1   A Logical Problem

A prison warden has three prisoners summoned and announces to them the following:

" I have here five discs differing only in color: three white and rwo black. Without letting you know which I have chosen,I shall fasten one of them to each of you between his shoulders; outside, that is, your direct visual field.
At that point, you will be left at your leisure to consider your companions and their respective discs. The first to be able to deduce his own color will be the one to benefit. His conclusion, moreover, must be founded upon logical *and not simply probabilistic* reasons. "

How can the subiects solve the problem?

The Classical Solution

Case 1   Two blacks one white: A is white, **B, C are black**

Step 1.1   A *announces 'I know my color'*   *Reasoning*: _____

Step 1.2   B, C *announce 'I know my color'*   *Reasoning*: _____

Case 2   Two whites one black: **A is black**, B, C are white

Step 2.1   A, B, C *announce 'I do not know my color'*

Step 2.2   B, C *announce 'I know my color'*

Reasoning of B: If I were black, we would be in Case 1, therefore C would have announced *'I know my color'* at Step 2.1.

Reasoning of C: _____

Step 2.3 A *announces 'I know my color'*

Reasoning of A: If I were either white or black, my announcement at Step 2.1 would have been *'I do not know my color'* ; and the same holds for B and C. Therefore, if we are all white we would all have, after Step 2.1, the same information we had at the beginning. So B, C could not have announced *'I know my color'* at Step 2.2.

Alternative reasoning of A:
After Step 2.1 everybody knows that at most one is black (by Case 1).
Since B, C announce *'I know my color'* at Step 2.2. they must have used that information to reach their conclusion; if I were white no conclusion would be reached, therefore I must be black.

Case 3   Three whites: **A, B, C are white**

Step 3.1   A, B, C *announce 'I do not know my color'*

Step 3.2   A, B, C *announce 'I do not know my color'*

Step 3.3   A, B, C *announce 'I know my color'*

Reasoning of A: If I were black, we would be in Case 2 so B, C
could not have announced *'I do not know my color'* at Step 2.2.

Reasoning of B: _____   *Reasoning of C*: _____

Σχετικό εκπαιδευτικό υλικό

Fagin – Halpern – Moses – Vardi   *Reasoning About Knowledge*
      Ενότητα 1.1. The Muddy Children Puzzle

Προτεινόμενες ασκήσεις

1 Συμπληρώστε τον συλλογισμό των B, C στο Step 1.2.

2 α Εξηγείστε γιατί οι παίκτες δεν είναι σε θέση να βρούν το χρώμα τους μετά το Step 3.1.

β Εξηγείστε για ποιό λόγο οι παίκτες είναι σε θέση να βρούν το χρώμα τους μετά το Step 3.2, παρόλο που η πληροφορία που πήραν σε αυτό το βήμα (A, B, C do not know their color) μπορεί να θεωρηθεί ως ήδη γνωστή, μετά το Step 3.1.

3 *Reasoning About Knowledge*   Exercise 1.3.

4 Εξετάστε το Muddy Children Puzzle όταν k = 2 (δύο λερωμένα παιδάκια).
Πόσες επαναλήψεις της ερώτησης του πατέρα χρειάζονται για να καταλάβουν όλα τα παιδάκια άν είναι λερωμένα; Ποιούς συλλογισμούς κάνουν;