Theorem 3.1.1 *For all formulas $\phi, \psi \in L_n$, structures $M \in M_n$, and agents $i = 1, \dots, n$,*

(a) *if ϕ is an instance of a propositional tautology, then $M \models \phi$,*

(b) $M \models (Ki\phi \wedge Ki(\phi \Rightarrow \psi)) \Rightarrow Ki\,\psi$,

(c) *if $M \models \phi$ and $M \models \phi \Rightarrow \psi$ then $M \models \psi$,*

(d) *if $M \models \phi$ then $M \models Ki\phi$.*

Axiom system K_n:

A1. *All tautologies of propositional calculus*

A2. $(Ki\phi \wedge Ki(\phi \Rightarrow \psi)) \Rightarrow Ki\,\psi$, $i = 1, \dots, n$ *(Distribution Axiom)*

R1. *From ϕ and $\phi \Rightarrow \psi$ infer ψ* *(modus ponens)*

R2. *From ϕ infer $Ki\phi$, $i = 1, \dots, n$* *(Knowledge Generalization)*

Proposition *For all formulas $\phi, \psi \in L_n$ where $\phi \Rightarrow \psi$ is a propositional tautology, if $K_n \vdash \phi$ then $K_n \vdash \psi$.*

Lemma *For all formulas $\phi, \psi \in L_n$, structures $M \in M_n$, and agents $i = 1, \dots, n$,*

A *if* $M \models \phi \Rightarrow \psi$ *then* $M \models Ki\phi \Rightarrow Ki\,\psi$

B *if* $K_n \vdash \phi \Rightarrow \psi$ *then* $K_n \vdash Ki\phi \Rightarrow Ki\,\psi$

***Prove B*:**

$\phi \Rightarrow \psi$

$Ki\,(\phi \Rightarrow \psi)$ **R2**

$Ki\,(\phi \Rightarrow \psi) \Rightarrow (Ki\phi \Rightarrow Ki\,\psi)$

A2 και Proposition

$Ki\phi \Rightarrow Ki\,\psi$ **R1**

Σχετικό εκπαιδευτικό υλικό

Huth – Ryan Logic in Computer Science 2nd Ed.

Ενότητες: 1.1 , 1.3 , 1.4 , 1.5.1 , 1.5.2

Fagin – Halpern – Moses – Vardi Reasoning About Knowledge

Ενότητα 2.1. The Possible-Worlds Model

Ενότητα 2.4. The Propertie of Knowledge , μέχρι και το Theorem 2.4.1.

Προτεινόμενες ασκήσεις