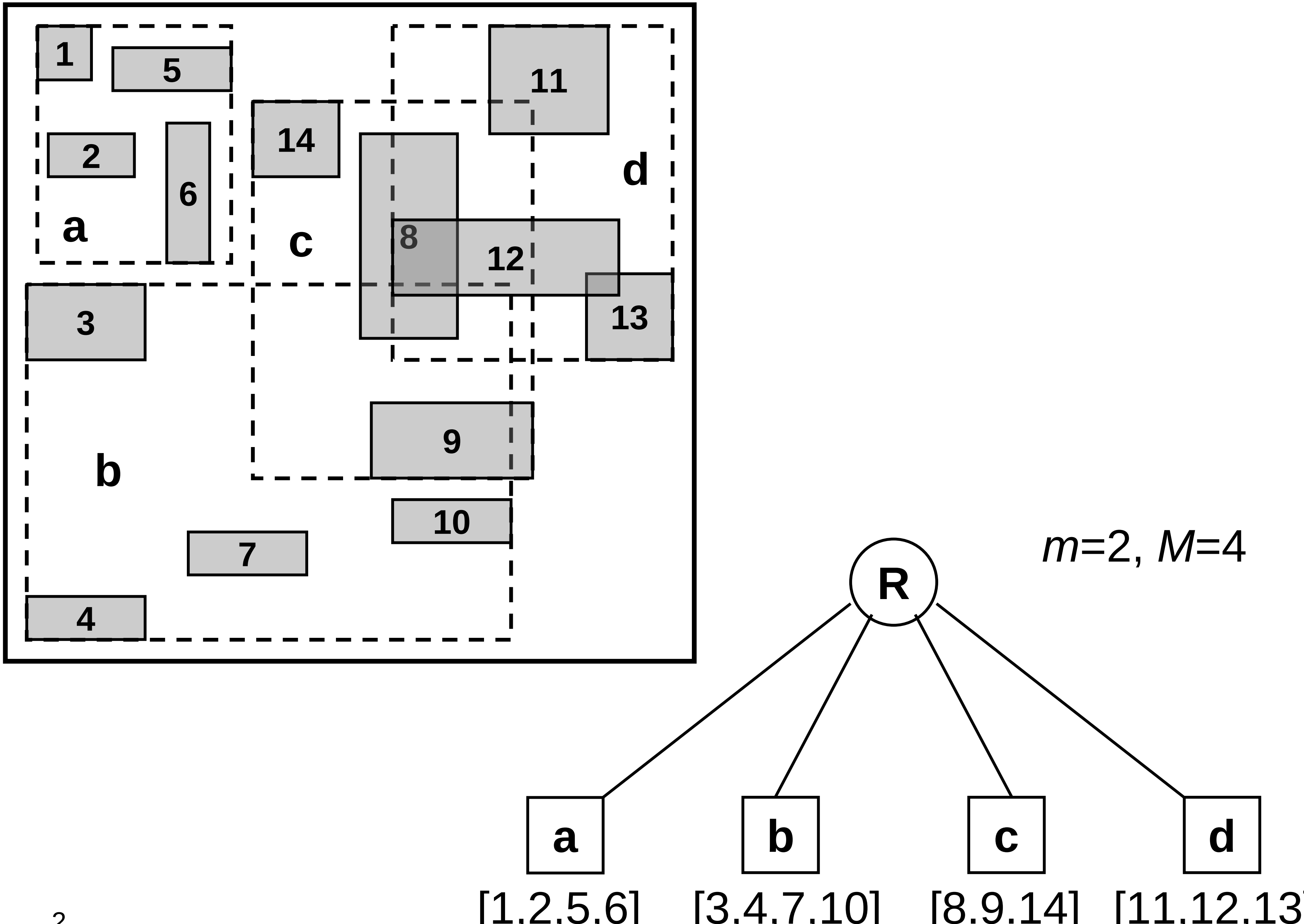
R-Tree

- An R-tree is a depth-balanced tree
  - Each node corresponds to a disk page
  - Leaf node: an array of leaf entries
    - A leaf entry: (mbb, oid)
  - Non-leaf node: an array of node entries
    - A node entry: (dr, nodeid)


1
5
11
14
2
6
d
a
c
8
12
13
3
9
b
10
7
4
m=2, M=4
R
a
b
c
d
[1,2,5,6]
[3,4,7,10]
[8.9.14]
[11,12,13]

Properties

- The number of entries of a node (except for the root) in the tree is between *m* and *M* where $m \in [0, M/2]$
  - *M*: the maximum number of entries in a node, may differ for leaf and non-leaf nodes $M = \lfloor size(P)/size(E) \rfloor$ *P*: disk page *E*: entry
  - The root has at least 2 entries unless it is a leaf
- All leaf nodes are at the same level
- An R-tree of depth *d* indexes at least m^{d+1} objects and at most M^{d+1} objects, in other words, $\lfloor \log_M N - 1 \rfloor \le d \le \lfloor \log_m N - 1 \rfloor$

Search with R-tree

- Given a point q, find all mbbs containing q
- A recursive process starting from the root

  $result = \emptyset$

  For a node N

  if N is a leaf node, then $result = result \cup \{N\}$

  else // N is a non-leaf node

  for each child N' of N

  if the rectangle of N' contains q

  then recursively search N'

Time complexity of search

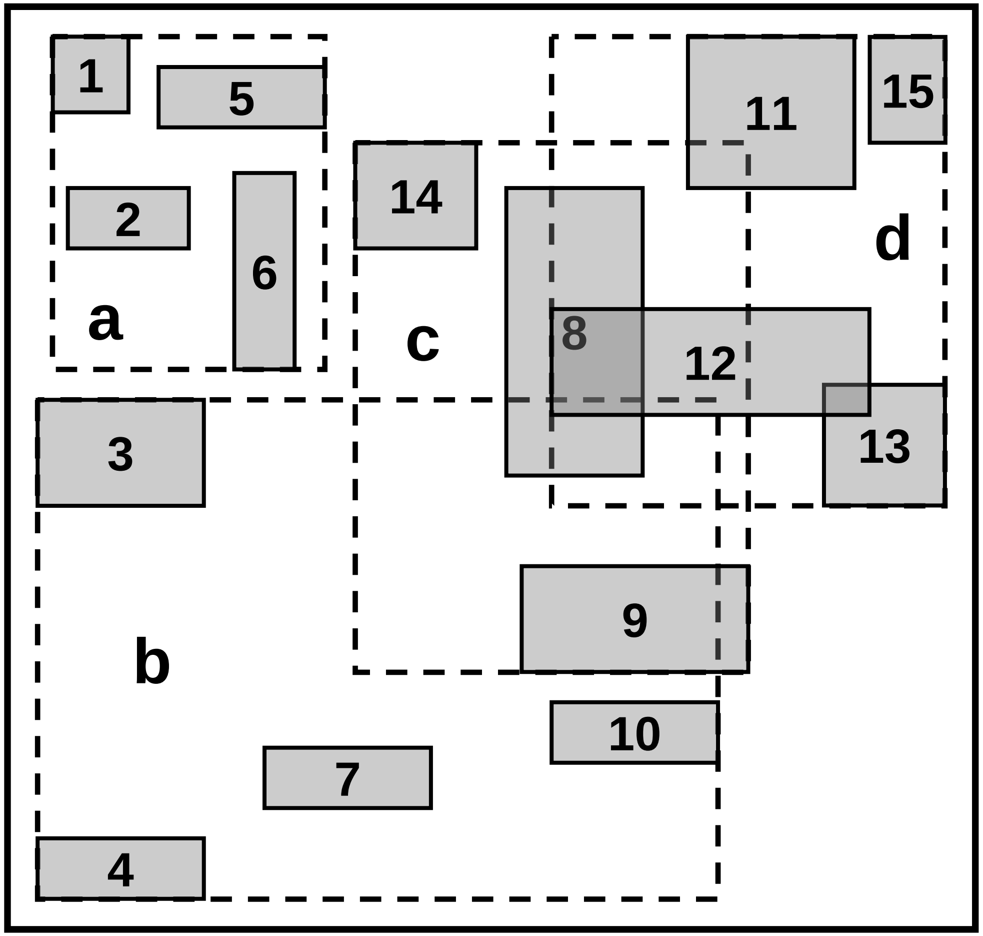
- If mbbs do not overlap on *q*, the complexity is $O(\log_m N)$.
- If mbbs overlap on *q*, it may not be logarithmic, in the worst case when all mbbs overlap on *q*, it is $O(N)$.

Insertion – choose a leaf node

- Traverse the R-tree top-down, starting from the root, at each level
  - If there is a node whose directory rectangle contains the mbb to be inserted, then search the subtree
  - Else choose a node such that the enlargement of its directory rectangle is minimal, then search the subtree
  - If more than one node satisfy this, choose the one with smallest area,
- Repeat until a leaf node is reached

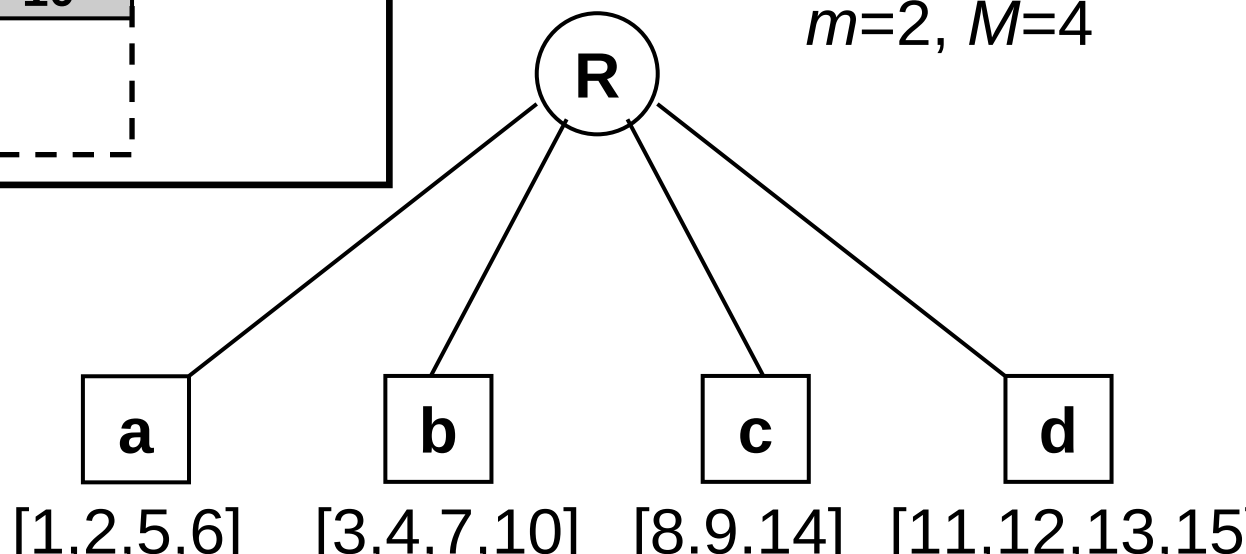
Insertion – insert into the leaf node

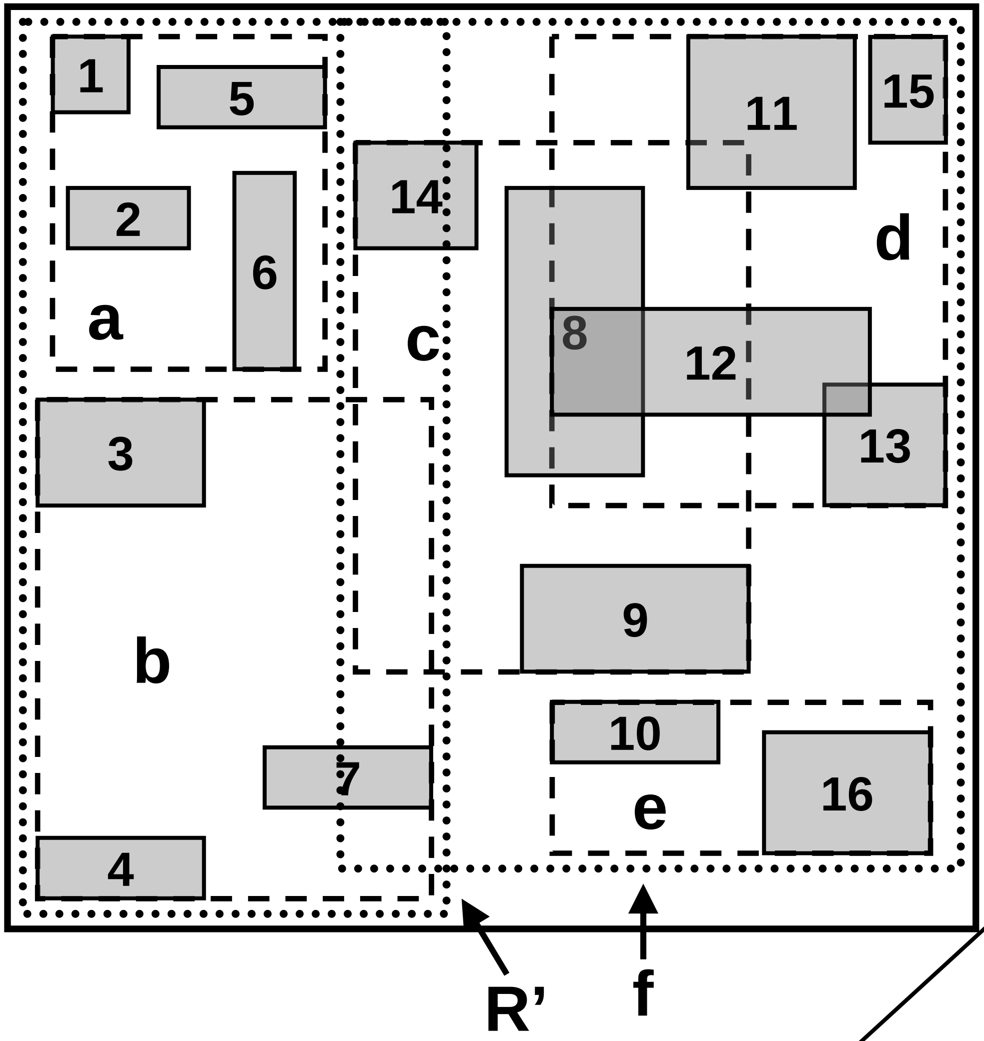
- If the leaf node is not full, an entry [mbb, oid] is inserted
- Else // the leaf node is full
  - Split the leaf node
  - Update the directory rectangles of the ancestor nodes if necessary



Insert object 15

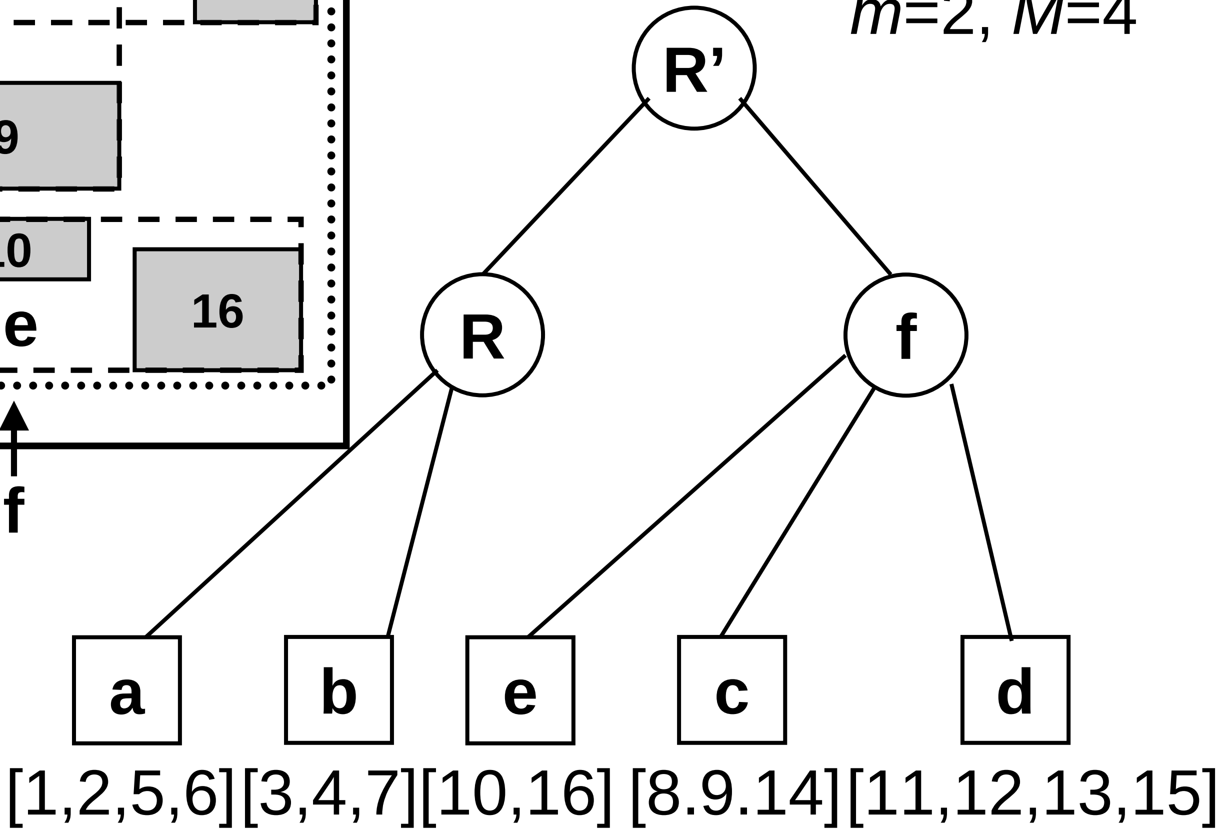
m=2, M=4





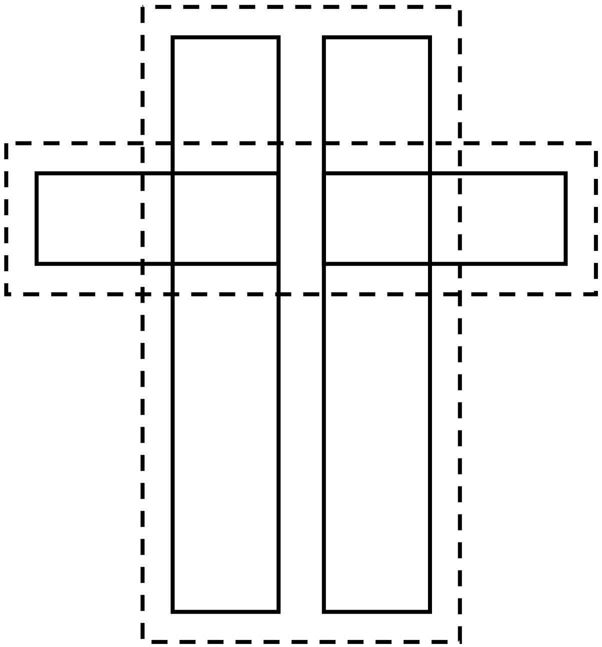
Insert object 16

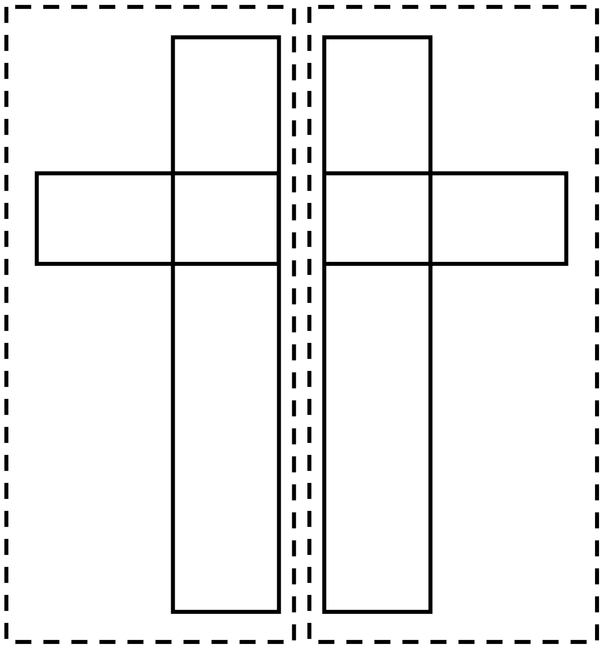
m=2, M=4



Split - goal

- The leaf node has M entries, and one new entry to be inserted, how to partition the M+1 mbbs into two nodes, such that
  - 1. The total area of the two nodes is minimized
  - 2. The overlapping of the two nodes is minimized
- Sometimes the two goals are conflicting
  - Using 1 as the primary goal





Split - solution

- Optimal solution: check every possible partition, complexity $O(2^{M+1})$
- A quadratic algorithm:
  - Pick two "seed" entries e_1 and e_2 far from each other, that is to maximize $\text{area}(\text{mbb}(e_1,e_2)) - \text{area}(e_1) - \text{area}(e_2)$ here $\text{mbb}(e_1,e_2)$ is the mbb containing both e_1 and e_2, complexity $O((M+1)^2)$
  - Insert the remaining (M-1) entries into the two groups

Quadratic split cont.

- A greedy method
- At each time, find an entry *e* such that *e* expands a group with the minimum area, if tie
  - Choose the group of small area
  - Choose the group of fewer elements
- Repeat until no entry left or one group has (M-m+1) entries, all remaining entries go to another group
- If the parent is also full, split the parent too. The recursive adjustment happens bottom-up until the tree satisfies the properties required. This can be up to the root.