ V^T\ V\ W\ \Sigma\ V^T = R\ [tx]$

$R = U\ W^{-1}\ V^T$ & $[tx] = V\ W\ \Sigma\ V^T$

Εκτίμηση θεσης

- Πως βρίσκω τα



Σχέση ανάμεσα σε 3D και 2D

Two normalized cameras project the 3D world onto their respective image planes. Let the 3D coordinates of a point **P** be (x_1, x_2, x_3) and (x'_1, x'_2, x'_3) relative to each camera's coordinate system.

Since the cameras are normalized, the corresponding image coordinates are

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \frac{1}{x_3} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad \begin{pmatrix} y'_1 \\ y'_2 \end{pmatrix} = \frac{1}{x'_3} \begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix}$$

A homogeneous representation of the two image coordinates is then given by

$$\begin{pmatrix} y_1 \\ y_2 \\ 1 \end{pmatrix} = \frac{1}{x_3} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad \begin{pmatrix} y'_1 \\ y'_2 \\ 1 \end{pmatrix} = \frac{1}{x'_3} \begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix}$$

which also can be written more compactly as

$$\mathbf{y} = \frac{1}{x_3} \tilde{\mathbf{x}} \qquad \mathbf{y}' = \frac{1}{x'_3} \tilde{\mathbf{x}}'$$

Τελικοί υπολογισμοί

$$y_1' = \frac{x_1'}{x_3'} = \frac{\mathbf{r}_1\left(\tilde{\mathbf{x}} - \mathbf{t}\right)}{\mathbf{r}_3\left(\tilde{\mathbf{x}} - \mathbf{t}\right)} = \frac{\mathbf{r}_1\left(\mathbf{y} - \mathbf{t}/x_3\right)}{\mathbf{r}_3\left(\mathbf{y} - \mathbf{t}/x_3\right)} \qquad \mathbf{R} = \begin{pmatrix} -\mathbf{r}_1- \\ -\mathbf{r}_2- \\ -\mathbf{r}_3- \end{pmatrix}$$

$$x_3 = \frac{\left(\mathbf{r}_1 - y_1'\,\mathbf{r}_3\right)\mathbf{t}}{\left(\mathbf{r}_1 - y_1'\,\mathbf{r}_3\right)\mathbf{y}} \qquad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_3 \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$